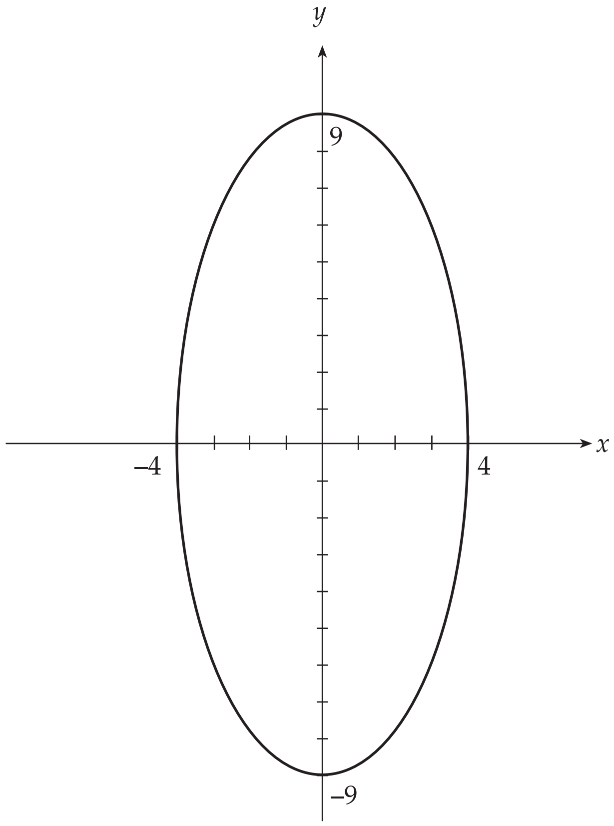
Part I: Multiple-choice questions

Question 1



The equation of the above ellipse is:

A $\frac{x^2}{2} + \frac{y^2}{3} = 1$

B $\frac{x^2}{4} + \frac{y^2}{9} = 1$

C $\frac{x^2}{16} + \frac{y^2}{81} = 1$

D $\frac{x^2}{4} - \frac{y^2}{9} = 1$

E $\frac{x^2}{16} - \frac{y^2}{81} = 1$

Question 2

The exact value of $\text{Sin}^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$ is:

A $-\frac{\pi}{3}$

B $-\frac{\pi}{6}$

C $\frac{\pi}{3}$

D $-\frac{2\pi}{3}$

E $\frac{7\pi}{6}$

Question 3

If $\cos(x) = -\frac{2}{3},\ \frac{\pi}{2} \le x \le \pi,$ then $\tan(x)$ is equal to:

A $\frac{\sqrt{5}}{2}$

B $-\frac{\sqrt{5}}{2}$

C $\frac{2}{\sqrt{5}}$

D $-\frac{2}{\sqrt{5}}$

E $\frac{\sqrt{13}}{2}$

Question 4

If $z^4 - 81 = 0$ then z is equal to:

A $\pm 8i$

B $\pm 3i$

C $\pm 3,\ \pm 3i$

D ± 3

E ± 9

Question 5

If $z = 2\text{cis}\left(\frac{5\pi}{6}\right)$ and $w = 5\text{cis}\left(\frac{\pi}{3}\right)$, then $\text{Arg}(zw)$ is equal to:

A $\frac{7\pi}{6}$

B $\frac{\pi}{6}$

C $\frac{5\pi}{18}$

D $-\frac{\pi}{6}$

E $-\frac{5\pi}{6}$

Question 6

If $z = 2 + 3i$ and $w = 2 - i$ then $\bar{z}w$ is:

A $7 + 4i$

B $4 + 3i$

C $4 - 3i$

D $1 - 8i$

E 7

Question 7

If $z = -\sqrt{3} + i$ then z^4 is equal to:

A $16\text{cis}\left(-\frac{2\pi}{3}\right)$

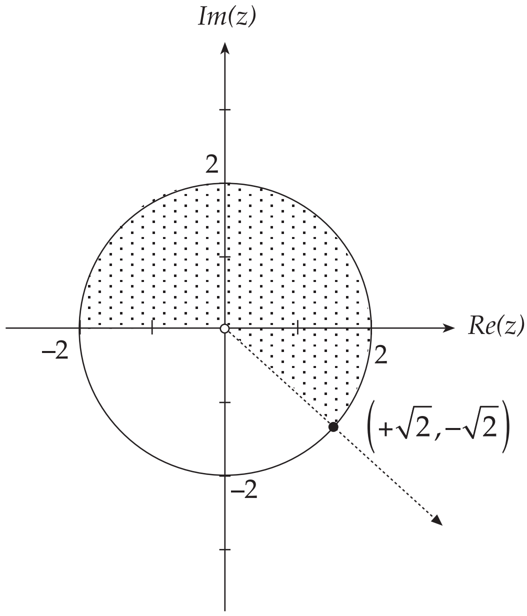
B $16\text{cis}\left(\frac{625\pi}{1296}\right)$

C $16\text{cis}\left(\frac{5\pi}{6}\right)$

D $10\text{cis}\left(\frac{5\pi}{6}\right)$

E $9 + i$

Question 8



Given that $z \in C$, the shaded region is best described by:

A $\{z : z\bar{z} \le 2\} \cup \{|z-2| < |z+2|\}$

B $\{z : |z| \le 2\} \cap \{|z-2i| \ge |z+2|\}$

C $\{z : z\bar{z} \le 2\} \cap \left\{\text{Arg}(z) \ge \frac{\pi}{4}\right\}$

D $\{z : |z| \le 2\} \cap \left\{\text{Arg}(z) \ge -\frac{\pi}{4}\right\}$

E $\{z : z\bar{z} > 2\} \cup \left\{\text{Arg}(z) < -\frac{\pi}{4}\right\}$

Question 9

If $\frac{3x-1}{x^2+x-6}$ is written as the partial fractions $\frac{A}{x+3}+\frac{B}{x-2}$ then,

A A = 1 and B = 2

B A = $3x$ and B = 0

C A = –2 and B = 3

D A = 2 and B = 1

E A = –2 and B = –1

Question 10

The derivative of $x\text{Tan}^{-1}(x)$ with respect to x is $\frac{x}{x^2+1}+\text{Tan}^{-1}(x)$. It follows that an anti-derivative of $\text{Tan}^{-1}(x)$ is:

A $\int \text{Tan}^{-1}(x)dx-\frac{x}{x^2+1}$

B $x\text{Tan}^{-1}(x)-\frac{1}{2}\log_e\left(x^2+1\right)$

C $x\text{Tan}^{-1}(x)+\int\frac{x}{x^2+1}dx$

D $x\text{Tan}^{-1}(x)+\log_e\left(x^2+1\right)$

E $\int x\text{Tan}^{-1}(x)dx-\int\frac{x}{x^2+1}dx$

Question 11

Using an appropriate substitution $\int_1^3 (2x+1)\sqrt{2x-1}\,dx$ is equal to:

A $\frac{1}{2}\int_1^5 (u+2)\sqrt{u}\,du$

B $\int_1^5 (u+2)\sqrt{u}\,du$

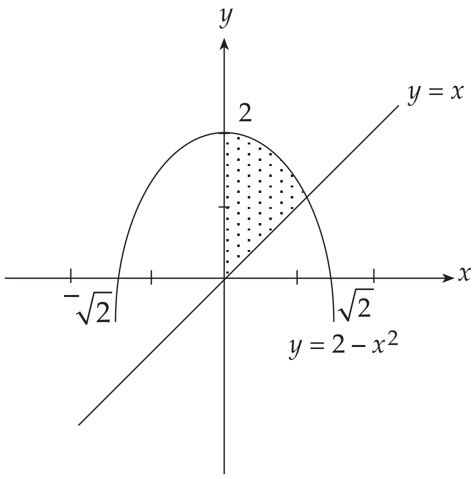
C $\int_1^3 (u+2)\sqrt{u}\,du$

D $\frac{1}{2}\int_1^3 (u+2)\sqrt{u}\,du$

E $\frac{1}{2}\int_1^3 (u-2)\sqrt{u}\,du$

Question 12

A volume of revolution is found by rotating the area enclosed by the line $y = x$, the parabola $y = 2 - x^2$ and the y-axis about the x-axis.



The volume of revolution formed is equivalent to:

A $\pi\int\left(\left(2-x^2\right)^2 - x^2\right)dx$

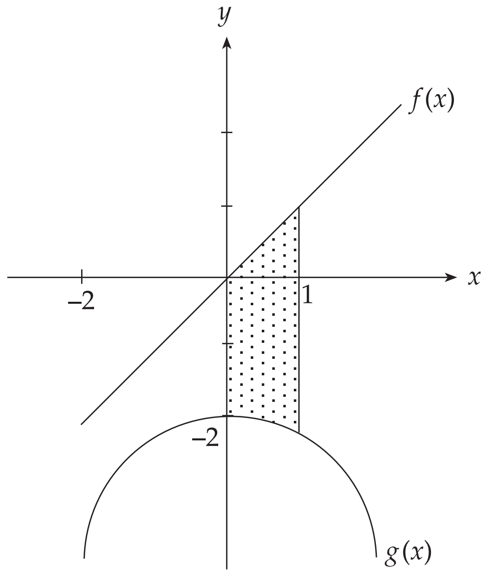
B $\pi\int_0^1\left(4+x^4-x^2\right)dx$

C $\pi\int_0^1\left(2-x^2\right)^2 dx$

D $\pi\int_1^2\left(2-x-x^2\right)dx$

E $\pi\int_0^1\left(\left(2-x^2\right)^2 - x^2\right)dx$

Question 13



Given $g(x) = \dfrac{-2}{\sqrt{4 - x^2}}$ and $f(x) = x$. The shaded region in the diagram is bounded by the y-axis, $g(x)$, $f(x)$ and the line $x = 1$.

The exact value of the area of the shaded region is:

A $\dfrac{1}{2} + \dfrac{\pi}{6}$

B $\dfrac{\pi}{6} - \dfrac{1}{2}$

C $\dfrac{\pi}{3} - \dfrac{1}{2}$

D $\dfrac{\pi}{12} + \dfrac{1}{2}$

E $\dfrac{1}{2} + \dfrac{\pi}{3}$

Question 14

The value of $\int_0^1 \left(e^{x^2} + e^{-x^2}\right)dx$, correct to four decimal places, is:

A 4.9626

B 3.8271

C 2.9253

D 2.9626

E 8.8126

Question 15

Using the midpoint rule and one interval, $\int_0^1 \cos(2x)dx$ would be approximated by:

A $\cos(2)$

B $\dfrac{\cos(0)+\cos(2)}{2}$

C $\cos(1)$

D $\dfrac{1}{2}\cos(2)$

E $\dfrac{1}{2}\cos(1)$

Question 16

A tank initially contains 100 litres of water in which is dissolved 20 kg of salt. A salt solution which contains 0.1 kg of salt per litre of water is poured into the tank at a rate of 2 litres/minute. An amount of the solution is simultaneously withdrawn from the tank at the same rate.

Assuming that the mixture is uniform and x kg is the amount of salt in the tank at t minutes, the differential equation that models this situation is:

A $\dfrac{dx}{dt} = 0.1 - x$

B $\dfrac{dx}{dt} = 0.2 - x$

C $\dfrac{dx}{dt} = (0.2 - 2x) \times \dfrac{20}{100}$

D $\dfrac{dx}{dt} = 0.2 - 2x$

E $\dfrac{dx}{dt} = 0.2 - \dfrac{x}{50}$

Question 17

The solution of the equation $\frac{d^2y}{dx^2} = \cos(2x)$ with $\frac{dy}{dx} = 0$ and $y = 0$ at $x = 0$ is:

A $y = \frac{1}{4}\cos(2x)$

B $y = -\frac{1}{4}\cos(2x) + \frac{1}{4}$

C $y = \frac{1}{4}\cos(2x) - \frac{1}{4}$

D $y = -\frac{1}{4}\cos(2x)$

E $y = -\frac{1}{4}\cos(2x) + \frac{x}{2} + \frac{1}{4}$

Question 18

The angle between $\underset{\sim}{a} = 2\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k}$ and $\underset{\sim}{b} = \underset{\sim}{i} + 3\underset{\sim}{j} - 2\underset{\sim}{k}$ is

A 90°

B 60°

C 120°

D 0°

E 30°

Question 19

A boat travels 9 kilometres due north, 6 kilometres in a direction 60° west of north and then 7 kilometres due east. If $\underset{\sim}{i}$ and $\underset{\sim}{j}$ are unit vectors in the directions east and north respectively, the position vector of the final position of the boat relative to its starting position is exactly:

A $(7 - 3\sqrt{3})\underset{\sim}{i} + 12\underset{\sim}{j}$

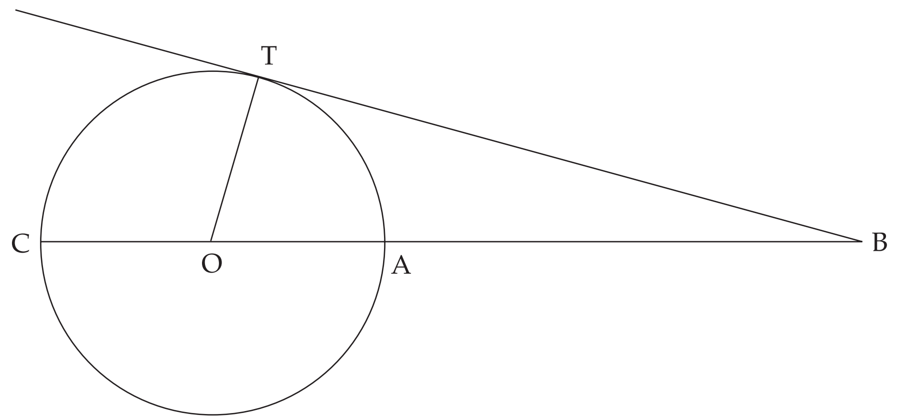
B $11\underset{\sim}{i} + 3(3 + 3\sqrt{3})\underset{\sim}{j}$

C $(4 + 3\sqrt{3})\underset{\sim}{i} + 12\underset{\sim}{j}$

D $3\underset{\sim}{k} + 3(3 - \sqrt{3})\underset{\sim}{j}$

E $15\underset{\sim}{i} + 13\underset{\sim}{j}$

Question 20



O is the centre of the circle and BT is a tangent.

Let $\overrightarrow{OT} = \underset{\sim}{c}$ and $\overrightarrow{OB} = \underset{\sim}{b}$.

The dot product $\overrightarrow{OT}.\overrightarrow{TB}$ is equal to:

A 90

B $\sqrt{c^2 + b^2}$

C 0

D $\underset{\sim}{c}.\underset{\sim}{b}$

E $\underset{\sim}{c}.(\underset{\sim}{c} + \underset{\sim}{b})$

Question 21

The magnitude of $4\underset{\sim}{a} + \underset{\sim}{b}$ if $\underset{\sim}{a} = 2\underset{\sim}{i} + \underset{\sim}{j} + 5\underset{\sim}{k}$ and $\underset{\sim}{b} = -4\underset{\sim}{i} + \underset{\sim}{j}$ is:

A 6

B $2\sqrt{11}$

C $\sqrt{569}$

D $4\sqrt{30}$

E 21

Question 22

Given $\underset{\sim}{a} = 6\underset{\sim}{i} - 2\underset{\sim}{j} + 6\underset{\sim}{k}$ and $\underset{\sim}{b} = -6\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ the vector resolute of $\underset{\sim}{b}$ in the direction of $\underset{\sim}{a}$ is equal to:

A $-\dfrac{13}{\sqrt{19}}\left(6\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}\right)$

B $-\dfrac{13}{\sqrt{19}}\left(-6\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}\right)$

C $\dfrac{1}{2\sqrt{19}}\left(6\underset{\sim}{i} - 2\underset{\sim}{j} + 6\underset{\sim}{k}\right)$

D $-\dfrac{13}{\sqrt{19}}$

E $-\dfrac{13}{19}\left(3\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k}\right)$

Question 23

The position of a body at time t, $t \geq 0$, is given by $\underset{\sim}{r}(t) = (t-1)\underset{\sim}{i} + 5(t-1)^2\underset{\sim}{j}$. The Cartesian equation of the body's path is represented by:

A $y = 5x^2, x \geq 0$

B $y = 5x^2, x \geq -1$

C $x = \pm\sqrt{5y}, y \geq 0$

D $x = \sqrt{5y}, y \leq -1$

E $y = \sqrt{5x}, x \geq -1$

Question 24

The position of a particle is given by: $\underset{\sim}{r}(t) = (2\sin 2t + 1)\underset{\sim}{i} + 3e^t\underset{\sim}{j}$.
The speed of the particle at $t = 0$ is:

A $4\underset{\sim}{i} + 3\underset{\sim}{j}$

B 25

C $4\cos 2t\,\underset{\sim}{i} + 3e^t\underset{\sim}{j}$

D 5

E $\underset{\sim}{i} + 3\underset{\sim}{j}$

Question 25

If a particle of mass 4 kg is acted on by two forces $\underset{\sim}{F}_1 = 2\underset{\sim}{i} + \underset{\sim}{j}$ and $\underset{\sim}{F}_2 = 8\underset{\sim}{i} - 5\underset{\sim}{k}$, the acceleration of the particle is:

A $10\underset{\sim}{i} + \underset{\sim}{j} - 5\underset{\sim}{k}$

B $6\underset{\sim}{i} - \underset{\sim}{j} - 5\underset{\sim}{k}$

C $2.5\underset{\sim}{i} + 0.25\underset{\sim}{j} - 1.25\underset{\sim}{k}$

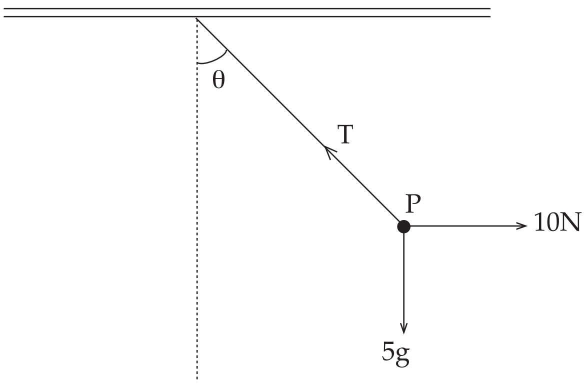
D 2.806

E $\sqrt{126}$

Question 26

A light inelastic string suspended from a horizontal beam has a particle, P, of mass 5 kg attached to it. A horizontal force of 10 N is applied to the particle. This means that the string is θ radians from the vertical.

θ is approximately:



A 11.5346

B 0.2013

C 1.3695

D 78.4654

E 0.4636

Question 27

A particle moves in a straight line with acceleration of $a = x + 6$ at time t. Given that at $x = 1$, $v = 7$, the velocity of the particle is:

A $\pm(x + 6)$

B $x^2 + 12x$

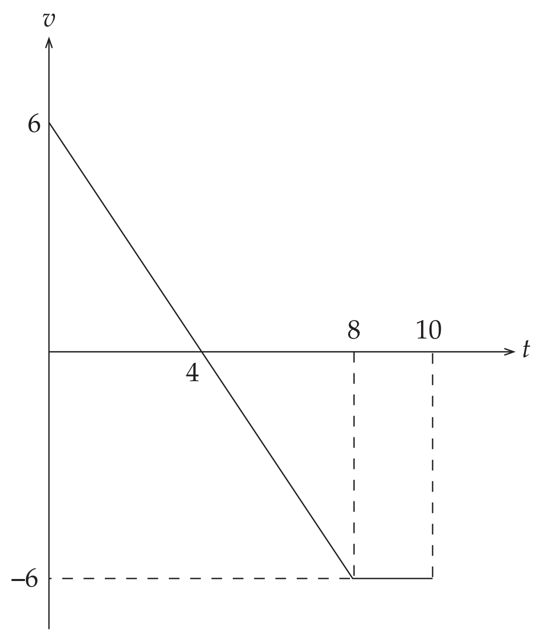
C $x + 6$

D $x^2 + 12x + 36$

E 49

Question 28

The following is the velocity-time graph of a particle over a 10 second interval.



The total distance travelled during the 10 second interval is:

A –12

B –24

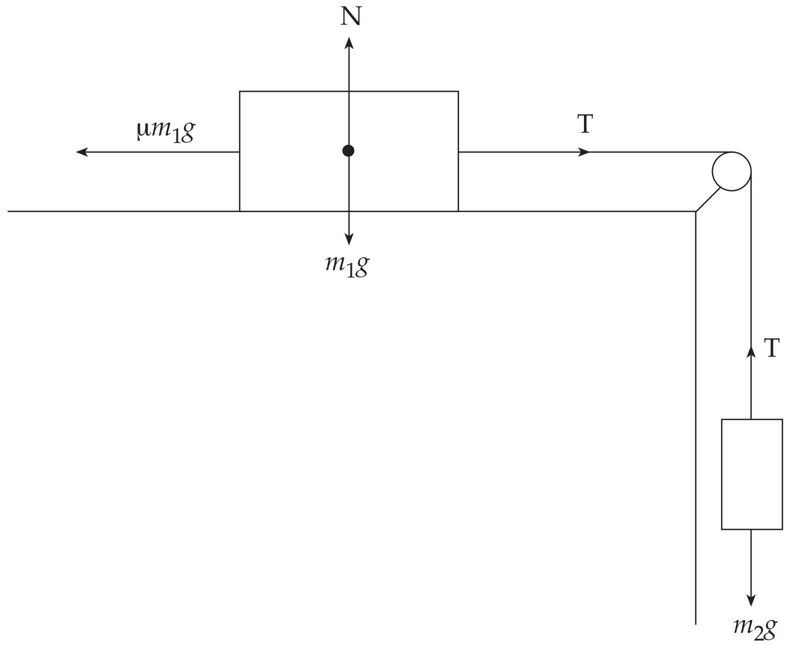
C 12

D 36

E 60

Question 29

A block of mass m_1 rests on a rough horizontal table. The coefficient of friction between the block and the table is μ. The block is connected to a second block of mass m_2 by a light inelastic string which passes over a smooth pulley. The mass m_2 hangs vertically.



If the block of mass m_1 is accelerating to the right, then,

A $T - \mu m_1 g > m_2 g$

B $T > m_2 g$

C $m_2 < \mu m_1$

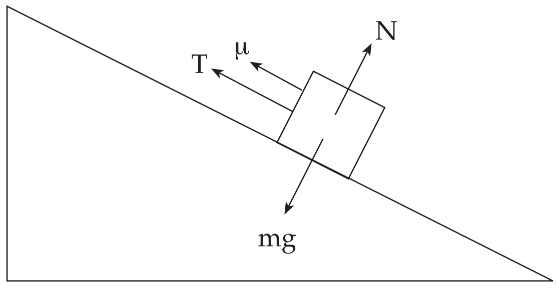
D $m_2 > \mu m_1$

E $m_1 g = m_2 g$

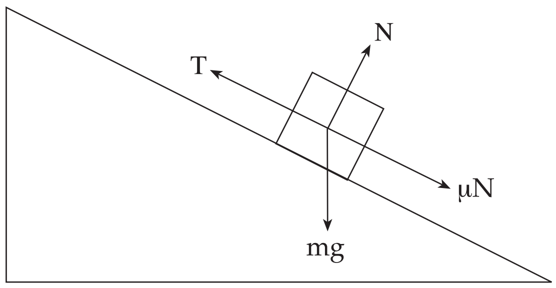
Question 30

Terry is dragging a large box of mass *m* up a ramp. The force she applies to the box is T newtons. The coefficient of friction between the box and the ramp is μ. The normal force exerted by the ramp on the box is N newtons. The diagram which correctly shows all the forces acting on the box is:

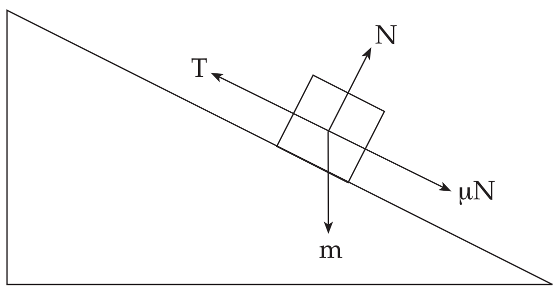
A



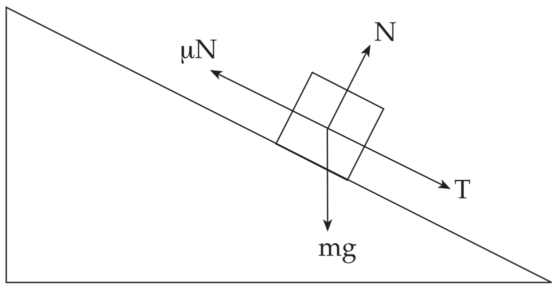
B



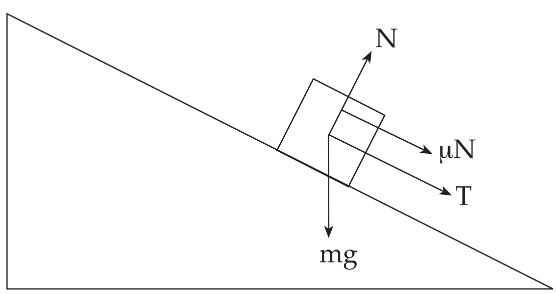
C



D



E



Part II: Short answer questions

Question 1

Let $f: R \to R$ where $f(x) = \dfrac{-1}{1+x^2}$.

a **i** Use calculus to find $f'(x)$.

ii Find $f'(1)$.

(2 + 1 = 3 marks)

b Hence, use Euler's method to estimate $f(1.01)$.

(1 mark)

Total = 4 marks

Question 2

Use calculus to find the exact value of $\int_1^{\sqrt{2}} \frac{1}{\sqrt{4-x^2}}\,dx$.

Total = 2 marks

Question 3

A particle moves so that its position vector at time t is given by $\underset{\sim}{r}(t) = 4\cos t\,\underset{\sim}{i} + 4\sin t\,\underset{\sim}{j} + 3t\,\underset{\sim}{k}$, with $t \geq 0$.

a Find its initial speed.

(3 marks)

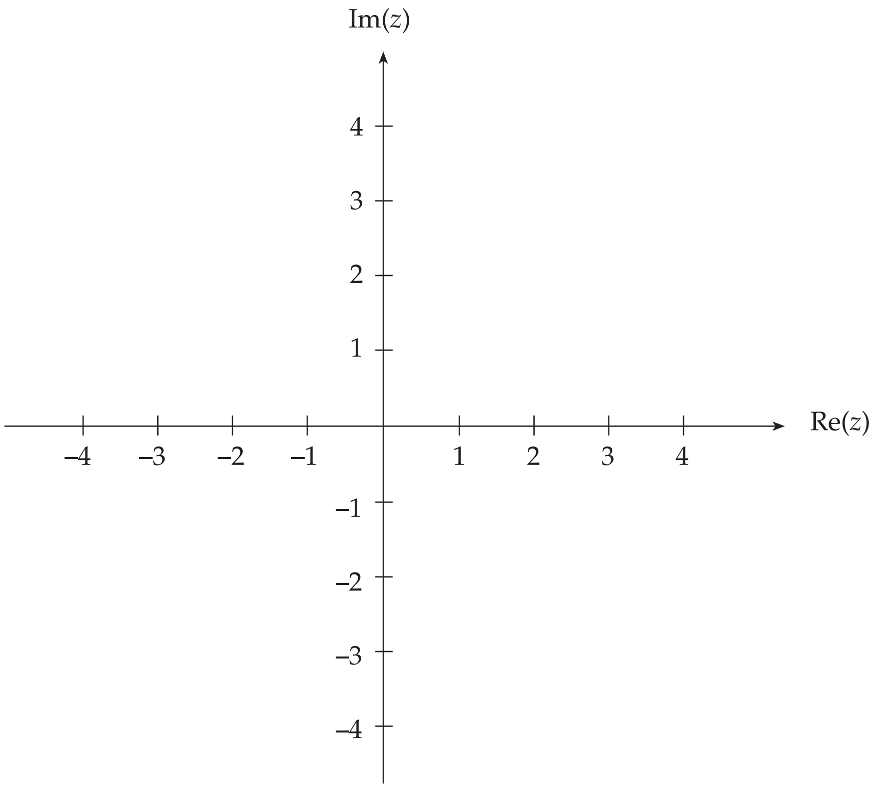
b Show that the velocity of the particle is always perpendicular to its acceleration.

(3 marks)

Total = 6 marks

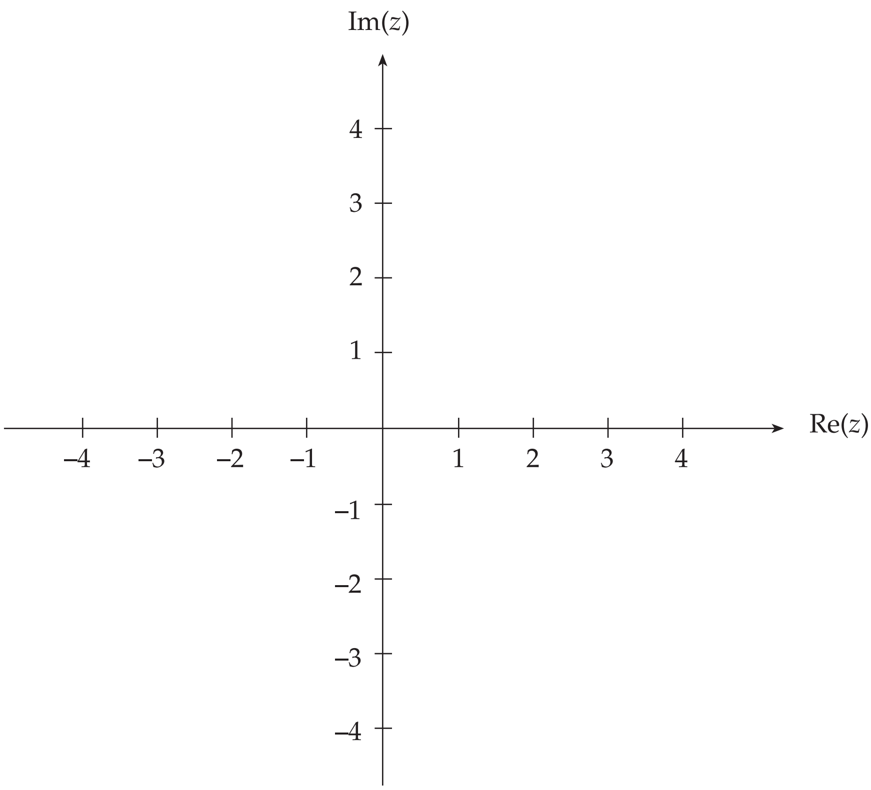
Question 4

a Shade the region of the complex plane specified by $\{z : |z - 2 + i| \le 2\}$.



(2 marks)

b Shade the region of the complex plane specified by $\left\{z : 0 \le Arg(z+1) \le \frac{\pi}{4}\right\}$.



(2 marks)

Total = 4 marks

Question 5

Consider the expression $(\cos\theta + i\sin\theta)^3$ where θ is a real number.

a Use the expression to show that $\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$.

(3 marks)

b Hence, or otherwise, find a similar expression for $\sin 3\theta$.

(1 mark)

Total = 4 marks