2003 Specialist Mathematics GA 3: Written examination 2

GENERAL COMMENTS

The number of students who sat for the 2003 examination was 6231, 143 more than the number (6088) in 2002. As in 2002, students had to answer five questions worth a total of 60 marks, with each question worth from 8 to 14 marks.

Most of the data suggests that, on average, the level of student performance was similar to that on the 2002 paper. The mean and median scores, out of a possible 60, were 26.3 and 27 respectively, compared with 26.1 and 25 in 2002. On the other hand, only 1.3 per cent of students scored 90 per cent or more of the marks, compared with 2.7 per cent last year and 1.6 per cent in 2001. The number of students who scored full marks was 4, compared with 9 last year and just 2 in 2001. Tthere were sufficient 'available' marks for students to show what they could do, but it was slightly more challenging to get near full marks than in 2002. At the lower end, about 5 per cent of students scored 4 or less marks.

The average score for the five questions, expressed as a percentage of the marks available, was about 44 per cent, 51 per cent, 38 per cent, 50 per cent and 34 per cent respectively. A smaller percentage for the last question was to be expected, both because the final part of the question was quite challenging and some students find it difficult to sustain their performance throughout the examination. However, the averages are disappointing and unexpected for the first and third questions. Less than 5 per cent of students scored full marks on Questions 3cii and 5b; and only between 5 per cent and 10 per cent scored full marks on Questions 4eii and 5d. The latter two results were not unexpected, but the former were more surprising.

As is usually the case, in several Questions (2a, 2d, 3a, 3ci, 4ei and 5b) students were asked to show (prove; verify) given results. It needs to be consistently emphasised to students that, in such questions, all steps need to be shown in order to gain full credit. On the other hand, students need to be reminded that, even if they cannot establish a given result, they are entitled to use that result in the remainder of the question, if it is required. Thus, for example, the given results of 2a can (and should) be used throughout the remainder of Question 2, and the result of Question 4ei is almost certain to be needed in Question 4eii.

There was less scope for graphics calculators in 2003 than on the 2002 paper, but many students used the technology effectively in the most obvious cases: Questions 4b and 4d. In Question 4b, the graphs of a sec function and its inverse were required. Most students were able to do the first one but many were unable to successfully draw the inverse. It should be noted that the rule for the inverse is not needed to draw it (hence the reason for making the rule for the inverse part c of the question). First, students should know that the graph of an inverse is a reflection in the line $y = x$ of the original graph. Second, the technology can help: most graphics calculators have a feature, usually called DrawInv or similar, which will draw the inverse of a current function (as opposed to plotting the rule direct) – this feature was clearly not known to students. In Question 4d, an integral expression for the area A between the curves and its value to three decimal places was required, but the question was worth just 2 marks. It should be clear that a numerical integration using technology was called for. Of the approximately 62 per cent of students that correctly wrote the integral expression required, almost 75 per cent used calculator numerical integration successfully.

The intent of instructions at the beginning of the paper concerning exact answers and the use of calculus is generally understood by students. It is worth stressing however, that where an exact answer is requested, full marks cannot be obtained if the examiners see only a decimal approximation. Furthermore, appropriate working must be shown when an exact answer is required, and this is also true in questions worth more than 1 mark. In multi-mark questions, the examiners need to see the steps students use to reach a solution. Marks are allocated to methods and procedures, not just numerical or algebraic answers.

Although the phrase 'Use calculus to …' was not employed on the 2003 paper (instead specific terms like 'derivative' and 'differentiation' were used in some places), the intention of such instructions require students to show a formal derivative or antiderivative in the course of their working. Thus if Question 4eii had read 'Use calculus to find the value of A correct to three decimal places', it would not have sufficed to have used the numerical integration feature of a graphics calculator to obtain the answer. Intermediate steps involving antidifferentiation would have been essential.

SPECIFIC INFORMATION

Question 1

a

Marks	0	1	2	3	4	Average
%	17	10	14	11	48	**2.63**

Answer: 2985 (N)

A significant number of students did not use degree mode on their calculator when evaluating trigonometric expressions. A small proportion were unable to make significant progress when their force components on the sled were incorrect (usually the normal reaction pointing up); or when they omitted the weight force component down the plane; or where they introduced spurious forces on the tractor, assuming they used a force diagram at all. Common errors included: cos and sin interchanged in the equations of motion; the omission of g from the weight components; sign problems in the parallel component leading to an answer of 2385 N; and other algebraic or numeric substitution mistakes.

b

Marks	0	1	2	Average
%	72	3	25	**0.52**

Answer: $1200g\sin(8°) - F$

Students who wrote $T = 1200g\sin(8°) + F$ were ineligible for any subsequent marks. While it was anticipated that some would not pick up the change in direction of friction (the sled and tractor are parked with the tractor's brakes applied, so the tendency is for the system to slide down the slope), it was quite unexpected that most would make this error.

ci

Marks	0	1	Average
%	77	23	**0.23**

Answer: 589 (N)

Most students who obtained 2 marks in part b also obtained this mark.

cii

Marks	0	1	Average
%	88	12	**0.12**

Answer: 0 (N)

Of the students who obtained 2 marks in part b, about half obtained an answer of –110 N (or 110 N) rather than 0 N, although there were some well-reasoned solutions from a few students. Most students assumed $F = \mu N$, which is only true if the friction is limiting. In this case, $F < \mu N$; which was the point of the question.

Question 2

a

Marks	0	1	2	3	Average
%	22	2	2	74	**2.28**

Answer: As given.

Generally quite well done, although a very small proportion of students were not familiar with the formulas for trigonometric sums or differences, despite them being provided on the formula sheet. Some students managed to get the required results without the correct formulas, or despite simplification errors.

b

Marks	0	1	2	3	Average
%	29	19	27	25	**1.47**

Answer: $\frac{-\pi}{2}$

Not as well done as might be expected. There was much careless treatment of signs and brackets. Many students made sign errors during or before manipulation of $v - u$ typically, $\left(\frac{\sqrt{6}-\sqrt{2}}{2}\right)i$ or dropped i altogether. Some students who correctly found $v - u$ continued: 'Arg$(v - u) = \text{Tan}^{-1}\left(\frac{\cdots}{0}\right)$ = undefined'. A few correctly stated that $v - u$ had no real component, but incorrectly concluded that the argument was $\frac{\pi}{2}$.

A number of students thought that Arg(v – u) = Arg(v) – Arg(u) = $-\frac{\pi}{12}-\frac{\pi}{12}=-\frac{\pi}{6}$ (the answer to part c) – these students then invariably got part c wrong.

A small number wrote 'Arg(v – u) = $\text{Tan}^{-1}\left(\frac{\sqrt{2}}{\sqrt{6}}\right)=\text{Tan}^{-1}\left(\frac{-1}{\sqrt{3}}\right)=-\frac{\pi}{6}$; showing a degree of facility with surds if not with complex numbers.

c

Marks	0	1	2	Average
%	51	13	36	**0.84**

Answer: $-\frac{\pi}{6}$

Over half of the students subtracted arguments, though some made sign errors. However, a not uncommon, but rarely successful, approach was to use the Cartesian forms for u and v and attempt to simplify $\frac{v}{u}$ (surprising since the formula sheet gives a simplified expression for the ratio of two complex numbers in polar form). Students need to practise questions where they must choose the most appropriate form for complex numbers. Here, polar form is clearly superior. A number of students divided the arguments to get –1 (or sometimes +1) and concluded that the answer was 0 or 1 or 2π or 1^c.

d

Marks	0	1	2	3	Average
%	53	14	13	20	**0.99**

Answer: $a=-\left(\frac{\sqrt{6}+\sqrt{2}}{2}\right)$

Some students 'fudged' their attempt to show that b = 1. Few students used polar form for u and v in this part; those that did almost always were partially or completely successful. Most students used Cartesian form and then found the surd manipulation too challenging.

Question 3

a

Marks	0	1	2	3	Average
%	29	31	7	33	**1.44**

Answer: As given.

Most students were able to successfully employ the standard i-j-k system, though a small proportion did not make the connection with the three dimensional coordinate system used.

The most common correct method was to show an opposite pair of sides parallel and equal, and an adjacent pair of sides equal; a smaller number of students showed that the lengths of each of the four sides were the same. Some students went on to show that the figure was not a square by proving that adjacent sides did not meet at right angles – it should be appreciated that this is unnecessary as all squares are rhombuses.

Many students did not recognise the requirement to show that the figure was a rhombus. The most typical misconceptions were: showing one pair of opposite sides parallel and equal (a parallelogram); showing two pairs of opposite sides parallel and equal (still only a parallelogram); or showing that the diagonals intersect at right angles (a necessary but not sufficient condition).

A small number of students showed that the diagonals were perpendicular and that a pair of opposite sides were parallel (and equal), which is sufficient. A handful showed that the diagonals were perpendicular, and that a pair of adjacent sides were equal in length, but this is not sufficient (a kite has this property).

b

Marks	0	1	2	3	Average
%	20	9	26	45	**1.95**

Answer: 109.5°

Reasonably well done, with the typical error relating to signs (getting the dot product equal to +11 rather than –11). A worrying number of students could not work out – 8 + 5 – 8 correctly. Some students gave the answer correct to the nearest degree (when the nearest tenth of a degree was required). A few students attempted to use the cosine rule.

ci

Marks	0	1	2	3	4	Average
%	34	25	20	8	13	**1.43**

Answer: $p = \frac{1}{\sqrt{2}}, r = -\frac{1}{\sqrt{2}}$

Not well done by most students. Many students were able to correctly find the two scalar products and equate each to zero. However, showing that $q = 0$ was beyond most and 'fudged' by others. Of those that succeeded (or used the result), many lost sight of the unit vector requirement; and of those that did know what to do, many forgot the condition $p > 0$.

cii

Marks	0	1	2	3	Average
%	96	1	1	2	**0.10**

Answer: $\frac{10}{3\sqrt{2}}$ or $\frac{5\sqrt{2}}{3}$

A very poorly done question. Very few students realised that a scalar (or vector) resolute was required, or what connection this had with the previous part. Most students who attempted this question in any significant sense made the incorrect assumption that E was the midpoint of OB.

Students are clearly not very well versed in recognising situations in which a scalar (or vector) resolute might be helpful. More work needs to be done explaining to students what scalar and vector resolutes are, and hence what they can be used for, rather than just on how to find them.

Question 4

a

Marks	0	1	Average
%	15	85	**0.85**

Answer: 2

A well done question with no clear pattern to incorrect student work.

b

Marks	0	1	2	Average
%	13	27	60	**1.46**

Answer: f: concave up (asymptote $x = 3$), f^{-1}: reflection of f in $y = x$; both curves pass through (0, 0) and (2, 2).

Most students correctly drew the graph of f, but some struggled with the graph of f^{-1}. The most common error was to take f^{-1} to be $\frac{1}{f}$ and so produce the reciprocal of the sec graph. Given that inverse circular functions are a key component of the course, and that the content of Mathematical Methods in assumed knowledge this is a major error/misconception. Some students ignored the domain of f for their sketch.

c

Marks	0	1	2	Average
%	28	4	68	**1.40**

Answer: $\frac{6}{\pi}$

Reasonably well done, although most students went straight into the routine of finding the rule for f^{-1} by swapping x and y in the rule for f and solving for y. There was insufficient realisation that what was needed was to find a and note that as $f(2) = 2$, then $f^{-1}(2) = 2$. A few students gave a decimal approximation instead of the exact value.

d

Marks	0	1	2	Average
%	39	16	45	**1.06**

Answer: 1.939

Reasonably well done by those who had correctly graphed the functions in part b with the correct point of intersection at (2, 2).

ei

Marks	0	1	2	3	Average
%	35	17	18	30	**1.43**

Answer: As given; use quotient and chain rules OR simplify the log expression and use chain rule.

Inadequate algebraic manipulation and simplification was the main difficulty with some poor setting out of the work (parentheses in particular were spread around). Most students were aware that a combination of quotient rule and chain rule was required. A few students used the result 'log(quotient) = difference in logs' to simplify the initial expression prior to differentiation.

eii

Marks	0	1	2	3	4	Average
%	67	10	13	3	7	**0.74**

Answer: $12-\left(\frac{24}{\pi}\right)\log_e\left(2+\sqrt{3}\right)$

A few students were able to apply the result in part ei to help them integrate $f(x)$; but only a handful were able to re-express the area required in terms of $f(x)$ (for example, by using symmetry). Those students who retained the Cos^{-1} term in their integrand either gave up or then differentiated rather than integrate.

Question 5

a

Marks	0	1	2	3	4	Average
%	23	7	9	6	55	**2.62**

Answer: As given.

Quite well done, except for those students who had problems with negative signs. There were a number who changed the fraction $\frac{1}{(100-y)}$ into $\frac{-1}{(y-100)}$ and integrated this to give $-\log_e(y-100)$. This is not correct for the interval of integration. Presumably, these students were attempting to make the antidifferentiation step simpler (although many managed to convert logarithms of negative numbers into a correct final answer – they were not awarded full marks). This was also a problem in Question 4b of Examination 1. Students must be more aware of the need to use logarithms with care in the context of integration.

b

Marks	0	1	2	3	Average
%	54	40	2	4	**0.55**

Answer: As given, with $A = 38$.

Most students ignored the instruction: 'Verify, by differentiation …' and proceeded to attempt (rarely successfully) to solve the differential equation by integration. Students must learn to read and follow such instructions. The examiners were assessing the ability of students to use a specific solution technique, different from that used in part a.

Many students had difficulty with the 'initial' condition $y = 48$ at $t = T$, with the parameters b and T together just too much for others.

c

Marks	0	1	2	3	Average
%	48	18	19	15	**1.00**

Answer: Slightly curved, concave down from (0, 5) to (T, 48); slightly curved, concave up from (T, 48) to (4T, 36).

Rather poorly answered, given that most of the information was contained in the previous two parts. The main difficulty was with the concavity of the two sections of the curve – often both were drawn with the same concavity (usually concave up). Many students were unable to correctly label the key points {(0, 5), (T, 48), (4T, 36)}, either explicitly or by scaling on the axes. The involvement of multiple parameters (a, b and T) was too difficult for many students.

d

Marks	0	1	2	3	4	Average

%	76	6	6	5	7	**0.60**

Answer: 4.76

Only a few students were able to obtain two equations, the first by substitution of (T, 48) in the part a rule, the second by substitution of ($4T$, 36) in the part b rule. The most common error involved equating the two expressions for y and proceeding erroneously from there. Of those students that did get two equations, only a small number were able to correctly manipulate them.

Published by the Victorian Curriculum and Assessment Authority
41 St Andrews Place, East Melbourne 3002