Year 2004

VCE

Specialist Mathematics

Trial Examination 1

Suggested Solutions

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Question 1

$y = \dfrac{x^3 + b^3}{x^2} = \dfrac{x^3}{x^2} + \dfrac{b^3}{x^2} = x + \dfrac{b^3}{x^2}$

The denominator is zero when $x = 0$, so $x = 0$ is a vertical asymptote.
As $x \to \pm\infty$ $y \to x$, so $y = x$ is an oblique asymptote.
Both $x = 0$ and $y = x$ are asymptotes.

Answer: D.

Question 2

The ellipse has centre $(-3, 1)$ semi-major axis is 3 , semi-minor is 1, the general form is

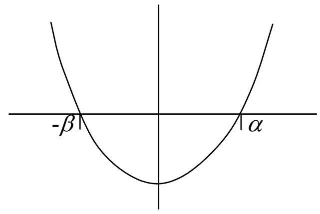
$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

with $h = -3$ $k = 1$ $a = 3$ and $b = 1$ so the equation is $\dfrac{(x+3)^2}{9} + \dfrac{(y-1)^2}{1} = 1$.

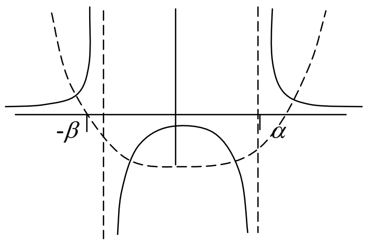
Answer: E.

Question 3

Given $f(x) = a(x-\alpha)(x+\beta)$ where $a > 0$ this graph crosses the x-axis at $x = \alpha$ and $x = -\beta$.



the graph of $\dfrac{1}{f(x)}$



has vertical asymptotes at $x = \alpha$ and $x = -\beta$. So alternatives A. and B. are incorrect.

$f(x) = a\left(x^2 + (\beta - \alpha)x + \alpha\beta\right)$

$f'(x) = a\left(2x + (\beta - \alpha)\right) = 0$ when

$2x + (\beta - \alpha) = 0$

$2x = (\alpha - \beta)$

$x = \dfrac{\alpha - \beta}{2}$

So $\dfrac{1}{f(x)}$ has a local maximum at $x = \dfrac{\alpha - \beta}{2}$.

Answer: C.

Question 4

The range of $y = \text{Cos}^{-1}x$ *is* $[0, \pi]$

The range of $f(x) = a + b\,\text{Cos}^{-1}(cx)$ is dilated by b and translated by a,

its range is $[a, a + b\pi]$

(the value of c has no effect on the range).

Answer: E.

Question 5

Given $|\underset{\sim}{a}| = 7 \quad |\underset{\sim}{b}| = 3 \quad \underset{\sim}{a}.\underset{\sim}{b} = 2$, now

$$|\underset{\sim}{a} - \underset{\sim}{b}|^2 = (\underset{\sim}{a} - \underset{\sim}{b}).(\underset{\sim}{a} - \underset{\sim}{b})$$

$$= \underset{\sim}{a}.\underset{\sim}{a} - \underset{\sim}{b}.\underset{\sim}{a} - \underset{\sim}{a}.\underset{\sim}{b} + \underset{\sim}{b}.\underset{\sim}{b}$$

$$= |\underset{\sim}{a}|^2 - 2\underset{\sim}{a}.\underset{\sim}{b} + |\underset{\sim}{b}|^2$$

$$= 49 - 4 + 9$$

$$|\underset{\sim}{a} - \underset{\sim}{b}|^2 = 54$$

so $\quad |\underset{\sim}{a} - \underset{\sim}{b}| = \sqrt{54}$

Answer: C.

Question 6

Given $P(2, -8, 10) \quad Q(x, 4, -5)$ then

$\overrightarrow{OP} = 2\underset{\sim}{i} - 8\underset{\sim}{j} + 10\underset{\sim}{k} \quad \overrightarrow{OQ} = x\underset{\sim}{i} + 4\underset{\sim}{j} - 5\underset{\sim}{k}$

$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = (x-2)\underset{\sim}{i} + 12\underset{\sim}{j} - 15\underset{\sim}{k}$

Checking each alternative:

A When $x = 2$ $\overrightarrow{PQ} = 12\underset{\sim}{j} - 15\underset{\sim}{k}$ so it is parallel to the YZ plane.
Alternative A is correct.

B When $x = -1$, $\overrightarrow{OP} = -2\ \overrightarrow{OQ}$, so $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ are linearly dependent
Alternative B is incorrect.

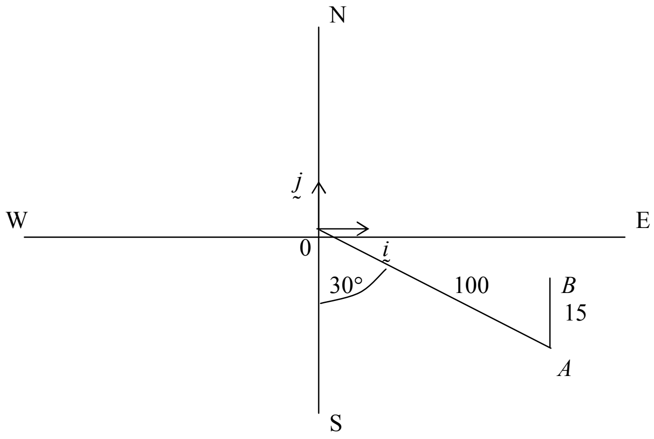
C When $x = -3$, $|\overrightarrow{OQ}| = \sqrt{9 + 16 + 25} = \sqrt{50} = 5\sqrt{2}$

D When $x = 3$, $|\overrightarrow{OQ}| = \sqrt{9 + 16 + 25} = \sqrt{50} = 5\sqrt{2}$. So alternatives C and D are correct.

E If $\overrightarrow{OP}$ is perpendicular to $\overrightarrow{OQ}$,
$\overrightarrow{OP} \bullet \overrightarrow{OQ} = 0$
$\therefore 2x - 32 - 50 = 0$ So alternative E is correct.
$\therefore 2x = 82 \Rightarrow x = 41$

Answer: B.

Question 7



$\overrightarrow{OA} = 100\sin 30°\underset{\sim}{i} - 100\cos 30°\underset{\sim}{j} = 50\underset{\sim}{i} - 50\sqrt{3}\underset{\sim}{j}$

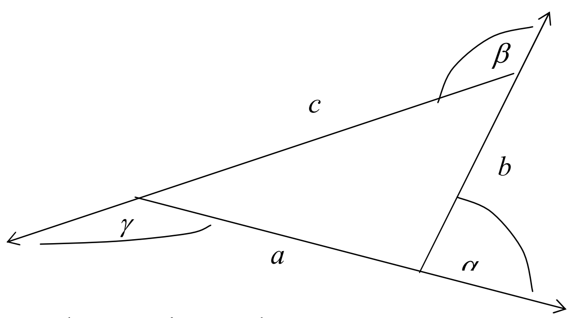
$\overrightarrow{AB} = 15\underset{\sim}{k}$

$\therefore \overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = 50\underset{\sim}{i} - 50\sqrt{3}\underset{\sim}{j} + 15\underset{\sim}{k}$

Answer: B.

Question 8

If $\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c} = \underset{\sim}{0}$ the vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ form the sides of a triangle.



The angles given are the exterior angles.
So the interior angles are $\pi - \alpha$, $\pi - \beta$ and $\pi - \gamma$

$\Rightarrow \pi - \alpha + \pi - \beta + \pi - \gamma = \pi$

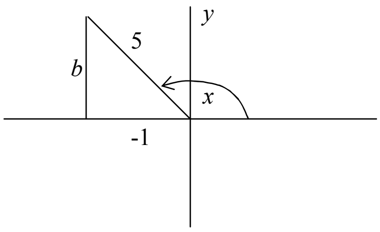
$\Rightarrow 3\pi - \pi = \alpha + \beta + \gamma$

$\Rightarrow \alpha + \beta + \gamma = 2\pi$

Answer: E.

Question 9

Given $\sec(x) = -5$ so $\sec(x) = \dfrac{1}{\cos(x)} = -5$ so $\cos(x) = -\dfrac{1}{5}$, but $\dfrac{\pi}{2} < x < \pi$ so x is the 2nd quadrant.



by Pythagorus' theorem: $1^2 + b^2 = 5^2$

$$b^2 = 25 - 1 = 24$$

$$b = \sqrt{24} = \sqrt{4 \times 6} = 2\sqrt{6}$$

Now $\operatorname{cosec}(x) = \dfrac{1}{\sin(x)} = \dfrac{1}{\frac{2\sqrt{6}}{5}} = \dfrac{5}{2\sqrt{6}} \times \dfrac{\sqrt{6}}{\sqrt{6}} = \dfrac{5\sqrt{6}}{12}$

since $\sin(x)$ is positive in the 2nd quadrant.

Answer: E.

Question 10

Let $\underset{\sim}{a} = 2\underset{\sim}{i} - 3\underset{\sim}{j} + 4\underset{\sim}{k}$, the scalar resolute of $\underset{\sim}{a}$ in the direction of $\underset{\sim}{b}$ is $\underset{\sim}{a}.\hat{\underset{\sim}{b}} = 2\sqrt{6}$.
The vector resolute of $\underset{\sim}{a}$ perpendicular to $\underset{\sim}{b}$ is $\underset{\sim}{a} - (\underset{\sim}{a}.\hat{\underset{\sim}{b}})\hat{\underset{\sim}{b}} = \underset{\sim}{j} + 2\underset{\sim}{k}$.

$$2\underset{\sim}{i} - 3\underset{\sim}{j} + 4\underset{\sim}{k} - 2\sqrt{6}\ \hat{\underset{\sim}{b}} = \underset{\sim}{j} + 2\underset{\sim}{k}$$

$$2\sqrt{6}\,\hat{\underset{\sim}{b}} = \left(2\underset{\sim}{i} - 3\underset{\sim}{j} + 4\underset{\sim}{k}\right) - (\underset{\sim}{j} + 2\underset{\sim}{k})$$

$$2\sqrt{6}\,\hat{\underset{\sim}{b}} = 2\underset{\sim}{i} - 4\underset{\sim}{j} + 2\underset{\sim}{k}$$

so $\hat{\underset{\sim}{b}} = \dfrac{1}{\sqrt{6}}(\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k})$

Answer: C.

Question 11

$P(z) = z^3 + az^2 + bz + c$ given that $P(\alpha - i\beta) = 0$ and a, b and c are real then

A $P(\alpha + i\beta) = 0$ is correct by the conjugate root theorem.

B $P(z)$ has three roots (1 pair of complex conjugates and one root). This is correct.

C This is correct.

D Now $u = \alpha + i\beta \quad v = \alpha - i\beta$

$u + v = 2\alpha \qquad uv = \alpha^2 - i^2\beta^2 = \alpha^2 + \beta^2$ so

$z^2 - (u + v)z + uv = 0$

$z^2 -$ (sum of the roots) $z +$ (product of the roots) $= 0$

so $z^2 - 2\alpha z + (\alpha^2 + \beta^2) = 0$ is a factor of $P(z)$.

Option D is INCORRECT.

E is correct since

$$z^3 + az^2 + bz + c = 0$$
$$= \left(z^2 - 2\alpha z + (\alpha^2 + \beta^2)\right)(z + d) = 0$$

so $d(\alpha^2 + \beta^2) = c$

$$d = \frac{c}{\alpha^2 + \beta^2} \text{ and } P(-d) = 0$$

Answer: D.

Question 12

The shaded region is the intersection of the inside of a circle with centre at the origin and radius 2 , with the region above the line $y = -x$.

The circle is $z\bar{z} \le r^2 = 4$ (where r is the radius).

since if $z = x + iy$ $\bar{z} = x - iy$ so

$$z\bar{z} = x^2 - i^2y^2 = x^2 + y^2 = r^2$$

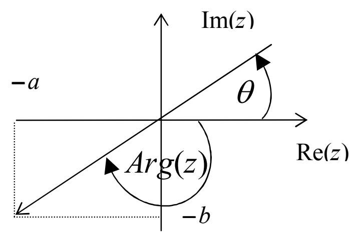
The line is $y \ge -x$, if $z = x + iy$

$\text{Re}(z) = x$ and $\text{Im}(z) = y$

so $\text{Im}(y) + \text{Re}(z) \ge 0$

Answer: A.

Question 13



Now $z = -a - bi$

a and b are real positive

$-\pi < Arg(z) \le \pi$

$$\tan\theta = \frac{b}{a} > 0 \quad \theta = \text{Tan}^{-1}\left(\frac{b}{a}\right)$$

$$Arg(z) = -\pi + \theta$$

$$= -\pi + \text{Tan}^{-1}\left(\frac{b}{a}\right)$$

Answer: D.

Question 14

Solving $z^3 + a^3 i = 0$ since $i^2 = -1$

$z^3 = -a^3 i = a^3 i^3$, one answer is:

$z = ai$ where $a > 0$

There are three answers one of which is D are all equally spaced around the circle. The roots are D, H and L

Answer: B.

Question 15

$\underset{\sim}{r}(t) = \cos^2(t)\underset{\sim}{i} + \cos(2t)\underset{\sim}{j}$

$x = \cos^2(t)$

$y = \cos(2t) = 2\cos^2(t) - 1$

$y = 2x - 1$, this is a straight line.

Answer: A.

Question 16

$y = \text{Cos}^{-1}\left(\dfrac{5x}{4}\right) = \text{Cos}^{-1}\left(\dfrac{u}{4}\right)$ where $u = 5x$ Chain Rule

$\dfrac{dy}{du} = \dfrac{-1}{\sqrt{16-u^2}} \quad \dfrac{du}{dx} = 5$

$\dfrac{dy}{dx} = \dfrac{-5}{\sqrt{16-25x^2}}$, now at $x = 0$ $m_T = \left.\dfrac{dy}{dx}\right|_{x=0} = f'(0) = \dfrac{-5}{4}$

at $x = 0$ $y = \text{Cos}^{-1}(0) = \dfrac{\pi}{2}$ so $P\left(0, \dfrac{\pi}{2}\right)$ and the gradient of the normal is $\dfrac{4}{5}$,

so the equation of the normal is: $y - \dfrac{\pi}{2} = \dfrac{4}{5}(x - 0)$

or $y = \dfrac{4x}{5} + \dfrac{\pi}{2}$

Answer: A.

Question 17

$y = \text{Tan}^{-1}\left(\frac{3}{4x}\right) = \text{Tan}^{-1} u$ where $u = \frac{3}{4x} = \frac{3}{4}x^{-1}$

$\frac{dy}{du} = \frac{1}{1+u^2} \qquad \frac{du}{dx} = -\frac{3}{4}x^{-2} = \frac{-3}{4x^2}$

$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$ (chain rule)

$= -\frac{3}{4x^2}\left(\frac{1}{1+\frac{9}{16x^2}}\right)$

$= -\frac{3}{4x^2}\left(\frac{1}{\frac{16x^2+9}{16x^2}}\right)$

$= -\frac{3}{4x^2}\left(\frac{16x^2}{16x^2+9}\right)$

$= \frac{-12}{9+16x^2}$

Answer: A.

Question 18

$\int_0^{\frac{\pi}{4}} \cos^3(2x)\sin(2x)\,dx$

let $\quad u = \cos(2x) \qquad \frac{du}{dx} = -2\sin(2x)$

changing terminals, when

$x = \frac{\pi}{4} \quad u = \cos\left(\frac{\pi}{2}\right) = 0$ and when $x = 0 \quad u = \cos(0) = 1$

$= \int_1^0 u^3 \times -\frac{1}{2}\frac{du}{dx}.dx = -\frac{1}{2}\int_1^0 u^3\,du \quad = \frac{1}{2}\int_0^1 u^3\,du$

Answer: E.

Question 19

$$y=(A+Bx)e^{-2x} \text{ satisfies } \frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=0$$

$$\frac{dy}{dx}=Be^{-2x}-2(A+Bx)e^{-2x}$$

$$=\left((B-2A)-2Bx\right)e^{-2x}$$

$$\frac{d^2y}{dx^2}=-2\left((B-2A)-2Bx\right)e^{-2x}-2Be^{-2x}$$

$$=\left((4A-4B)+4Bx\right)e^{-2x}$$

so $\dfrac{d^2y}{dx^2}+4\dfrac{dy}{dx}+4y$

$$=\left(4A-4B+4Bx+4B-8A-8Bx+4A+4Bx\right)e^{-2x}=0$$

If $B=0$ and $A=3$ or $A=5$ so alternatives A. and B. both satisfy the differential equation.
If $A=0$ and $B=2$ or if $A=4$ and $B=2$ so alternatives C. and D. both satisfy the differential equation.
Alternative E. **does not** satisfy the differential equation.

Answer: E.

Question 20

$y_1=\dfrac{\cos(2x)}{2x-1}$ using a graphic calculator, in the Radians mode we find the graph crosses the x-axis at $x\approx 2.36$, the area is:

$$\int_2^{2.36}\frac{\cos(2x)}{2x-1}\,dx+\int_{2.36}^{2}\frac{\cos(2x)}{2x-1}\,dx$$

$$=A_1+A_2$$

but $A_1=-0.0376$ and $A_2=0.0796$ the shaded area is:

$$A=\left|A_1\right|+A_2$$

$$=0.0376+0.0796$$

$$=0.1173$$

Answer: D.

Question 21

Rearranging to $\frac{dy}{dx} = \frac{\cos(2x)}{2x-1}$ with the calculator in the radians mode, using a PRGM Euler

$Y_1 = \frac{\cos(2x)}{2x-1}$ $x_0 = 0$ $y_0 = -1$ $h = 0.2$

$y(0.4) = -1.507$

Answer: D.

Question 22

$$\frac{d}{dx}\left(x \log_e(4+x)\right) = \log_e(4+x) + \frac{x}{4+x}$$

so it follows that:

$$\int\left(\log_e(4+x) + \frac{2x}{4+x}\right) dx = x \log_e(4+x)$$

$$\begin{aligned}\int \log_e(4+x)\, dx &= x\log_e(4+x) - \int \frac{x}{4+x} dx \\ &= x \log_e(4+x) - \int\left(\frac{x+4-4}{4+x}\right) dx \\ &= x \log_e(4+x) - \int\left(1 - \frac{4}{4+x}\right) dx \\ &= x \log_e(4+x) - x + 4\log_e(4+x) \\ &= (x+4)\log_e(x+4) - x\end{aligned}$$

Answer: C.

Question 23

Since for

$x < 0 \quad \frac{dy}{dx} < 0$ and $0 < x < 1 \quad \frac{dy}{dx} > 0$

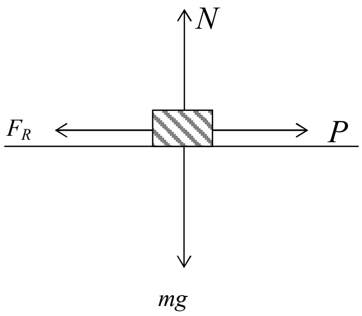
while at $x = 0 \quad \frac{dy}{dx} = 0$ the graph has a local minimum at $x = 0$

and at $x = 4 \quad \frac{d^2y}{dx^2} = 0$

The graph has a stationary point of inflexion at $x = 4$

Answer: B.

Question 24



$m = 12$ kg $\quad P = 11.65$ N

$\mu = 0.1$

$\text{N} = mg$

$\text{N} = 12 \times 9.8$

$= 117.6$ N

Now $\mu\text{N} = 0.1 \times 117.6$

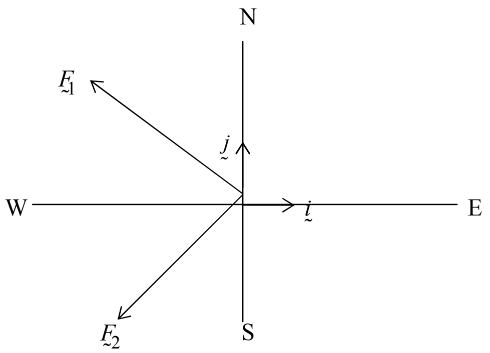
$= 11.76$ N

since $\mu\text{N} = 11.76 > P = 11.65$

The suitcase remains at rest.

Answer: E.

Question 25



$$\underset{\sim}{F}_1 = -3\underset{\sim}{i} + 3\underset{\sim}{j} \qquad \left|\underset{\sim}{F}_1\right| = 3\sqrt{2}$$

$$\underset{\sim}{F}_2 = -4\underset{\sim}{i} - 4\underset{\sim}{j} \qquad \left|\underset{\sim}{F}_2\right| = 4\sqrt{2}$$

$$\underset{\sim}{R} = \underset{\sim}{F}_1 + \underset{\sim}{F}_2 = -7\underset{\sim}{i} - \underset{\sim}{j} = 2\underset{\sim}{a}$$

so $\qquad \underset{\sim}{a} = -\frac{1}{2}(7\underset{\sim}{i} - \underset{\sim}{j})$

$$\left|\underset{\sim}{a}\right| = \frac{1}{2}\sqrt{49+1} = \frac{\sqrt{50}}{2} = \frac{5\sqrt{2}}{2}$$

Answer: B.

Question 26

Resolving vertically gives: $T_1 \sin\beta + T_2 \sin\alpha - mg = 0$
so alternative B. is correct.

Resolving horizontally gives: $T_1 \cos\beta - T_2 \cos\alpha = 0$

so $\qquad T_1 \cos\beta = T_2 \cos\alpha$

$\dfrac{T_1}{T_2} = \dfrac{\cos\alpha}{\cos\beta} = \dfrac{\frac{L_2}{\alpha}}{\frac{L_1}{\alpha}} = \dfrac{L_2}{L_1}$ so alternatives A. and C. are correct.

The vector equation $\underset{\sim}{T}_1 + \underset{\sim}{T}_2 + m\underset{\sim}{g} = \underset{\sim}{0}$ is true.

Alternative D. is correct. Alternative E. is INCORRECT.
The shorter string carries more tension.

Answer: E.

Question 27

Let $y_1 = 4e^{-\frac{x}{2}}$ *and* $y_2 = 3\sin\left(\frac{x}{2}\right)$ the volume is

$$V = \pi \int_0^b (y_1{}^2 - y_2{}^2)\,dx = \pi \int_0^b \left(16e^{-x} - 9\sin^2\left(\frac{x}{2}\right)\right)dx$$

Answer: A.

Question 28

Since $v > 0$ for $0 \le t < c$ and $v = 0$ for $t = c$

$v < 0$ for $t > c$ Alternative B. is correct.

Answer: B.

Question 29

Given the initial conditions $t = 0$ $x = 0$ $v = v_0$

Now $\frac{dv}{dx} = 1$ so $v\frac{dv}{dx} = a = v$ Alternative A. is correct.

Since $\frac{dv}{dx} = 1$ integrating *wrt* x

$v = \int 1\,dx = x + c$ but $v = v_0$ when $x = 0$

so $c = v_0$ and $v = x + v_0$ Alternative B. is correct.

Since $a = \frac{dv}{dt} = v$ this has the solution $v = v_0 e^t$ Alternative C. is correct.

Alternative D. is the **constant** acceleration formula $v^2 = v_0^2 + 2vx$ is FALSE.

Alternative E. is correct since $v = \frac{dx}{dt} = v_0 e^t$ integrating *wrt* t

$x = \int v_0 e^t\,dt = v_0 e^t + c$

but when $t = 0$ $x = 0$ so $0 = v_0 + c$ so $c = -v_0$

and $x = v_0\left(e^t - 1\right)$

Answer: D.

Question 30

Resolving perpendicular to the incline gives

(1) $N - mg\cos\theta = 0$

resolving downwards parallel to the plane gives

(2) $mg\sin\theta - \mu N = ma$

from (1) $N = mg\cos\theta$ into (2)

$mg\sin\theta - \mu mg\cos\theta = ma$

$a = g(\sin\theta - \mu\cos\theta)$ Alternative A. and B. are correct.

The acceleration is constant, using $u = 0$ $t = T$ $s = D$

$v^2 = u^2 + 2as$ gives

$v^2 = 0 + 2g(\sin\theta - \mu\cos\theta)D$

so $v = \sqrt{2gD(\sin\theta - \mu\cos\theta)}$ Alternative C. is correct.

Using: $s = \left(\frac{u+v}{2}\right)t$ gives

$D = \left(\frac{0+V}{2}\right)T = \frac{VT}{2}$

so $T = \frac{2D}{V}$

Alternative D. is correct.

Alternative E. is false, terminal or limiting velocity occurs in a situation when falling vertically with air resistance proportional to the velocity.

Answer: E.

Question 1

Let $u = \sqrt{x} = x^{\frac{1}{2}}$ $\frac{du}{dx} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$ so $2\ du = \frac{1}{\sqrt{x}}\ dx$

terminals $x = 4 \quad u = 2$

$x = 0 \quad u = 0$

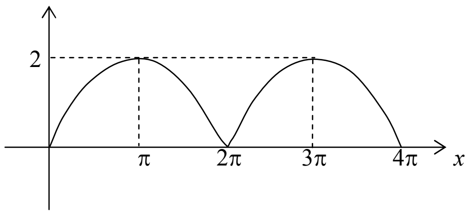
$$\int_0^4 \frac{\cos\sqrt{x}}{\sqrt{x}}\, dx \quad = 2\int_0^2 \cos(u)\ du$$

$$= 2\sin u\Big]_0^2 = 2\sin(2) - 2\sin(0)$$

$$= 2\sin(2)$$

Question 2

i. $x = 2t - \sin(2t) \quad y = 1 - \cos(2t) \quad 0 \le t \le 2\pi$



ii. $\underset{\sim}{r}(t) = (2t - \sin(2t))\underset{\sim}{i} + (1 - \cos(2t))\underset{\sim}{j} \quad 0 \le t \le 2\pi$

$\dot{\underset{\sim}{r}}(t) = (2 - 2\cos(2t))\underset{\sim}{i} + 2\sin(2t)\underset{\sim}{j}$

$|\dot{\underset{\sim}{r}}(t)| = \sqrt{(2 - 2\cos 2t)^2 + (2\sin 2t)^2}$

$= \sqrt{4 - 8\cos(2t) + 4\cos^2(2t) + 4\sin^2(2t)}$

$= \sqrt{8 - 8\cos(2t)}$

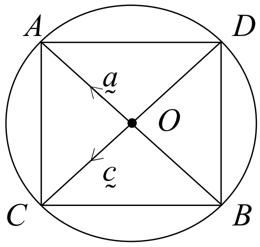
$= \sqrt{8(1 - \cos(2t))}$ since $\cos(2A) = 1 - 2\sin^2(A)$

$= \sqrt{8(2\sin^2(t))}$ and $2\sin^2(A) = 1 - \cos(2A)$

$= 4\sin t$

so $c = 4$

Question 3



Now $\overrightarrow{OA} = \overrightarrow{BO} = \underset{\sim}{a}$ and $\overrightarrow{OC} = \overrightarrow{DO} = \underset{\sim}{c}$

$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC} = \underset{\sim}{c} - \underset{\sim}{a}$

$\overrightarrow{AD} = \overrightarrow{AO} + \overrightarrow{OD} = -\underset{\sim}{a} - \underset{\sim}{c}$

$\overrightarrow{CB} = \overrightarrow{CO} + \overrightarrow{OB} = -\underset{\sim}{a} - \underset{\sim}{c} \Rightarrow \overrightarrow{AD} = \overrightarrow{CB}$

consider

$$\begin{aligned}
&\overrightarrow{AC} \bullet \overrightarrow{CB} \\
&= (\underset{\sim}{c} - \underset{\sim}{a}) \bullet (-\underset{\sim}{a} - \underset{\sim}{c}) \\
&= -\underset{\sim}{c}.\underset{\sim}{a} + \underset{\sim}{a}.\underset{\sim}{a} - \underset{\sim}{c}.\underset{\sim}{c} + \underset{\sim}{a}.\underset{\sim}{c} \\
&= |\underset{\sim}{a}|^2 - |\underset{\sim}{c}|^2 \\
&= 0 \text{ since } |\underset{\sim}{a}| = |\underset{\sim}{c}| \text{ (both are radii of the circle)}
\end{aligned}$$

so $\quad \overrightarrow{AC}$ is perpendicular to $\overrightarrow{CB}$ and $\overrightarrow{AD} = \overrightarrow{CB}$

$ABCD$ is rectangle

Question 4

i. $u = 2\sqrt{2}(1 - i) \qquad Arg(u) = -\dfrac{\pi}{4}$

so $u = 4cis\left(-\dfrac{\pi}{4}\right)$

And $arg(u^8) = 8 \times -\dfrac{\pi}{4} = -2\pi$

But $Arg(u^8) = -2\pi + 2\pi = 0$

ii. $u^2 = \left(4\, cis\left(-\dfrac{\pi}{4}\right)\right)^2$

$= 16\, cis\left(-\dfrac{\pi}{2}\right)$

$= -16i$

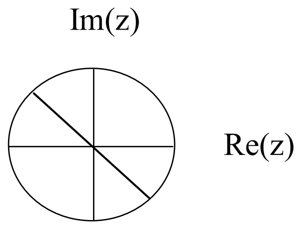
iii. $z^3 + 16iz = 0$

$z(z^2 + 16i) = 0$

$z = 0$ or $z^2 = -16i$ hence from **i.** and **ii.**

$z = 2\sqrt{2}\,(1 - i) = 4\, cis\left(-\dfrac{\pi}{4}\right) = z_1$

$z = 2\sqrt{2}(-1 + i) = 4\, cis\left(\dfrac{3\pi}{4}\right) = z_2$



note that $z_2 = -z_1$ these are **not** conjugates.

Question 5

a. $\underset{\sim}{a} = -2\underset{\sim}{i} + y\underset{\sim}{j} + 5\underset{\sim}{k}$

$$|\underset{\sim}{a}| = \sqrt{4 + y^2 + 25} = 6$$

$$\sqrt{29 + y^2} = 6$$

$$29 + y^2 = 36$$

$$y^2 = 7$$

$y = \pm\sqrt{7}$ both $\pm$ are both acceptable answers.

b. $\cos\beta = -\dfrac{\sqrt{7}}{6} = \dfrac{y}{|\underset{\sim}{a}|} = \dfrac{y}{\sqrt{29 + y^2}}$

$$6y = -\sqrt{7}\left(\sqrt{29 + y^2}\right)$$

$$36y^2 = 7\left(29 + y^2\right)$$

$$= 7 \times 29 + 7 \times y^2$$

$$29y^2 = 7 \times 29$$

$y = \pm\sqrt{7}$ but

$y = -\sqrt{7}$ since $y < 0$ it is an obtuse angle, there is only one answer.

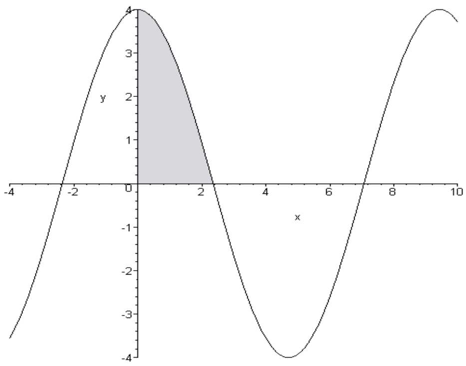
Question 6

to find the first x intercept

$$\cos\left(\frac{2x}{3}\right) = 0$$

$$\frac{2x}{3} = \frac{\pi}{2}$$

$$x = \frac{3\pi}{4}$$



Now the volume $V_x = \pi \int_a^b y^2 \, dx$

$$= \pi \int_0^{\frac{3\pi}{4}} 16 \cos^2\left(\frac{2x}{3}\right) dx$$

$$= 8\pi \int_0^{\frac{3\pi}{4}} \left(1 + \cos\left(\frac{4x}{3}\right)\right) dx$$

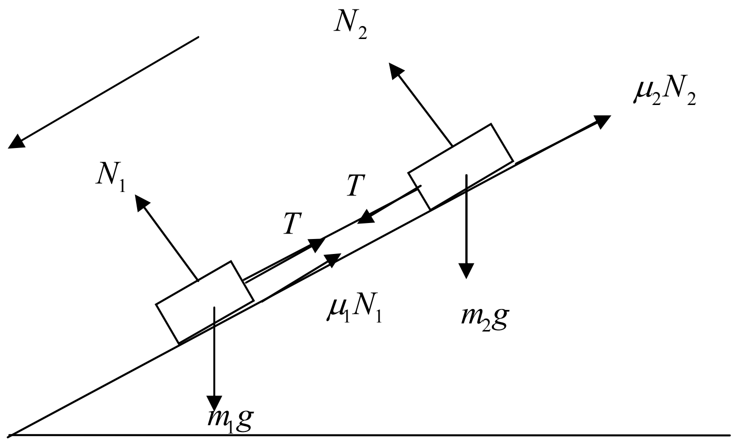
$$= 8\pi \left[x + \frac{3}{4}\sin\left(\frac{4x}{3}\right)\right]_0^{\frac{3\pi}{4}}$$

$$= 8\pi\left[\frac{3\pi}{4} + \frac{3}{4}\sin(\pi) - \left(0 + \frac{3}{4}\sin 0\right)\right]$$

$$= 6\pi^2$$

Question 7

i.



ii. isolating the particles and resolving around each individually,

resolving perpendicular to the plane around the mass m_1

$N_1 - m_1 g\cos\alpha = 0$ so that (1) $N_1 = m_1 g\cos\alpha$

resolving perpendicular to the plane around the mass m_2

$N_2 - m_2 g\cos\alpha = 0$ so that (2) $N_2 = m_2 g\cos\alpha$

resolving parallel and up the plane around the mass m_1 gives

$T + \mu_1 N_1 - m_1 g\sin\alpha = 0$ using (1) gives (3) $T = m_1 g(\sin\alpha - \mu_1\cos\alpha)$

resolving parallel and up the plane around the mass m_2 gives

$\mu_2 N_2 - T - m_2 g\sin\alpha = 0$ using (2) gives (4) $T = m_2 g(\mu_2\cos\alpha - \sin\alpha)$

Equating (3) and (4)

$m_1(\sin\alpha - \mu_1\cos\alpha) = m_2(\mu_2\cos\alpha - \sin\alpha)$

$(m_1 + m_2)\sin\alpha = (m_1\mu_1 + m_2\mu_2)\cos\alpha$

$\tan\alpha = \dfrac{m_1\mu_1 + m_2\mu_2}{m_1 + m_2}$ shown.

END OF SUGGESTED SOLUTIONS
2004 Specialist Mathematics Trial Examination 1

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