Part 1: Multiple-choice questions

Question 1

If $u = 3 - i$ and $v = 3 + i$ then $\frac{u}{v}$ is equal to

A. $\frac{5}{4 - 3i}$

B. $\frac{4 - 3i}{5}$

C. $\frac{4 - 3i}{4}$

D. $-i$

E. -1

Question 2

The angle between the vectors $\underset{\sim}{a} = 2\underset{\sim}{i} - \underset{\sim}{j}$ and $\underset{\sim}{b} = 3\underset{\sim}{i} + 2\underset{\sim}{j}$ is closest to

A. 1°

B. 7°

C. 30°

D. 60°

E. 86°

Question 3

If $f(x) = 2x^2 + 7x - 4$, then the graph of $\frac{1}{f(x)}$ has

A. asymptotes at $x = -4$ and $x = \frac{1}{2}$

B. asymptotes at $x = 4$ and $x = -\frac{1}{2}$

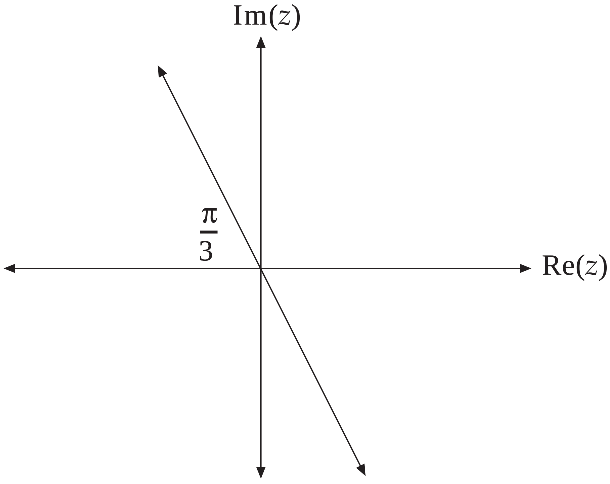
C. x-intercepts at $x = -4$ and $x = \frac{1}{2}$

D. a maximum at the point $\left(-\frac{7}{4}, \frac{81}{8}\right)$

E. a minimum at the point $\left(-\frac{7}{4}, -\frac{8}{81}\right)$

Question 4

The Argand diagram shown is the graph of



A. $\left\{z\colon \text{Arg } z = \frac{\pi}{3}\right\}$

B. $\left\{z\colon \text{Arg } z = -\frac{\pi}{3}\right\}$

C. $\left\{z\colon \text{Arg } z = \frac{2\pi}{3}\right\}$

D. $\left\{z\colon \text{Im } z - \sqrt{3}\,\text{Re } z = 0\right\}$

E. $\left\{z\colon \text{Im } z + \sqrt{3}\,\text{Re } z = 0\right\}$

Question 5

$(1+i)^5$ can be expressed in polar form as

A. $2\sqrt{2}\text{cis}\left(\frac{5\pi}{4}\right)$

B. $4\sqrt{2}\text{cis}\left(-\frac{3\pi}{4}\right)$

C. $32\text{cis}\left(\frac{\pi}{4}\right)$

D. $32\text{cis}\left(-\frac{3\pi}{4}\right)$

E. $4\sqrt{2}\text{cis}\left(\frac{\pi}{4}\right)$

Question 6

If $\text{Sec}^{-1}x = 4.2$, then the value of x is

A. undefined

B. –0.49

C. –0.87

D. –1.15

E. –2.04

Question 7

An antiderivative of $\frac{2}{1-3x}$, $x > \frac{1}{3}$ is

A. $-\frac{2}{3}\log_e(1-3x),\ x < \frac{1}{3}$

B. $-\frac{2}{3}\log_e(1-3x),\ x > \frac{1}{3}$

C. $2\log_e(1-3x),\ x > \frac{1}{3}$

D. $\frac{6}{(1-3x)^2},\ x > \frac{1}{3}$

E. $-\frac{3}{2}\log_e(1-3x),\ x > \frac{1}{3}$

Question 8

The derivative of $x\mathrm{Sin}^{-1}(2x)$ with respect to x is $\frac{2x}{\sqrt{1-4x^2}} + \mathrm{Sin}^{-1}(2x)$. It follows that an antiderivative of $\mathrm{Sin}^{-1}(2x)$ is

A. $\int x\,\mathrm{Sin}^{-1}(2x)dx - \int \frac{2x}{\sqrt{1-4x^2}}\,dx$

B. $\int x\,\mathrm{Sin}^{-1}(2x)dx + \int \frac{2x}{\sqrt{1-4x^2}}\,dx$

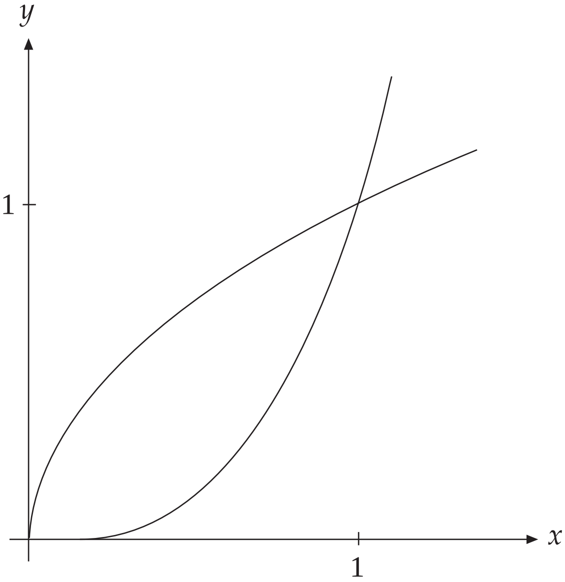
C. $x\,\mathrm{Sin}^{-1}(2x) - \int \frac{2x}{\sqrt{1-4x^2}}\,dx$

D. $x\,\mathrm{Sin}^{-1}(2x) + \int \frac{2x}{\sqrt{1-4x^2}}\,dx$

E. $x\,\mathrm{Sin}^{-1}(2x) - \frac{2x}{\sqrt{1-4x^2}}$

Question 9

The region enclosed by the curves with equations $f(x) = \sqrt{x}$ and $g(x) = x^3$ is rotated about the x-axis. The **exact value** of the solid obtained is given by



A. $\int_0^1 \pi\left(\sqrt{x} - x^3\right)^2 dx$

B. $\int_0^1 \pi\, x\, dx$

C. $\int_0^1 \pi\, x^6\, dx$

D. $\int_0^1 \pi\, x\, dx - \int_0^1 \pi\, x^6 dx$

E. $\int_0^1 \pi\sqrt{x}\, dx - \int_0^1 \pi\, x^3 dx$

Question 10

The position vector, $\underset{\sim}{r}(t)$, in metres, of a particle at time t is given by $\underset{\sim}{r}(t) = 2t\,\underset{\sim}{i} + 5\cos(2t)\,\underset{\sim}{j}$. The Cartesian equation of the path followed by the particle is

A. $x = 2t$

B. $y = 5\cos(2t)$

C. $y = 5x$

D. $y = 5\cos(x)$

E. $y = 5\cos\left(\frac{x}{2}\right)$

Question 11

The trapezoidal rule with two equal intervals is used to approximate the area enclosed by the curve, $f(x) = \sqrt{x} - 1$ and the x-axis between the lines $x = 2$ and $x = 4$. The value of this estimate, correct to three decimal places is

A. 1.439

B. 1.448

C. 1.452

D. 2.878

E. 3.439

Question 12

A unit vector parallel to $2\underset{\sim}{i} - 3\underset{\sim}{j}$ is

A. $\sqrt{13}\left(2\underset{\sim}{i} - 3\underset{\sim}{j}\right)$

B. $\frac{1}{\sqrt{13}}\left(2\underset{\sim}{i} - 3\underset{\sim}{j}\right)$

C. $-\frac{1}{\sqrt{13}}\left(2\underset{\sim}{i} + 3\underset{\sim}{j}\right)$

D. $\frac{1}{\sqrt{5}}\left(2\underset{\sim}{i} - 3\underset{\sim}{j}\right)$

E. $\frac{1}{\sqrt{13}}\left(2\underset{\sim}{i} + 3\underset{\sim}{j}\right)$

Question 13

The equation of the hyperbola shown below is

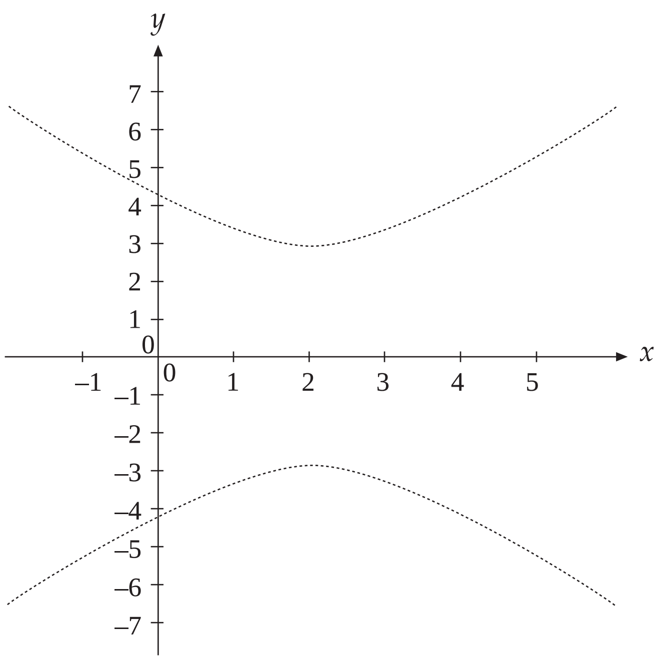
A. $\frac{y^2}{3} - \frac{(x-2)^2}{4} = 1$

B. $\frac{y^2}{9} - \frac{(x+2)^2}{4} = 1$

C. $\frac{y^2}{9} - \frac{(x-2)^2}{4} = 1$

D. $\frac{x^2}{4} - \frac{y^2}{9} = 1$

E. $\frac{x^2}{4} - \frac{y^2}{3} = 1$



Question 14

$\dfrac{5x}{(x-1)^2}$ expressed in partial fractions has the form

A. $\dfrac{a}{x-1}+\dfrac{b}{x-1}$

B. $\dfrac{a}{(x-1)^2}+\dfrac{b}{(x-1)^2}$

C. $\dfrac{a}{x-1}+\dfrac{b}{(x-1)^2}$

D. $\dfrac{a}{x-1}+\dfrac{b}{x+1}$

E. $\dfrac{a}{x-1}+\dfrac{bx+c}{(x-1)^2}$, $b \neq 0$

Question 15

The vector projection of $\underset{\sim}{a}$ perpendicular to $\underset{\sim}{b}$ is given by

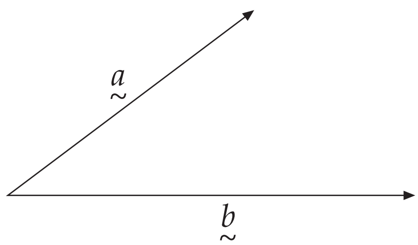
A. $\left(\underset{\sim}{a}\,.\,\hat{\underset{\sim}{b}}\right)\hat{\underset{\sim}{b}}$

B. $\underset{\sim}{a}-\left(\underset{\sim}{a}\,.\,\hat{\underset{\sim}{b}}\right)\underset{\sim}{b}$

C. $\underset{\sim}{a}-\left(\underset{\sim}{a}\,.\,\hat{\underset{\sim}{b}}\right)\hat{\underset{\sim}{b}}$

D. $\left(\underset{\sim}{a}\,.\,\hat{\underset{\sim}{b}}\right)\hat{\underset{\sim}{b}}-\underset{\sim}{a}$

E. $\left(\underset{\sim}{a}\,.\,\underset{\sim}{b}\right)\hat{\underset{\sim}{b}}-\underset{\sim}{a}$



Question 16

Which of the following is **not** a solution of the differential equation $\dfrac{d^2y}{dx^2}+y=4e^x$

A. $y=2e^x-\sin x$

B. $y=2e^x-\cos x$

C. $y=e^{2x}+\cos x$

D. $y=2e^x+\sin x$

E. $y=2e^x+\cos x$

Question 17

If $y = \text{Cos}^{-1}\left(\frac{7}{x}\right)$, $x > 7$, then $\frac{dy}{dx} =$

A. $-\frac{1}{\sqrt{49 - x^2}}$

B. $-\frac{7x}{\sqrt{x^2 - 49}}$

C. $-\frac{x}{\sqrt{x^2 - 49}}$

D. $\frac{7x}{\sqrt{x^2 - 49}}$

E. $\frac{7}{x\sqrt{x^2 - 49}}$

Question 18

The graph of $y = \text{cosec}\left(x - \frac{\pi}{4}\right) + 2$, for $0 \le x \le 2\pi$, has turning points at

A. $\left(\frac{\pi}{4}, 3\right)$ and $\left(\frac{\pi}{4}, 3\right)$

B. $\left(\frac{3\pi}{4}, 1\right)$ and $\left(\frac{3\pi}{4}, -1\right)$

C. $\left(\frac{3\pi}{4}, 3\right)$ and $\left(\frac{7\pi}{4}, 1\right)$

D. $\left(\frac{\pi}{4}, 1\right)$ and $\left(\frac{5\pi}{4}, -1\right)$

E. $\left(\frac{\pi}{4}, 3\right)$ and $\left(\frac{5\pi}{4}, 1\right)$

Question 19

An object is moving such that its position vector $\underset{\sim}{r}(t)$ is given by $\underset{\sim}{r}(t) = 4\sin(2t)\,\underset{\sim}{i} + 3t\,\underset{\sim}{j}$. The magnitude of the velocity vector in m / s at $t = \frac{\pi}{6}$ seconds is

A. $\sqrt{41}$

B. $\sqrt{57}$

C. $\sqrt{12 + \frac{\pi^2}{4}}$

D. 5

E. 7

Question 20

Euler's method, with a step size of 0.2 is used to solve the differential equation $\frac{dy}{dx} = x\log_e x$ with $y = 3$ at $x = 1$. The value obtained for y at $x = 1.4$, correct to four decimal places, is

A. 3.0365

B 3.0438

C. 3.0665

D. 3.0875

E. 3.1380

Question 21

The radius of a circular oil slick is increasing at a rate of 2 m / min. When the radius is 8 m, the rate of increase in m^2 / min of the area of the oil slick is

A. 4π

B. 8π

C. 16π

D. 32π

E. 128π

Question 22

Luiji leaves his campsite, C, and walks in a direction of NE for 4 km to B. He then heads N50°W until he reaches his destination at D. Luiji can tell from his compass that his destination of D is at a bearing of N20°W relative to his previous campsite at C. The distance, d km from C to D is given by

A. $\dfrac{d}{\sin 45°} = \dfrac{4}{\sin 50°}$

B. $\dfrac{d}{\sin 95°} = \dfrac{4}{\sin 20°}$

C. $\dfrac{d}{\sin 95°} = \dfrac{4}{\sin 40°}$

D. $\dfrac{d}{\sin 85°} = \dfrac{4}{\sin 50°}$

E. $\dfrac{d}{\sin 85°} = \dfrac{4}{\sin 30°}$

Question 23

The region enclosed by the curve $y = \dfrac{3}{1+x^2}$, the straight line $y = 1$ and the y-axis, is rotated about the y-axis to form a solid of revolution. The volume of this solid, in cubic units is given by

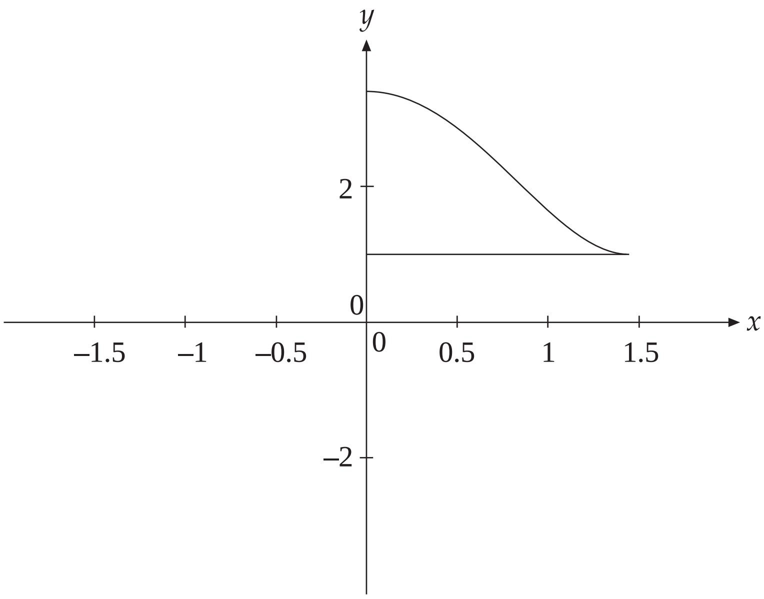
A. $\pi\displaystyle\int_0^{\sqrt{2}} \left(\frac{9}{\left(1+x^2\right)^2}\right) dx$

B. $\pi\displaystyle\int_0^{\sqrt{2}} \left(\frac{9}{\left(1+x^2\right)^2} - 1\right) dx$

C. $\pi\displaystyle\int_0^{\sqrt{2}} \left(\frac{3}{y} - 1\right) dy$

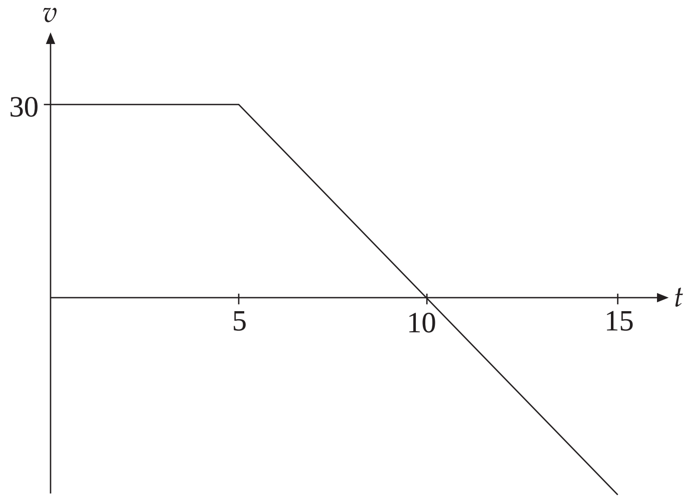
D. $\pi\displaystyle\int_1^{3} \left(\frac{3}{y} - 1\right) dy$

E. $\pi\displaystyle\int_1^{3} \left(\frac{3}{\left(1+y^2\right)^2}\right)^6 dy$



Question 24

The velocity-time graph for a particle moving in a straight line is shown below. The total distance, in metres, travelled by the particle in 15 seconds is



A. 275

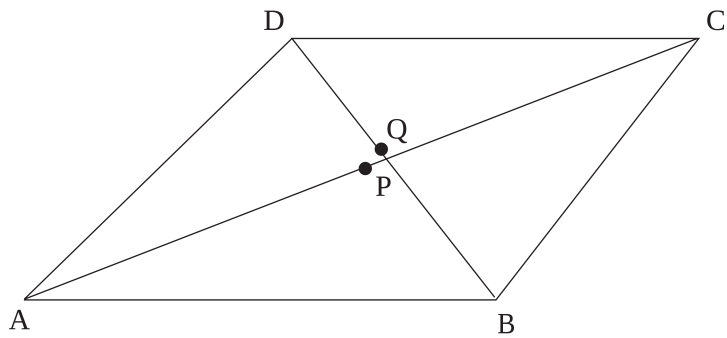
B. 175

C. 30

D. 50

E. 300

Question 25



If P is the mid-point of $\overrightarrow{AC}$ and Q is the mid-point of $\overrightarrow{BD}$, in order to prove that the diagonals of a parallelogram bisect each other, it needs to be shown that

A. $\overrightarrow{AP} . \overrightarrow{AQ} = 0$

B. $\overrightarrow{AP} . \overrightarrow{PQ} = 0$

C. $\overrightarrow{AP} = \overrightarrow{AQ}$

D. $|\overrightarrow{BD}| = \frac{1}{2}|\overrightarrow{AC}|$

E. $\overrightarrow{AP} = \overrightarrow{BQ}$

Question 26

A body of mass 2 kg is acted on by two forces, one of magnitude 3 Newtons acting due South and the other of magnitude 4 Newtons acting due East. The magnitude, in m/s^2, of the resulting acceleration is

A. 2.5

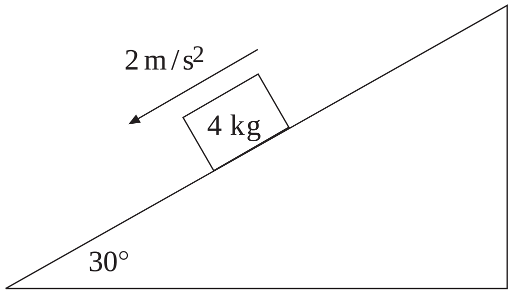
B. $2\sqrt{5}$

C. $2\sqrt{7}$

D. 14

E. 24

Question 27



A block of mass 4.0 kg slides down a rough slope inclined at 30° to the horizontal. The block is sliding with an acceleration of 2 m/s^2. The magnitude of the acceleration due to gravity is 9.8 m/s^2. The magnitude of the frictional force between the block and the rough surface is

A. 11.6

B. 19.6

C. 25.9

D. 27.6

E. 31.2

Question 28

A person of mass 64 kg stands in a lift accelerating downwards at a magnitude of 1.5 m/s^2. The magnitude of the acceleration due to gravity is 9.8 m/s^2. The reaction force, in Newtons, of the lift floor on the person is closest to

A. 96

B. 531

C. 544

D. 627

E. 723

Question 29

A particle of mass 3 kg is moving with a velocity of $3\underset{\sim}{i} + 4\underset{\sim}{j}$ m/s. The magnitude in kg m/s, of the momentum of the particle is

A. 9

B. 12

C. 15

D. 21

E. 75

Question 30

A box of mass 5 kg is pulled at a constant speed along a rough horizontal surface, by a force of 5 newtons acting at an angle of 30° to the horizontal. The acceleration due to gravity is 9.8 m/s^2. The magnitude of the frictional force is closest to

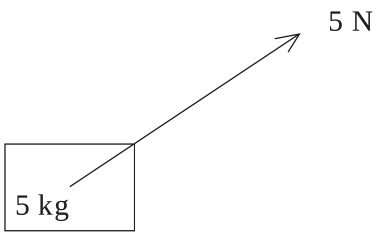
A. 0

B. 2.5

C. 4.3

D. 5

E. 9.3



PART II

Short-answer Questions

Question 1

Using the substitution $u = \sqrt{x}$, find $\int \frac{\sqrt{x}}{x-4} \, dx$

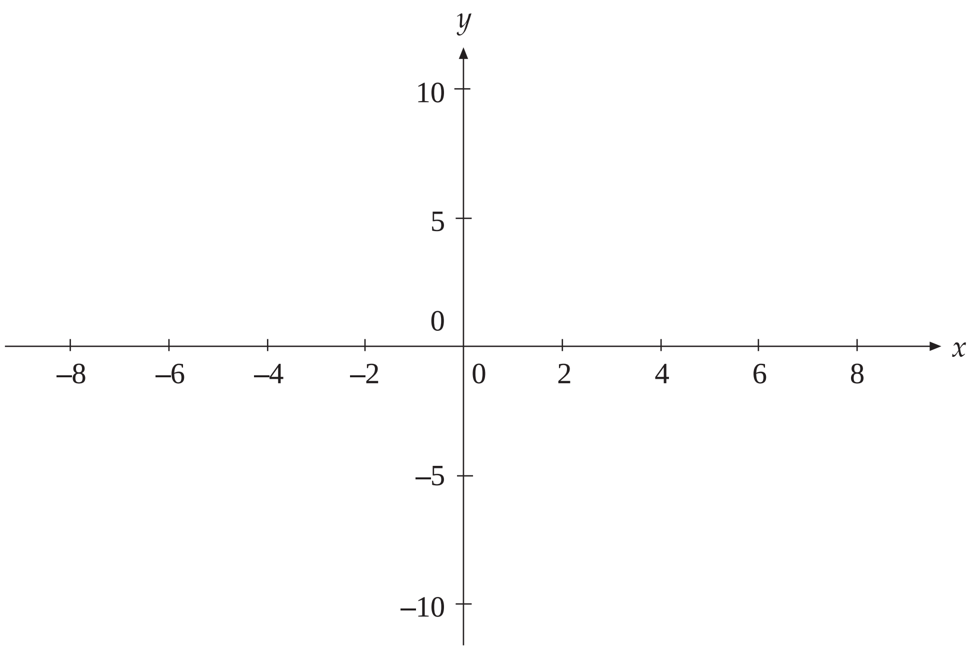
4 marks

Question 2

a. Show that the subset of the complex plane defined by $\{z: |z+1| + |z-1| = 6\}$ can be expressed in Cartesian form as $\frac{x^2}{9} + \frac{y^2}{8} = 1$

3 marks

b. Sketch $\{z: |z+1| + |z-1| < 6\}$, shading the required region.



2 marks

Question 3

A cricket ball is thrown upwards from the top of a 15 metre high building, with a speed of 4 m/s. Find, correct to one decimal place, the speed of the cricket ball when it strikes the ground.

2 marks

Question 4

a. Show that $\int (\tan x)dx = -\log_e|\cos x| + c$

1 mark

b. Hence find $\int \tan^3(x)dx$

2 marks

Question 5

The rate of cooling of a body is proportional to the excess of its temperature above that of its surroundings. This relationship can be expressed in the form:

$$\frac{dT}{dt} = -k(T - T_0),$$

where T_0 is the temperature of the surroundings, and T is the temperature of the body.

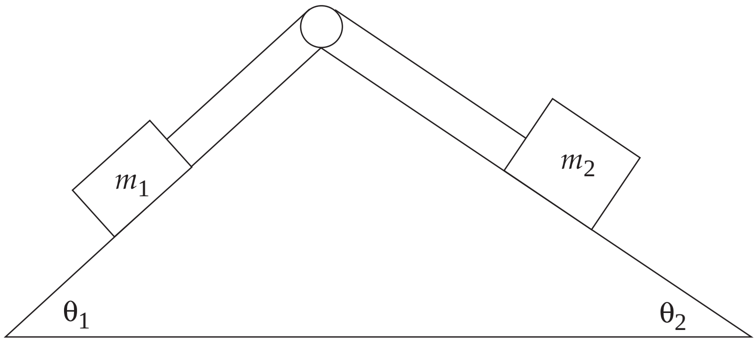
A body at a temperature of 25° is placed in a coolroom of temperature 10°. After 5 minutes the temperature of the body has cooled to 18°. Find the temperature of the body, correct to one decimal place, after a further 3 minutes have passed.

3 marks

Question 6

Two particles of mass m_1 and m_2 are connected by a light string that passes over a smooth pulley respectively. The coefficients of friction of the two surfaces are μ_1 and μ_2.

If the system is on the point of moving to the left, i.e. is about to slide down the plane, express $\frac{m_1}{m_2}$ in terms of $\theta_1, \theta_2, \mu_1$ and μ_2.



3 marks