MAV Specialist Mathematics Examination 1 Answers & Solutions

Part I (Multiple-choice) Answers

1. **B**	2. **D**	3. **A**	4. **E**	5. **B**
6. **E**	7. **B**	8. **C**	9. **D**	10. **D**
11. **A**	12. **B**	13. **C**	14. **C**	15. **C**
16. **C**	17. **E**	18. **C**	19. **D**	20. **B**
21. **D**	22. **E**	23. **D**	24. **E**	25. **C**
26. **A**	27. **A**	28. **B**	29. **C**	30. **C**

Question 1

$$\frac{3-i}{3+i} \times \frac{3-i}{3-i}$$

$$= \frac{9-6i+i^2}{10}$$

$$= \frac{8-6i}{10}$$

$$= \frac{4-3i}{5}$$ **[B]**

Question 2

$\underset{\sim}{a} = 2\underset{\sim}{i} - \underset{\sim}{j}$ $\quad |\underset{\sim}{a}| = \sqrt{2^2 + (-1)^2} = \sqrt{5}$

$\underset{\sim}{b} = 3\underset{\sim}{i} + 2\underset{\sim}{j}$ $\quad |\underset{\sim}{b}| = \sqrt{13}$

$$\cos\theta = \frac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{a}||\underset{\sim}{b}|}$$

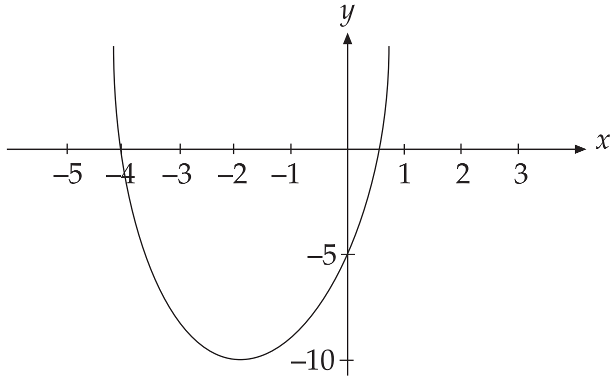
$$= \frac{2(3) - 1(2)}{\sqrt{5}\sqrt{13}}$$

$$= \frac{4}{\sqrt{65}}$$

$$\theta = \cos^{-1}\left(\frac{4}{\sqrt{65}}\right)$$

$\approx 60.255°$ **[D]**

Question 3



The graph of $f(x) = 2x^2 + 7x - 4$ is shown above.

Asymptotes for $\frac{1}{f(x)}$ will occur at the x-intercepts ($y = 0$). $x = -4$ and $x = \frac{1}{2}$ **[A]**

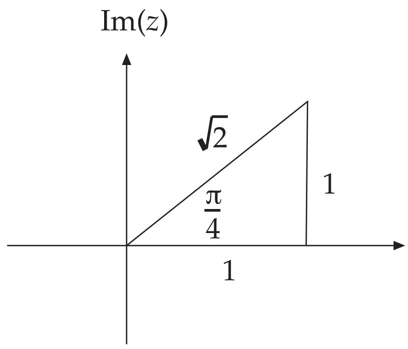
Question 4

$y = mx + c$ $\quad m = \tan\left(\frac{2\pi}{3}\right)$

$y = -\sqrt{3}x$ $\quad m = -\sqrt{3}$

$\text{Im } z + \sqrt{3}\text{ Re } z = 0$ **[E]**

Question 5



$$1 + i = \sqrt{2}\text{cis}\left(\frac{\pi}{4}\right)$$

$$\left(\sqrt{2}\text{cis}\frac{\pi}{4}\right)^5 = \left(\sqrt{2}\right)^5 \text{cis}\left(\frac{5\pi}{4}\right)$$

$$= 4\sqrt{2}\text{cis}\left(-\frac{3\pi}{4}\right)$$ **[B]**

Question 6

$\text{Sec}^{-1}x = 4.2$

$\text{Sec}\left(\text{Sec}^{-1}x\right) = \text{Sec}(4.2)$

$4.2 \notin \text{dom}\ [\text{Cos}(x)]$ but is acceptable for Sec x

$$x = \frac{1}{\cos(4.2)}$$
$$= -2.04$$

[E]

Question 7

$$\int \frac{2}{1-3x}dx,\ x > \frac{1}{3}$$

$$= \int \frac{-2}{3x-1}\,dx \qquad \text{Let } u = 3x - 1,\ \frac{du}{dx} = 3$$

$$= -\frac{2}{3}\int \frac{3}{3x-1}\frac{du}{dx}\,dx$$

$$= -\frac{2}{3}\log_e(3x-1),\ x > \frac{1}{3}$$

[B]

Question 8

$$\int\left(\frac{2x}{\sqrt{1-4x^2}} + \text{Sin}^{-1}(2x)\right)dx = x\text{Sin}^{-1}(2x)$$

$$\int\left(\text{Sin}^{-1}(2x)\right)dx = x\text{Sin}^{-1}(2x) - \int\left(\frac{2x}{\sqrt{1-4x^2}}\right)dx$$

[C]

Question 9

Required volume is the 'total' volume formed by rotating the 'outer' function, minus the hollowed section formed by rotating the 'inner' function.

$$\int_0^1 \pi\,(\sqrt{x})^2\,dx - \int_0^1 \pi\left(x^3\right)^2 dx$$

[D]

Question 10

Substituting $x = 2t$, into $y = 5\cos(2t)$

$y = 5\cos(x)$

[D]

Question 11

$$A = \frac{1}{2}\left((\sqrt{2}-1) + (\sqrt{3}-1)\right)1 + \frac{1}{2}\left((\sqrt{3}-1)+1\right)1$$

$$= \frac{1}{2}\left(\sqrt{2}+\sqrt{3}-2+\sqrt{3}\right)$$

$$\approx 1.439$$

[A]

Question 12

$$\left|2\underset{\sim}{i} - 3\underset{\sim}{j}\right| = \sqrt{4+9} = \sqrt{13}$$

Unit vector: $\frac{1}{\sqrt{13}}\left(2\underset{\sim}{i} - 3\underset{\sim}{j}\right)$

[B]

Question 13

$$\frac{y^2}{9} - \frac{(x-2)^2}{4} = 1$$

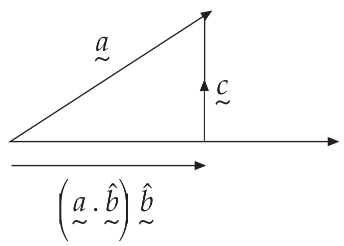
[C]

Question 14

$$\frac{a}{x-1} + \frac{b}{(x-1)^2}$$

[C]

Question 15



$$\underset{\sim}{c} = \underset{\sim}{a} - \left(\underset{\sim}{a}\,.\,\underset{\sim}{\hat{b}}\right)\underset{\sim}{\hat{b}}$$

[C]

Question 16

$$\frac{dy}{dx} = 2e^x - \cos x$$

$$\frac{d^2y}{dx^2} = 2e^x + \sin x$$

$$\text{LHS} = \frac{d^2y}{dx^2} + y$$
$$= 2e^x + \sin x + 2e^x - \sin x$$
$$= 4e^x$$
$$= \text{RHS}$$

Similarly for B, D and E.

Hence C is the only option not a solution to the equation.

[C]

Question 17

$y = \text{Cos}^{-1}\left(\frac{7}{x}\right)$ Let $u = \frac{7}{x} = 7x^{-1}$

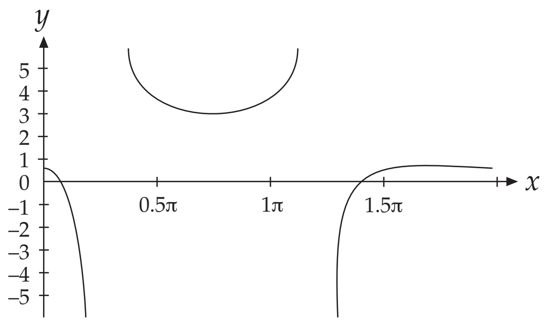
$y = \text{Cos}^{-1}u$ $\frac{du}{dx} = -7x^{-2}$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$
$$= -\frac{1}{\sqrt{1-u^2}}\frac{-7}{x^2}$$
$$= \frac{7}{x^2\sqrt{1-\frac{49}{x^2}}}$$
$$= \frac{7}{x^2\sqrt{\frac{x^2-49}{x^2}}}$$
$$= \frac{7}{x\sqrt{x^2-49}}$$

[E]

Question 18

$$y = \text{cosec}\left(x - \frac{\pi}{4}\right) + 2$$
$$= \frac{1}{\sin\left(x - \frac{\pi}{4}\right)} + 2$$



Turning points at $\left(\frac{3\pi}{4}, 3\right)$ and $\left(\frac{7\pi}{4}, 1\right)$ **[C]**

Question 19

$$\underset{\sim}{r}(t) = 4\sin(2t)\,\underset{\sim}{i} + 3t\,\underset{\sim}{j}$$

$$\dot{\underset{\sim}{r}}(t) = 8\cos(2t)\,\underset{\sim}{i} + 3\,\underset{\sim}{j}$$

$$\left|\dot{\underset{\sim}{r}}(t)\right| = \sqrt{64\cos^2(2t) + 9}$$

When $t = \frac{\pi}{6}$

$$\left|\dot{\underset{\sim}{r}}\left(\frac{\pi}{6}\right)\right| = \sqrt{64\cos^2\left(\frac{\pi}{3}\right) + 9}$$
$$= \sqrt{64\left(\frac{1}{2}\right)^2 + 9}$$
$$= 5$$

OR

$$\dot{\underset{\sim}{r}}\left(\frac{\pi}{6}\right) = 8\cos\left(\frac{\pi}{3}\right)\underset{\sim}{i} + 3\,\underset{\sim}{j}$$
$$= 4\,\underset{\sim}{i} + 3\,\underset{\sim}{j}$$

$$\left|\dot{\underset{\sim}{r}}\left(\frac{\pi}{6}\right)\right| = \sqrt{4^2 + 3^2}$$
$$= 5$$

[D]

Question 20

$$\frac{dy}{dx} = x\log_e x, \quad y_{n+1} = y_n + hf'(x_n), \quad h = 0.2$$

x	y
1	3
1.2	$3 + 0.2(1\log_e 1) = 3$
1.4	$3 + 0.2(1.2\log_e 1.2) = 3.0438$

[B]

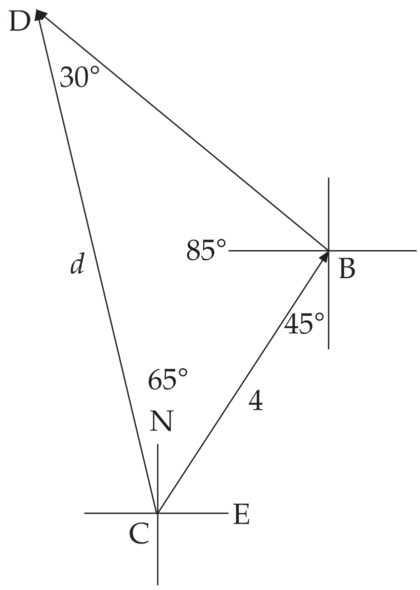
Question 21

$A = \pi r^2$ $\frac{dA}{dr} = 2\pi r$

$$\frac{dA}{dt} = \frac{dA}{dr}\frac{dr}{dt}$$
$$= 2\pi r(2)$$
$$= 4\pi r$$

When $r = 8$, $\frac{dA}{dt} = 32\pi$ **[D]**

Question 22



By the sine rule $\dfrac{d}{\sin 85°} = \dfrac{4}{\sin 30°}$ **[E]**

Question 23

$1 + x^2 = \dfrac{3}{y}$

$x^2 = \dfrac{3}{y} - 1$

$V = \pi\int_1^3 \left(\dfrac{3}{y} - 1\right) dy$ **[D]**

Question 24

The magnitude of the area beneath the curve gives the distance travelled.
Trapezium above t-axis:

$A = \dfrac{1}{2}(5 + 10)30 = 225$

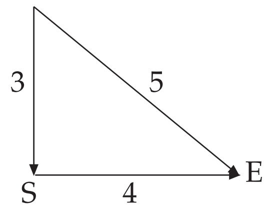
Triangle below t-axis: $A = \dfrac{1}{2} \times 5 \times 30 = 75$

Distance = 225 +75
= 300 metres **[E]**

Question 25

Given P and Q are the mid-points of the diagonals, it needs to be shown that P and Q coincide. $\overrightarrow{AP} = \overrightarrow{AQ}$ **[C]**

Question 26



$a = \dfrac{F}{m} = \dfrac{5}{2}$

$a = 2.5 \text{ m/s}^2$ **[A]**

Question 27

$mg \sin\theta - F_R = ma$

$Fr = mg \sin\theta - ma$

$Fr = 4(9.8) \sin 30° - 4(2)$

$= 11.6$ **[A]**

Question 28

Since lift is accelerating downward, the resultant force is downward, hence

$mg - N = ma$

$N = 64(9.8) - 64(1.5)$

$= 531.2$ newtons **[B]**

Question 29

$|\underset{\sim}{v}| = \sqrt{3^2 + 4^2}$

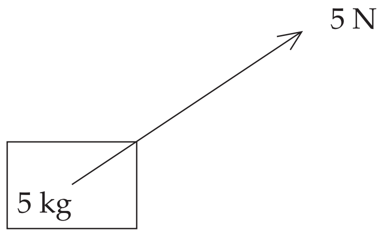
$= 5$

$|\underset{\sim}{p}| = m|\underset{\sim}{v}|$

$= 3 \times 5$

$= 15$ kg m/s **[C]**

Question 30



Since the box is moving with constant speed,

$F = 5\cos 30°$

≈ 4.3 Newton **[C]**

Part II
Short-answer Solutions

Question 1

$$\int \frac{\sqrt{x}}{x-4}dx$$

Let $u = \sqrt{x}$, $\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}} = \frac{1}{2u}$

$u^2 = x$

$$= \int 2u \times \frac{u}{u^2-4}\frac{du}{dx}dx$$ **M1**

$$= \int \frac{2u^2}{u^2-4}du$$

$\frac{2u^2}{u^2-4} = \frac{2(u^2-4)+8}{u^2-4}$ (or perform long division)

$= 2 + \frac{8}{u^2-4}$

$$= \int 2 + \frac{8}{u^2-4}du$$ **M1**

$$= \int 2 + \frac{8}{(u+2)(u-2)}du$$

$\frac{8}{(u+2)(u-2)} \equiv \frac{a}{(u+2)} + \frac{b}{(u-2)}$

$8 \equiv a(u-2) + b(u+2)$

$u = 2 \Rightarrow b = 2$

$u = -2 \Rightarrow a = -2$

$$= \int 2 + \frac{2}{u-2} - \frac{2}{u+2}du$$ **M1**

$$= 2u + 2\log_e\left(\frac{u-2}{u+2}\right) + c$$

$$= 2\sqrt{x} + 2\log_e\left(\frac{\sqrt{x}-2}{\sqrt{x}+2}\right) + c$$ **A1**

Question 2

a. $|z+1|+|z-1|=6$

$\sqrt{(x+1)^2+y^2}+\sqrt{(x-1)^2+y^2}=6$ **M1**

$\sqrt{(x+1)^2+y^2}=6-\sqrt{(x-1)^2+y^2}$

$x^2+2x+1+y^2=36-12\sqrt{(x-1)^2+y^2}+x^2-2x+1+y^2$

$12\sqrt{(x-1)^2+y^2}=36-4x$ **M1**

$3\sqrt{(x-1)^2+y^2}=9-x$

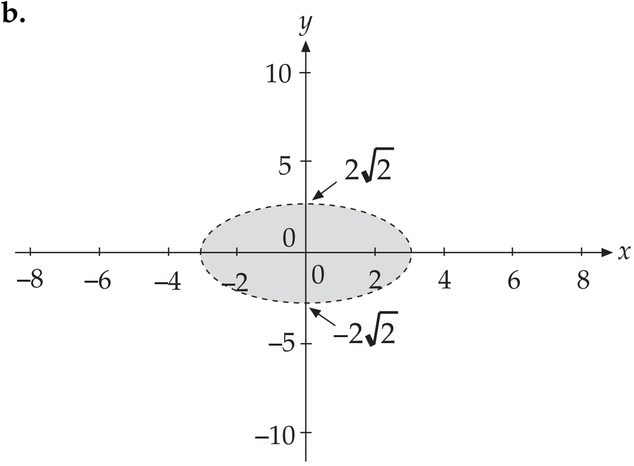
$9(x^2-2x+1+y^2)=81-18x+x^2$

$9x^2-18x+9+9y^2=81-18x+x^2$

$8x^2+9y^2=72$ **A1**

$\frac{x^2}{9}+\frac{y^2}{8}=1$

b.



Correct ellipse, with x-intercepts, $x=\pm 3$,

y-intercepts, $y=\pm 2\sqrt{2}$ **A1**

Shading inside ellipse. **A1**

Question 3

Method 1 (considering total motion)

$u=4\text{ m/s},\ g=-9.8\text{ m/s}^2,\ s=-15\text{m}$

$v^2-u^2=2as$ **M1**

$v^2=2as+u^2$

$v^2=2(-9.8)(-15)+16$

$v^2=310$

$v=17.6\text{ m/s}$ **A1**

Method 2 (considering upward then downward motion)

UP: $u=4\text{ m/s},\ g=-9.8\text{ m/s}^2,\ v=0$

$v^2-u^2=2as$

$s=\frac{v^2-u^2}{2a}=\frac{0-16}{-19.6}=0.82$ **A1**

DOWN: $u=0\text{ m/s},\ g=9.8\text{ m/s}^2,$

$s=15+0.82=15.82$

$v^2-u^2=2as$

$v^2=2(9.8)(15.82)$

$v=17.6\text{ m/s}$ **A1**

Question 4

a. $\int \tan x\,dx$

$=\int \frac{\sin x}{\cos x}\,dx$

Let $u=\cos x$, $\frac{du}{dx}=-\sin x$

$=\int -\frac{1}{u}\frac{du}{dx}\,dx$ **M1**

$=-\log_e|\cos x|+c$

b. $\int \tan^3(x)dx$

$= \int (\tan^2 x \times \tan x)dx$

$= \int (\sec^2 x - 1)\tan x dx$ **M1**

$= \int (\sec^2 x \tan x - \tan x)dx$

$\int \sec^2 x \tan x dx \quad$ Let $u = \tan x$, $\frac{du}{dx} = \sec^2 x$

$= \int u \frac{du}{dx} dx$

$= \frac{1}{2}\tan^2 x + c_1$

$= \frac{1}{2}\tan^2 x + \log_e|\cos x| + c$ **A1**

$\therefore \tan^3 x dx = \frac{1}{2}\tan^2 x + \log_e|\cos x| + c$

Question 5

$\frac{dT}{dt} = -k(T - T_0)$

$\frac{dt}{dT} = -\frac{1}{k(T-10)}$

$t = -\frac{1}{k}\int \frac{1}{T-10} dT$

$-kt = \log_e(T-10) + c$

$t = 0, T = 25 \Rightarrow c = -\log_e 15$

$-kt = \log_e\left(\frac{T-10}{15}\right)$

$e^{-kt} = \frac{T-10}{15}$

$T = 15e^{-kt} + 10$ **A1**

When $t = 5$ minutes, $T = 18$

$18 = 15e^{-5k} + 10$

$\frac{8}{15} = e^{-5k}$

$k = -\frac{1}{5}\log_e\left(\frac{8}{15}\right)$

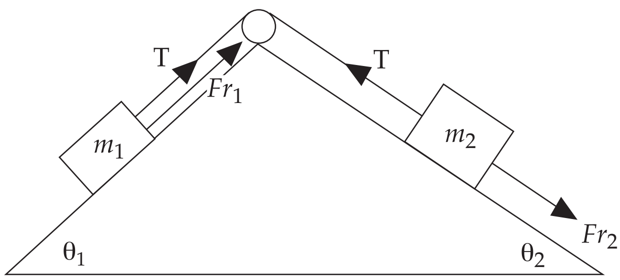
≈ 0.1257 **A1**

After a further 3 minutes, $t = 8$

$T = 15e^{-0.1257 \times 8} + 10$

$\approx 15.5°$ **A1**

Question 6



Consider mass 1:

$T = m_1 g \sin\theta_1 - Fr_1$

$Fr_1 = \mu_1 N_1$

$N_1 = m_1 g \cos\theta_1$

$\therefore Fr_1 = \mu_1 m_1 g \cos\theta_1$

Hence $T = m_1 g \sin\theta_1 - \mu_1 m_1 g \cos\theta_1$ **M1**

Consider mass 2:

$T = m_2 g \sin\theta_2 + Fr_2$

$Fr_2 = \mu_2 N_2$

$N_2 = m_2 g \cos\theta_2$

$\therefore Fr_2 = \mu_2 m_2 g \cos\theta_2$

$T = m_2 g \sin\theta_2 + \mu_2 m_2 g \cos\theta_2$ **M1**

Equating

$m_1 g \sin\theta_1 - \mu_1 m_1 g \cos\theta_1 = m_2 g \sin\theta_2 + \mu_2 m_2 g \cos\theta_2$

$m_1(\sin\theta_1 - \mu_1\cos\theta_1) = m_2(\sin\theta_2 + \mu_2\cos\theta_2)$

$\frac{m_1}{m_2} = \frac{\sin\theta_2 + \mu_2\cos\theta_2}{\sin\theta_1 - \mu_1\cos\theta_1}$ **A1**