Year 2004

VCE

Specialist Mathematics

Trial Examination 1

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VICTORIAN CERTIFICATE OF EDUCATION
2004

SPECIALIST MATHEMATICS

Trial Written Examination 1
(Facts, skills and applications)

Reading time: 15 minutes
Total writing time: 1 hour 30 minutes

PART I

MULTIPLE-CHOICE QUESTION BOOK

This examination has two parts: Part I (multiple-choice questions) and
Part II (short-answer questions).
Part I consists of this question book and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of a separate question and answer book.
You must complete both parts in the time allotted.
When you have completed one part continue immediately to the other part.

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
30	30	30

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and an approved scientific and/or graphics calculator (memory may be retained).
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question book of 23 pages, with a detachable sheet of miscellaneous formulas in the centrefold and two blank pages for rough working.
- Answer sheet for multiple-choice questions.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Check that your name and student number as printed on your answer sheet for multiple-choice questions are correct, and sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are not drawn to scale.

At the end of the examination

- Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer book (Part II).
- You may retain this question book.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
cosine rule:	$c^2=a^2+b^2-2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$

hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigonometric) functions

$\cos^2 x+\sin^2 x=1$

$1+\tan^2 x=\sec^2 x$

$\cot^2 x+1=\operatorname{cosec}^2 x$

$\sin(x+y)=\sin x\cos y+\cos x\sin y$

$\sin(x-y)=\sin x\cos y-\cos x\sin y$

$\cos(x+y)=\cos x\cos y-\sin x\sin y$

$\cos(x-y)=\cos x\cos y+\sin x\sin y$

$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x\tan y}$

$\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x\tan y}$

$\cos 2x=\cos^2 x-\sin^2 x=2\cos^2 x-1=1-2\sin^2 x$

$\sin 2x=2\sin x\cos x$

$\tan 2x=\frac{2\tan x}{1-\tan^2 x}$

Function	Sin^{-1}	Cos^{-1}	Tan^{-1}
Domain	$[-1,1]$	$[-1,1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0,\pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex Numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\,\mathrm{cis}\,\theta$

$|z| = \sqrt{x^2 + y^2} = r$ $\qquad -\pi < Arg\,z \le \pi$

$z_1 z_2 = r_1 r_2\,\mathrm{cis}(\theta_1 + \theta_2)$ $\qquad \dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\,\mathrm{cis}(\theta_1 - \theta_2)$

$z^n = r^n \mathrm{cis}\left(n\theta\right)$ (de Moivre's theorem)

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$ $\qquad \underset{\sim}{r_1} . \underset{\sim}{r_2} = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \dfrac{d\underset{\sim}{r}}{dt} = \dfrac{dx}{dt}\underset{\sim}{i} + \dfrac{dy}{dt}\underset{\sim}{j} + \dfrac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

sliding friction: $F \le \mu N$

constant (uniform) acceleration:

$v = u + at$ $\qquad s = ut + \dfrac{1}{2}at^2$ $\qquad v^2 = u^2 + 2as$ $\qquad s = \dfrac{1}{2}(u + v)t$

acceleration: $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\tfrac{1}{2}v^2\right)$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$
$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$
$$\frac{d}{dx}(\log_e x) = \frac{1}{x}$$
$$\frac{d}{dx}(\sin ax) = a\cos ax$$
$$\frac{d}{dx}(\cos ax) = -a\sin ax$$

$$\frac{d}{dx}(\tan ax) = a\sec^2 ax$$
$$\frac{d}{dx}(\mathrm{Sin}^{-1}x) = \frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}(\mathrm{Cos}^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}(\mathrm{Tan}^{-1}x) = \frac{1}{1+x^2}$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c\ ,\ n \neq -1$$
$$\int e^{ax}dx = \frac{1}{a}e^{ax} + c$$
$$\int \frac{1}{x}dx = \log_e x + c,\ for\ x > 0$$
$$\int \sin ax dx = -\frac{1}{a}\cos ax + c$$
$$\int \cos ax dx = \frac{1}{a}\sin ax + c$$
$$\int \sec^2 ax dx = \frac{1}{a}\tan ax + c$$
$$\int \frac{dx}{\sqrt{a^2-x^2}} = \mathrm{Sin}^{-1}\frac{x}{a} + c,\ a > 0$$
$$\int \frac{-1}{\sqrt{a^2-x^2}}dx = \mathrm{Cos}^{-1}\frac{x}{a} + c,\ a > 0$$
$$\int \frac{a}{a^2+x^2}dx = \mathrm{Tan}^{-1}\frac{x}{a} + c$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}(\frac{u}{v}) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

mid-point rule: $\int_a^b f(x)dx \approx (b-a)f(\frac{a+b}{2})$

trapezoidal rule: $\int_a^b f(x)dx \approx \frac{1}{2}(b-a)(f(a)+f(b))$

Euler's method If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x)$

Take the acceleration due to gravity to have magnitude g m/s^2, where $g = 9.8$

VCE SPECIALIST MATHEMATICS 2004

Trial Written Examination 1

ANSWER SHEET

NAME: ____

STUDENT NUMBER ____

SIGNATURE____

Instructions

- Write your name in the space provided above.
- Write your student number in the space provided above. Sign your name.
- Use a **PENCIL** for **ALL** entries.
  If you make a mistake, **ERASE** it - **DO NOT** cross it out.
- Marks will **NOT** be deducted for incorrect answers.
- **NO MARK** will be given if more than **ONE** answer is completed for any question.
- All answers must be completed like **THIS** example.

A	B	C	D	E

1	A	B	C	D	E	**16**	A	B	C	D	E
2	A	B	C	D	E	**17**	A	B	C	D	E
3	A	B	C	D	E	**18**	A	B	C	D	E
4	A	B	C	D	E	**19**	A	B	C	D	E
5	A	B	C	D	E	**20**	A	B	C	D	E
6	A	B	C	D	E	**21**	A	B	C	D	E
7	A	B	C	D	E	**22**	A	B	C	D	E
8	A	B	C	D	E	**23**	A	B	C	D	E
9	A	B	C	D	E	**24**	A	B	C	D	E
10	A	B	C	D	E	**25**	A	B	C	D	E
11	A	B	C	D	E	**26**	A	B	C	D	E
12	A	B	C	D	E	**27**	A	B	C	D	E
13	A	B	C	D	E	**28**	A	B	C	D	E
14	A	B	C	D	E	**29**	A	B	C	D	E
15	A	B	C	D	E	**30**	A	B	C	D	E

Please DO NOT fold, bend or staple this form

Instructions for Part I
Answer **all** questions in pencil, on the answer sheet provided for multiple-choice questions. A correct answer scores 1 mark, an incorrect answer scores 0.
Marks will not be deducted for incorrect answers.
No mark will be given for a question if more than one answer is completed for any question.

Question 1

The equations of all the asymptotes to the curve $y = \dfrac{x^3 + b^3}{x^2}$ are

A. $x = 0$ only

B. $x = 0$ and $x = -b$

C. $x = 0$ and $y = 0$

D. $x = 0$ and $y = x$

E. $x = 0$ and $y = x$ and $x = -b$

Question 2

The equation for the ellipse shown is

A. $\dfrac{(x-3)^2}{9} + (y+1)^2 = 1$

B. $\dfrac{(x+3)^2}{3} + (y-1)^2 = 1$

C. $\dfrac{(x-3)^2}{9} + \dfrac{(y-1)^2}{2} = 1$

D. $\dfrac{(x+3)^2}{9} + (y+1)^2 = 1$

E. $\dfrac{(x+3)^2}{9} + (y-1)^2 = 1$

Question 3

If $f(x)=a(x-\alpha)(x+\beta)$ where $a>0$ then the graph of $y=\dfrac{1}{f(x)}$ has

A. x-intercepts at $x=\alpha$ and $x=-\beta$

B. asymptotes at $x=\dfrac{1}{\alpha}$ and $x=-\dfrac{1}{\beta}$

C. a local maximum at the point $x=\dfrac{\alpha-\beta}{2}$

D. a local minimum at the point $x=\dfrac{\beta-\alpha}{2}$

E. a local maximum at the point $x=\dfrac{\beta-\alpha}{2}$

Question 4

The range of the function with the rule $f(x)=a+b\,\mathrm{Cos}^{-1}(cx)$ is

A. $[-c,c]$

B. $\left[-\dfrac{1}{c},\dfrac{1}{c}\right]$

C. $[a-b,a+b]$

D. $\left[a-\dfrac{b\pi}{2},a+\dfrac{b\pi}{2}\right]$

E. $[a,a+b\pi]$

Question 5

If $|\underset{\sim}{a}|=7$ and $|\underset{\sim}{b}|=3$ and $\underset{\sim}{a}.\underset{\sim}{b}=2$ then $|\underset{\sim}{a}-\underset{\sim}{b}|$ is equal to

A. 16

B. 4

C. $\sqrt{54}$

D. 6

E. 10

Question 6

Given the two points $P(2,-8,10)$ and $Q(x,4,-5)$ and the origin O then which of the following is **INCORRECT**?

A. When $x = 2$ the vector $\overrightarrow{PQ}$ is parallel to the YZ plane

B. When $x = -1$ the vectors $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ are linearly independent.

C. When $x = 3$ the length of the vector is $5\sqrt{2}$.

D. When $x = -3$ the length of the vector $\overrightarrow{OQ}$ is $5\sqrt{2}$.

E. When $x = 41$ the vector $\overrightarrow{OP}$ is perpendicular to the vector $\overrightarrow{OQ}$

Question 7

A student walks a distance of 100 m on a bearing south 30° east, to a lift. The lift travels vertically up a distance of 15 m. If $\underset{\sim}{i}$, $\underset{\sim}{j}$ and $\underset{\sim}{k}$ are unit vectors of magnitude one metre in the directions of east , north and vertically upwards respectively, then the position vector of the student relative to her initial position is given by

A. $50\underset{\sim}{i} - 50\sqrt{3}\,\underset{\sim}{j}$

B. $50\underset{\sim}{i} - 50\sqrt{3}\,\underset{\sim}{j} + 15\underset{\sim}{k}$

C. $50\sqrt{3}\underset{\sim}{i} - 50\,\underset{\sim}{j}$

D. $-50\underset{\sim}{i} + 50\sqrt{3}\,\underset{\sim}{j} + 15\underset{\sim}{k}$

E. $50\sqrt{3}\underset{\sim}{i} - 50\,\underset{\sim}{j} + 15\underset{\sim}{k}$

Question 8

If $\underset{\sim}{a}+\underset{\sim}{b}+\underset{\sim}{c}=\underset{\sim}{0}$ and the angle between the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is α, the angle between the vectors $\underset{\sim}{b}$ and $\underset{\sim}{c}$ is β, and the angle between the vectors $\underset{\sim}{c}$ and $\underset{\sim}{a}$ is χ, then $\alpha+\beta+\chi$ is equal to

A. 0

B. $\frac{\pi}{2}$

C. π

D. $\frac{3\pi}{2}$

E. 2π

Question 9

If $\sec(x)=-5$ and $\frac{\pi}{2}<x<\pi$ then $\operatorname{cosec}(x)$ equals

A. $-\frac{2\sqrt{6}}{5}$

B. $\frac{\sqrt{6}}{12}$

C. $-2\sqrt{6}$

D. $-\frac{5\sqrt{6}}{12}$

E. $\frac{5\sqrt{6}}{12}$

Question 10

The scalar resolute of the vector $2\underset{\sim}{i} - 3\underset{\sim}{j} + 4\underset{\sim}{k}$ in the direction $\underset{\sim}{b}$ is $2\sqrt{6}$.
The vector resolute of the vector $2\underset{\sim}{i} - 3\underset{\sim}{j} + 4\underset{\sim}{k}$ perpendicular to $\underset{\sim}{b}$ is $\underset{\sim}{j} + 2\underset{\sim}{k}$.
A unit vector parallel to $\underset{\sim}{b}$ is

A. $\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$

B. $2\underset{\sim}{i} - 2\underset{\sim}{j} + 6\underset{\sim}{k}$

C. $\frac{1}{\sqrt{6}}\left(\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}\right)$

D. $\frac{1}{\sqrt{14}}\left(\underset{\sim}{i} - 3\underset{\sim}{j} + 2\underset{\sim}{k}\right)$

E. $\frac{1}{\sqrt{11}}\left(\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k}\right)$

Question 11

If $P(z) = z^3 + az^2 + bz + c$ where a, b and c are real and
$P(\alpha - i\beta) = 0$ where α and β are real, then which of the following is **INCORRECT**?

A. $P(\alpha + i\beta) = 0$

B. $P(z)$ has three roots

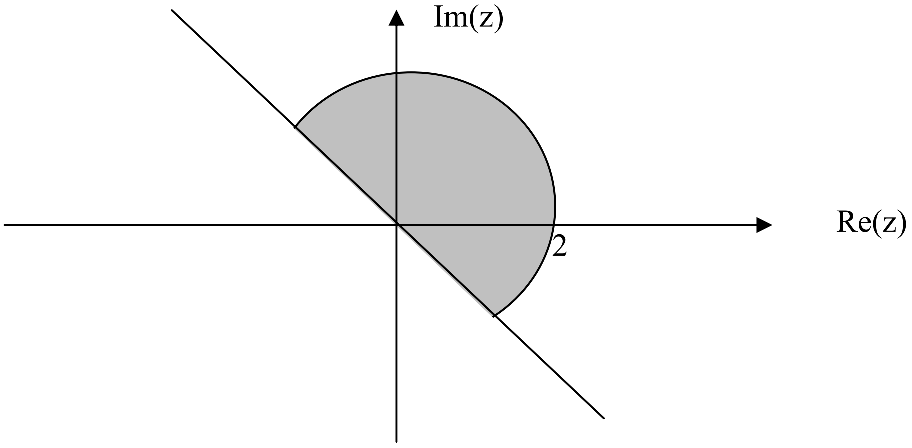
C. There is one pair of complex conjugate roots and one real root.

D. $z^2 + 2\alpha z + (\alpha^2 + \beta^2)$ is a factor of $P(z)$

E. $P\left(-\frac{c}{\alpha^2 + \beta^2}\right) = 0$

Question 12

The shaded region below with the boundaries included is best described by



A. $\{z : z\bar{z} \le 4\} \cap \{z : \mathrm{Im}(z) + \mathrm{Re}(z) \ge 0\}$

B. $\{z : z\bar{z} \le 2\} \cap \{z : \mathrm{Im}(z) + \mathrm{Re}(z) \ge 0\}$

C. $\{z : z\bar{z} \le 4\} \cap \{z : \mathrm{Im}(z) - \mathrm{Re}(z) \ge 0\}$

D. $\{z : z\bar{z} \le 2\} \cap \{z : \mathrm{Im}(z) - \mathrm{Re}(z) \ge 0\}$

E. $\{z : z\bar{z} \le 4\} \cap \{z : \mathrm{Im}(z) + \mathrm{Re}(z) \le 0\}$

Question 13

If $z = -a - bi$, where a and b are real positive numbers, then which one of the following represents $\mathrm{Arg}(z)$?

A. $\mathrm{Tan}^{-1}\left(\frac{b}{a}\right)$

B. $\mathrm{Tan}^{-1}\left(\frac{a}{b}\right)$

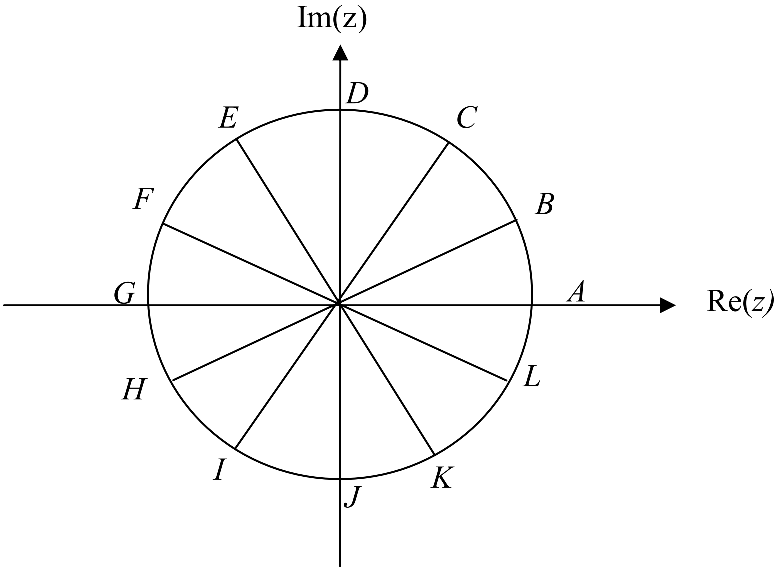
C. $\pi + \mathrm{Tan}^{-1}\left(\frac{b}{a}\right)$

D. $-\pi + \mathrm{Tan}^{-1}\left(\frac{b}{a}\right)$

E. $-\pi + \mathrm{Tan}^{-1}\left(\frac{a}{b}\right)$

Question 14

The diagram shows a circle of radius a on an Argand diagram.



The roots of the equation $z^3 + a^3 i = 0$ where a is a positive real number, are given by

A. A , E and I

B. D , H and L

C. G , C and K

D. J , B and F

E. L , I and E

Question 15

If the position vector of a particle is given by $\underset{\sim}{r}(t) = \cos^2(t)\underset{\sim}{i} + \cos(2t)\underset{\sim}{j}$
then the particle moves on

A. a straight line

B. a circle

C. an ellipse

D. a hyperbola

E. a parabola

Question 16

If $y = \text{Cos}^{-1}\left(\frac{5x}{4}\right)$ the equation of the normal to the curve at $x = 0$ is

A. $y = \frac{4x}{5} + \frac{\pi}{2}$

B. $y = -\frac{5x}{4} + \frac{\pi}{2}$

C. $y = \frac{\pi}{2} - \frac{x}{4}$

D. $y = \frac{\pi}{2} + 4x$

E. $y = -x$

Question 17

If $y = \text{Tan}^{-1}\left(\frac{3}{4x}\right)$ and $x \neq 0$ then $\frac{dy}{dx}$ is equal to

A. $\frac{-12}{16x^2 + 9}$

B. $\frac{3}{4(16x^2 + 9)}$

C. $\frac{16x^2 + 9}{4}$

D. $\frac{-4}{16x^2 + 9}$

E. $\frac{-4}{4(x^2 + 1)}$

Question 18

Using a suitable substitution $\int_0^{\frac{\pi}{4}} \cos^3(2x)\sin(2x)\,dx$ becomes

A. $\int_0^{\frac{\pi}{4}} u^3\,du$

B. $\int_1^0 u^3\,du$

C. $\int_0^1 u^3\,du$

D. $\frac{1}{2}\int_1^0 u^3\,du$

E. $\frac{1}{2}\int_0^1 u^3\,du$

Question 19

Given the differential equation $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$, which one of the following is **NOT** a solution?

A. $y = 3e^{-2x}$

B. $y = 5e^{-2x}$

C. $y = 2xe^{-2x}$

D. $y = (4 + 2x)e^{-2x}$

E. $y = x^2e^{-2x}$

Question 20

The area bounded by the curve $y = \frac{\cos(2x)}{2x-1}$ the x axis and $x = 2$ and $x = 3$ is closest to

A. 0.042

B. 0.0376

C. 0.0796

D. 0.1173

E. 0.2545

Question 21

Euler's method, with a step size of 0.2, is used to approximate the solution of the differential equation $\cos(2x)\frac{dx}{dy} = 2x - 1$ with $x = 0$ $y = -1$.When $x = 0.4$ the value obtained for y, correct to four decimal places, is

A. -1.3303

B. -1.380

C. -1.3816

D. -1.5070

E. -1.5333

Question 22

The derivative of $x\log_e(4+x)$ with respect to x is $\log_e(4+x) + \frac{x}{4+x}$

It follows that an antiderivative of $\log_e(4+x)$ is

A. $x\log_e(4+x) + \int \frac{x}{4+x}\,dx$

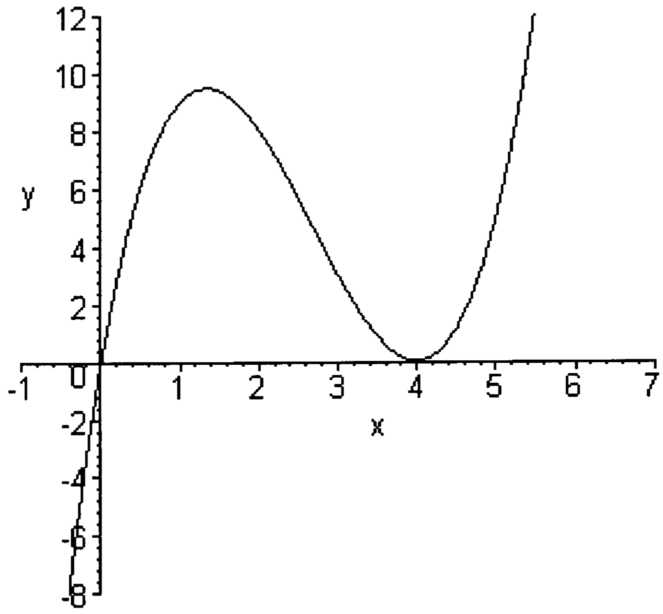
B. $x\log_e(4+x) - \frac{x}{2}\text{Tan}^{-1}\left(\frac{\sqrt{x}}{2}\right)$

C. $(x+4)\log_e(4+x) - x$

D. $\int \left(x\log_e(4+x) + \frac{x}{4+x} \right) dx$

E. $(x-4)\log_e(4+x) - x$

Question 23



The graph of $y = f'(x)$ is shown above.
Which of the following statements is true for the graph of $y = f(x)$?

A. The graph has a local maximum at $x = 0$ and a stationary point of inflexion at $x = 4$

B. The graph has a local minimum at $x = 0$ and a stationary point of inflexion at $x = 4$

C. The graph has a local maximum at $x = 4$ and a stationary point of inflexion at $x = 0$

D. The graph has a local minimum at $x = 4$ and a stationary point of inflexion at $x = 0$

E. The graph has a local minimum at $x = 4$, a local maximum at $x = 1$, and a stationary point of inflexion at $x = 0$

Question 24

A suitcase of mass 12 kilograms rests on a rough level ground. The suitcase is pulled with a horizontal force of 11.65 newtons. The coefficient of friction between the suitcase and the ground is 0.1. Which of the following statements is correct?

A. The suitcase is in limiting equilibrium.

B. The suitcase moves with constant acceleration.

C. The suitcase moves with constant velocity.

D. The suitcase is on the point of moving.

E. The suitcase remains at rest.

Question 25

A particle of mass 2 kg is acted upon by two forces. One force has a magnitude of $3\sqrt{2}$ newtons acting in the north-west direction, the other force has magnitude of $4\sqrt{2}$ newtons acting in the south-west direction. If $\underset{\sim}{i}$ and $\underset{\sim}{j}$ are unit vectors in the direction of east and north respectively, then the magnitude of the acceleration of the particle in m/s^2 is given by

A. $\dfrac{7\sqrt{2}}{2}$

B. $\dfrac{5\sqrt{2}}{2}$

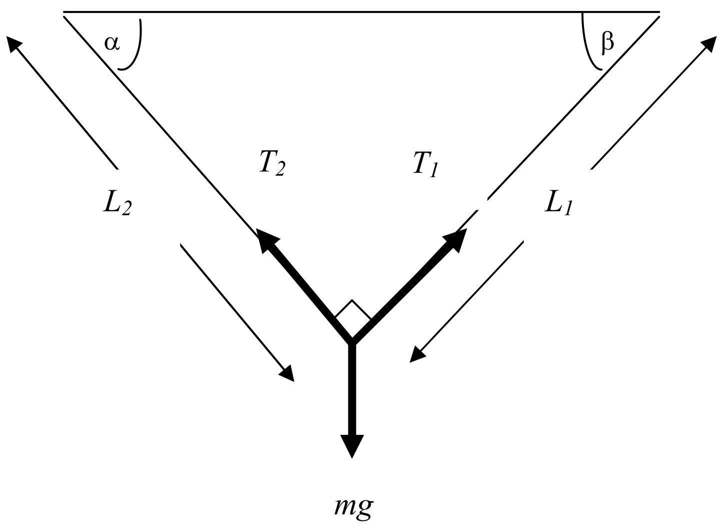
C. $7\sqrt{2}$

D. $5\sqrt{2}$

E. $-\dfrac{1}{2}\left(7\underset{\sim}{i}+\underset{\sim}{j}\right)$

Question 26

The diagram shows a particle of mass m kg hanging vertically at rest. It is connected by two light strings of lengths L_1 and L_2 m having tensions of T_1 and T_2 respectively. The strings are at right angles to each other and the other ends are connected to two fixed points on the same horizontal level. The strings make angles of α and β as shown. Which of the following is **INCORRECT**?



A. $\dfrac{T_1}{T_2} = \dfrac{L_2}{L_1}$

B. $T_1 \sin\beta + T_2 \sin\alpha = mg$

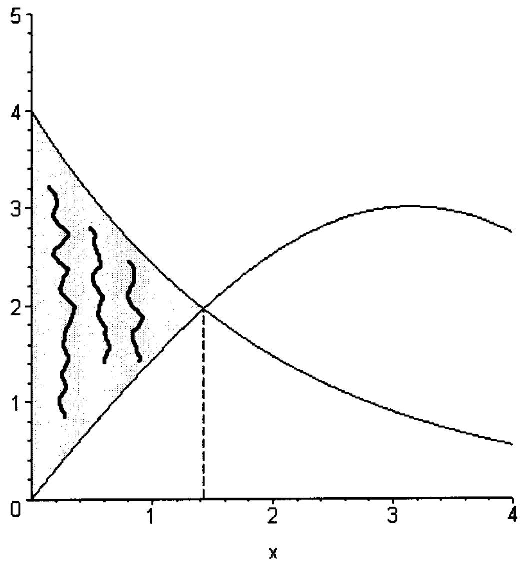
C. $T_1 \cos\beta = T_2 \cos\alpha$

D. $\underset{\sim}{T}_1 + \underset{\sim}{T}_2 + m\underset{\sim}{g} = \underset{\sim}{0}$

E. If $L_1 > L_2$ then $T_1 > T_2$

Question 27

The diagram shows the graphs of $y = 4e^{-\frac{x}{2}}$ and $y = 3\sin\left(\frac{x}{2}\right)$



If the curves intersect at the point (b, c) and the shaded area is rotated about the x axis to form a volume of revolution, the volume is given by

A. $\pi\int_0^b \left(16e^{-x} - 9\sin^2\left(\frac{x}{2}\right)\right)dx$

B. $\pi\int_0^b \left(9\sin^2\left(\frac{x}{2}\right) - 16e^{-x}\right)dx$

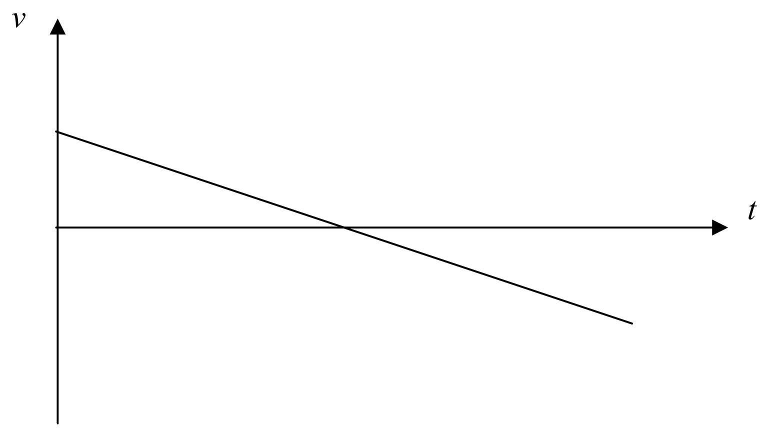
C. $\pi\int_0^b \left(4e^{-\frac{x}{2}} - 3\sin\left(\frac{x}{2}\right)\right)^2 dx$

D. $\pi\int_0^b \left(3\sin\left(\frac{x}{2}\right) - 4e^{-\frac{x}{2}}\right)^2 dx$

E. $\pi\int_0^b \left(16e^{-\frac{x^2}{4}} - 9\sin^2\left(\frac{x}{2}\right)\right)dx$

Question 28

Given the velocity time graph.



This could represent the motion of

A. a tennis ball as it falls from the second storey of a house and its rebound.

B. a tennis ball as it is thrown vertically upwards from ground level, and returning to the point of projection.

C. a train moving between two stations.

D. an express train as it passes one station slowing down to stop at the next station.

E. a car taking off from a set of lights approaching a freeway.

Question 29

A particle moves in a straight line so that at time t s, $t \geq 0$, its acceleration is a m/s^2, its velocity is v m/s and its displacement from a fixed point from an origin O on that line is x m.
If its initial velocity is v_0 m/s and the particle is initially at O and $\frac{dv}{dx} = 1$, then which of the following is **INCORRECT**?

A. $a = v$

B. $v = x + v_0$

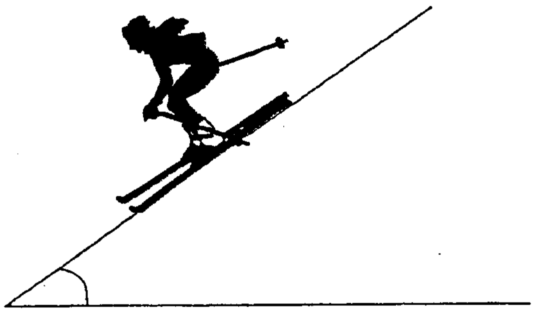
C. $v = v_0 e^t$

D. $v^2 = v_0^2 + 2vx$

E. $x = v_0\left(e^t - 1\right)$

Question 30

The diagram shows a skier travelling down an incline. The incline makes an angle of θ with the horizontal and the co-efficient of friction between the skier and the snow is μ. The skier has a mass of m kg and starts from rest. The skier moves down the incline a distance of D metres in a time of T seconds. At this time her speed is V m/s. Which one of the following statements is **INCORRECT**?



A. The acceleration of the skier is independent of her mass.

B. The acceleration of the skier is $g(\sin\theta - \mu\cos\theta)$

C. $V = \sqrt{2gD(\sin\theta - \mu\cos\theta)}$

D. $T = \dfrac{2D}{V}$

E. If the co-efficient of friction $\mu = 0$ the skier would approach a terminal or limiting velocity.

Instructions for Part II

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
Where an exact answer is required for a question, appropriate working must be shown.
Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$

Question 1

Find using calculus the exact value of $\int_0^4 \frac{\cos\sqrt{x}}{\sqrt{x}}dx$

2 marks

Question 2

A ball moves so that its position vector at a time t is given by

$$\underset{\sim}{r}(t) = (2t - \sin(2t))\underset{\sim}{i} + (1 - \cos(2t))\underset{\sim}{j} \qquad 0 \le t \le 2\pi$$

i. Sketch the path of the particle on the axes shown below. Label the axes.

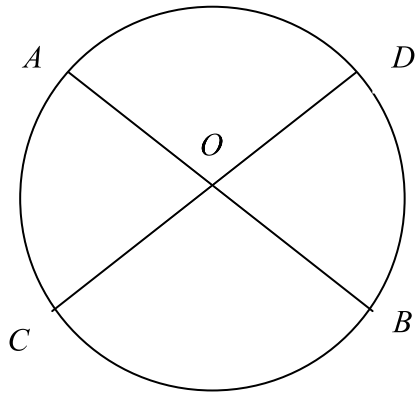
1 mark

ii. If the speed of the particle can be expressed in the form $c\sin t$, then find the value of c.

2 marks

Question 3

AB and *CD* are two diameters of a circle with centre *O*. Let $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OC} = \underset{\sim}{c}$



Prove that *ACBD* is a rectangle.

2 marks

Question 4

Let $u = 2\sqrt{2} - 2\sqrt{2}\,i$

i. Express u in polar form and find $\text{Arg}\left(u^8\right)$

1 mark

ii. Find u^2 giving the result in exact rectangular $x + yi$ form.

1 mark

iii. Hence or otherwise find all the solutions of $z^3 + 16zi = 0$ giving your answers in both exact rectangular and polar form.

2 marks

Question 5

Given the vector $\underset{\sim}{a} = -2\underset{\sim}{i} + y\underset{\sim}{j} + 5\underset{\sim}{k}$ find the exact value of the scalar y if

a. The length of the vector $\underset{\sim}{a}$ is 6

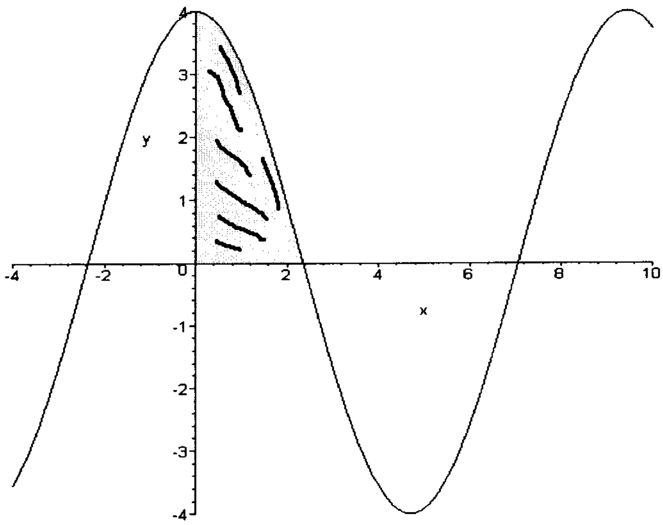
1 mark

b. The vector $\underset{\sim}{a}$ makes an angle of $\text{Cos}^{-1}\left(-\frac{\sqrt{7}}{6}\right)$ with the y axis.

2 marks

Question 6

The diagram shows the area bounded by the graph of $y = 4\cos\left(\frac{2x}{3}\right)$ and the co-ordinates axes.



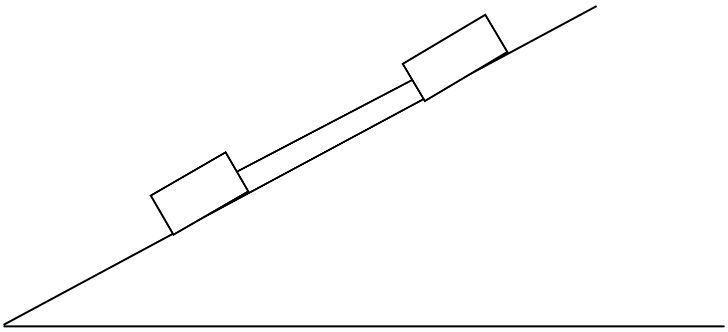
If this area is rotated about the x-axis, find, using calculus, the exact volume formed.

2 marks

Question 7

The diagram, shows two blocks of mass m_1 and m_2 kg with m_2 above m_1.
They are connected by a light rod and lie on a rough plane inclined at α to the horizontal. The coefficient of friction between the mass m_1 and the plane is μ_1 and the coefficient of friction between the mass m_2 and the plane is μ_2. The blocks are on the point of moving down the plane.

i. On the diagram mark in all the forces acting on the blocks.



1 mark

ii. By resolving the forces show that $\tan\alpha = \dfrac{\mu_1 m_1 + \mu_2 m_2}{m_1 + m_2}$

3 marks

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Question and Answer Book

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