Year 2005

VCE

Specialist Mathematics

Trial Examination 1

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VICTORIAN CERTIFICATE OF EDUCATION
2005

SPECIALIST MATHEMATICS

Trial Written Examination 1
(Facts, skills and applications)

Reading time: 15 minutes
Total writing time: 1 hour 30 minutes

PART I

MULTIPLE-CHOICE QUESTION BOOK

This examination has two parts: Part I (multiple-choice questions) and Part II (short-answer questions).
Part I consists of this question book and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of a separate question and answer book.
You must complete both parts in the time allotted.
When you have completed one part continue immediately to the other part.

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
30	30	30

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and an approved scientific and/or graphics calculator (memory may be retained).
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question book of 22 pages, with a detachable sheet of miscellaneous formulas in the centrefold and two blank pages for rough working.
- Answer sheet for multiple-choice questions.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Check that your name and student number as printed on your answer sheet for multiple-choice questions are correct, and sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are not drawn to scale.

At the end of the examination
- Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer book (Part II).
- You may retain this question book.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of triangle: $\frac{1}{2}bc\sin(A)$

sine rule: $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$

cosine rule: $c^2 = a^2 + b^2 - 2ab\cos(C)$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$

$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$

$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$

$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$

$\tan(2x) = \dfrac{2\tan(x)}{1-\tan^2(x)}$

function	Sin^{-1}	Cos^{-1}	Tan^{-1}
domain	$[-1,1]$	$[-1,1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0,\pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex Numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\,\text{cis}\,\theta$

$|z| = \sqrt{x^2+y^2} = r$

$-\pi < Arg\,z \le \pi$

$z_1 z_2 = r_1 r_2\,\text{cis}(\theta_1+\theta_2)$

$\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\,\text{cis}(\theta_1-\theta_2)$

$z^n = r^n\text{cis}(n\theta)$ (de Moivre's theorem)

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2+y^2+z^2} = r$

$\underset{\sim}{r_1}.\underset{\sim}{r_2} = r_1 r_2\cos\theta = x_1x_2 + y_1y_2 + z_1z_2$

$\dot{\underset{\sim}{r}} = \dfrac{d\underset{\sim}{r}}{dt} = \dfrac{dx}{dt}\underset{\sim}{i} + \dfrac{dy}{dt}\underset{\sim}{j} + \dfrac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

sliding friction: $F \le \mu N$

constant (uniform) acceleration:

$s = ut + \frac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \frac{1}{2}(u+v)t$

acceleration: $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\frac{1}{2}v^2\right)$

Calculus

$$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$$

$$\frac{d}{dx}\left(\log_e\left(x\right)\right)=\frac{1}{x}$$

$$\frac{d}{dx}\left(\sin\left(ax\right)\right)=a\cos\left(ax\right)$$

$$\frac{d}{dx}\left(\cos\left(ax\right)\right)=-a\sin\left(ax\right)$$

$$\frac{d}{dx}\left(\tan\left(ax\right)\right)=a\sec^2\left(ax\right)$$

$$\frac{d}{dx}\left(\text{Sin}^{-1}\left(x\right)\right)=\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\text{Cos}^{-1}\left(x\right)\right)=\frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\text{Tan}^{-1}\left(x\right)\right)=\frac{1}{1+x^2}$$

$$\int x^n dx=\frac{1}{n+1}x^{n+1}+c\ ,\ n\neq -1$$

$$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$$

$$\int \frac{1}{x}\,dx=\log_e\left(x\right)+c,\ for\ x>0$$

$$\int \sin\left(ax\right)dx=-\frac{1}{a}\cos\left(ax\right)+c$$

$$\int \cos\left(ax\right)dx=\frac{1}{a}\sin\left(ax\right)+c$$

$$\int \sec^2\left(ax\right)dx=\frac{1}{a}\tan\left(ax\right)+c$$

$$\int \frac{dx}{\sqrt{a^2-x^2}}=\text{Sin}^{-1}\left(\frac{x}{a}\right)+c,\ a>0$$

$$\int \frac{-1}{\sqrt{a^2-x^2}}\,dx=\text{Cos}^{-1}\left(\frac{x}{a}\right)+c,\ a>0$$

$$\int \frac{a}{a^2+x^2}dx=\text{Tan}^{-1}\left(\frac{x}{a}\right)+c$$

product rule: $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

quotient rule: $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

chain rule: $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

mid-point rule: $\int_a^b f(x)dx \approx (b-a)f\left(\dfrac{a+b}{2}\right)$

trapezoidal rule: $\int_a^b f(x)dx \approx \dfrac{1}{2}(b-a)(f(a) + f(b))$

Euler's method

If $\dfrac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x)$

Take the acceleration due to gravity to have magnitude g m/s^2, where $g = 9.8$

VCE SPECIALIST MATHEMATICS 2005
Trial Written Examination 1
ANSWER SHEET

NAME: ____

STUDENT NUMBER ____

SIGNATURE____

Instructions

- Write your name in the space provided above.
- Write your student number in the space provided above. Sign your name.
- Use a **PENCIL** for **ALL** entries.
  If you make a mistake, **ERASE** it - **DO NOT** cross it out.
- Marks will **NOT** be deducted for incorrect answers.
- **NO MARK** will be given if more than **ONE** answer is completed for any question.
- All answers must be completed like **THIS** example.

A	B	C	D	E

1	A	B	C	D	E	**16**	A	B	C	D	E
2	A	B	C	D	E	**17**	A	B	C	D	E
3	A	B	C	D	E	**18**	A	B	C	D	E
4	A	B	C	D	E	**19**	A	B	C	D	E
5	A	B	C	D	E	**20**	A	B	C	D	E
6	A	B	C	D	E	**21**	A	B	C	D	E
7	A	B	C	D	E	**22**	A	B	C	D	E
8	A	B	C	D	E	**23**	A	B	C	D	E
9	A	B	C	D	E	**24**	A	B	C	D	E
10	A	B	C	D	E	**25**	A	B	C	D	E
11	A	B	C	D	E	**26**	A	B	C	D	E
12	A	B	C	D	E	**27**	A	B	C	D	E
13	A	B	C	D	E	**28**	A	B	C	D	E
14	A	B	C	D	E	**29**	A	B	C	D	E
15	A	B	C	D	E	**30**	A	B	C	D	E

Please DO NOT fold, bend or staple this form

Instructions for Part I

Answer **all** questions, in pencil, on the answer sheet provided for multiple-choice questions.
A correct answer scores 1 mark, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No mark will be given for a question if more than one answer is completed for any question.

Question 1

If a is a positive real number, then the graph of $y = \dfrac{x^3 + a^3}{2x}$ has

A. no straight line asymptotes.

B. $x = 0$ as the only asymptote.

C. $y = 2x$ as its only asymptote.

D. $x = 0$ and $x = -a$ as its straight line asymptotes.

E. $x = 0$ and $y = \dfrac{x^2}{2}$ as its asymptotes.

Question 2

The graph shown could have the equation

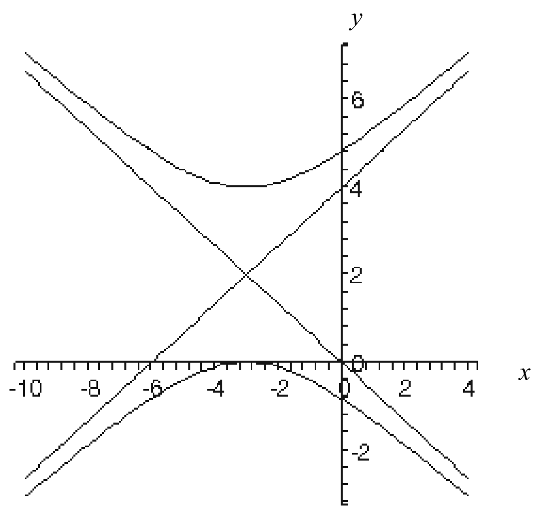
A. $\dfrac{(x+3)^2}{9} - \dfrac{(y-2)^2}{4} = 1$

B. $\dfrac{(y-2)^2}{4} - \dfrac{(x+3)^2}{9} = 1$

C. $\dfrac{(y-2)^2}{9} - \dfrac{(x+3)^2}{4} = 1$

D. $\dfrac{(y+2)^2}{9} - \dfrac{(x+3)^2}{4} = 1$

E. $\dfrac{(x-3)^2}{9} - \dfrac{(y-2)^2}{4} = 1$



Question 3

Let $f:\left(0,\frac{\pi}{2}\right)\to R$ where $f(x)=\operatorname{cosec}(x)-\cot(x)$

Which one of the following statements is true?

A. f has range $[0,1]$ and is identical to the function

$h:\left(0,\frac{\pi}{2}\right)\to R$ where $h(x)=\tan\left(\frac{x}{2}\right)$

B. f has range $[0,1]$ and is identical to the function

$h:\left(0,\frac{\pi}{2}\right)\to R$ where $h(x)=\cot\left(\frac{x}{2}\right)$

C. f has range $(0,1)$ and is identical to the function

$h:\left(0,\frac{\pi}{2}\right)\to R$ where $h(x)=\tan\left(\frac{x}{2}\right)$

D. f has range $(0,1)$ and is identical to the function

$h:\left(0,\frac{\pi}{2}\right)\to R$ where $h(x)=\cot\left(\frac{x}{2}\right)$

E. f has range $[1,\infty)$ and is identical to the function

$h:\left(0,\frac{\pi}{2}\right)\to R$ where $h(x)=\cot\left(\frac{x}{2}\right)$

Question 4

If a and b are positive real numbers, then the implied (maximal) domain of the function with rule $f(x)=\operatorname{Sin}^{-1}\left(\frac{b}{ax}\right)$ is

A. $x\neq 0$

B. $\left[\frac{-a}{b},\frac{a}{b}\right]$

C. $\left[\frac{-b}{a},\frac{b}{a}\right]$

D. $\left(-\infty,-\frac{b}{a}\right]\cup\left[\frac{b}{a},\infty\right)$

E. $\left(-\infty,-\frac{a}{b}\right]\cup\left[\frac{a}{b},\infty\right)$

Question 5

If $z = x + yi$ where x and y are non-zero real numbers, which of the following statements is **false?**

A. $|\overline{z}| = |z|$

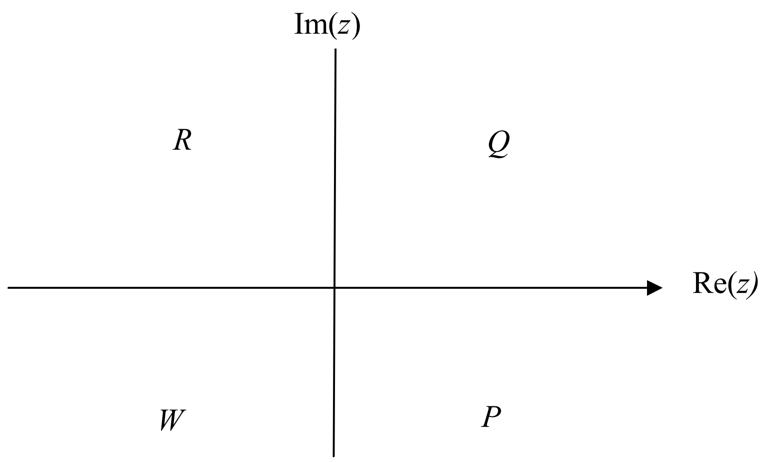
B. $\dfrac{1}{\overline{z}} = \dfrac{z}{|z|^2}$

C. $\dfrac{1}{z} + \dfrac{1}{\overline{z}} = \dfrac{z + \overline{z}}{|z|^2}$

D. $\text{Re}(z) = \dfrac{1}{2}(z + \overline{z})$

E. $\text{Im}(z) = \dfrac{i}{2}(z - \overline{z})$

Question 6



The point W on the Argand diagram above represents a complex number w where $\text{Arg}(w) = -\dfrac{3\pi}{4}$. The complex number $i\,\overline{w}$ is best represented by the point

A. W

B. P

C. Q

D. R

E. None of the above.

Question 7

If $P(z) = z^2 + bz + c$ and $P(\alpha + i\beta) = 0$ where b , c, α and β are all real non-zero numbers, then which of the following statements is **false**?

A. $P(\alpha - i\beta) = 0$

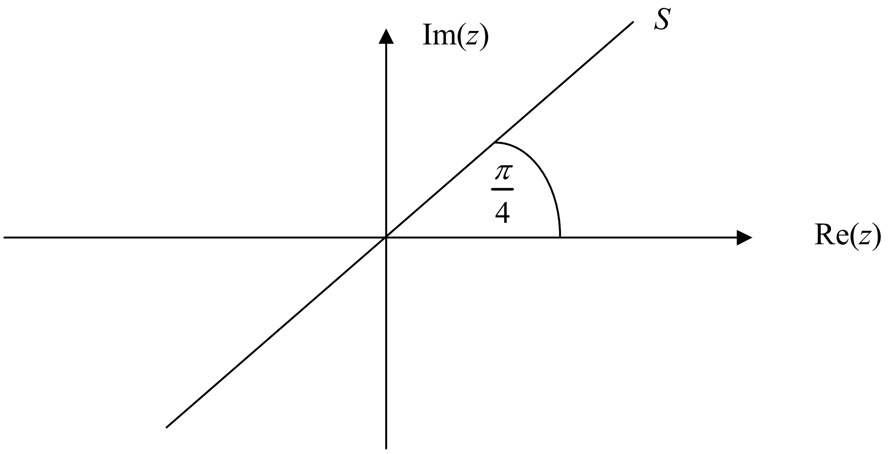
B. $b^2 < 4c$

C. $\alpha^2 + \beta^2 = c$

D. $b - 2\alpha = 0$

E. $P(z)$ has one pair of complex conjugates as its roots.

Question 8



Which of the following does **NOT** describe the line S shown in the above Argand diagram?

A. $\{z : \text{Arg}(z) = \frac{\pi}{4}\} \cup \{z : \text{Arg}(z) = -\frac{3\pi}{4}\}$

B. $\{z : \text{Re}(z) = \text{Im}(z)\}$

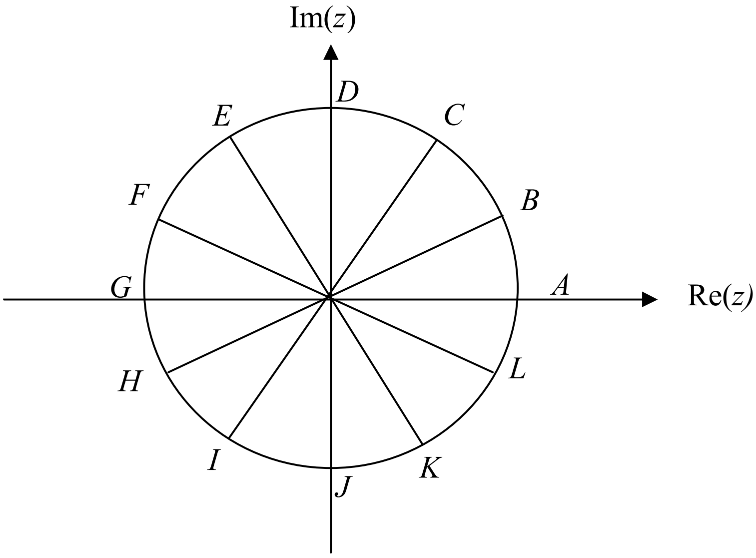
C. $\{z : |z - 2| = |z - 2i|\}$

D. $\{z : |z + 3| = |z + 3i|\}$

E. $\{z : |z + 1 - i| = |z - 1 + i|\}$

Question 9

The diagram shows a circle of radius a on an Argand diagram.



The roots of the equation $z^3 + a^3 = 0$ where a is a positive real number, are given by

A. G only.

B. A only.

C. G, C and K.

D. A, E and I.

E. D, H and L.

Question 10

Which of the following is an antiderivative of $\sec^2(3x)\tan^2(3x)$?

A. $3\tan^3(3x)$

B. $\tan^3(3x)$

C. $\frac{1}{3}\tan^3(3x)$

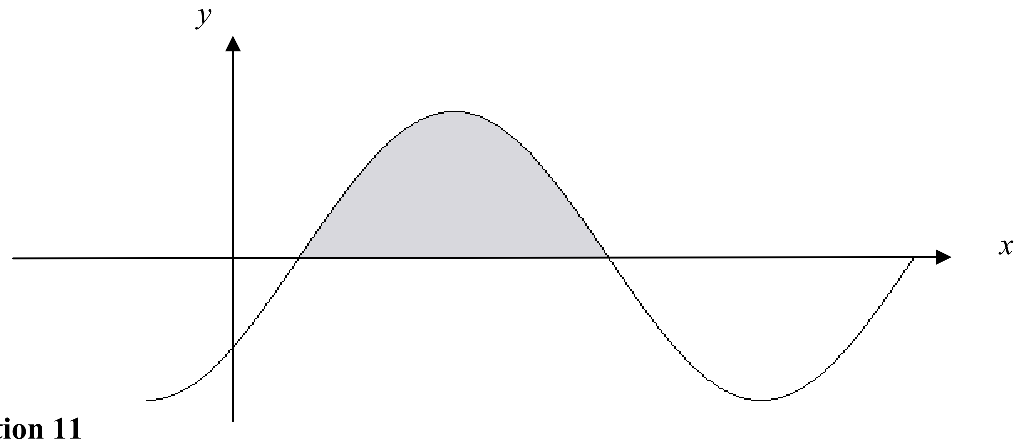
D. $\frac{1}{9}\tan^3(3x)$

E. $\frac{1}{3}\sec^3(3x)$

The following information relates to Questions 11 and 12.

The shaded area below is the area under one arch of the curve $y = a\sin(px)$.

It is the area bounded by the curve $y = a\sin(px)$ the x-axis and the first x-intercept, where a and p are positive real numbers.



Question 11

The shaded area is equal to

A. $\dfrac{2a}{p}$

B. $\dfrac{a}{p}$

C. $2\,a\,p$

D. $\dfrac{\pi a}{p}$

E. $a\,p$

Question 12

If the shaded area is rotated about the x-axis, then the volume formed is given by

A. $\dfrac{\pi a^2}{p}$

B. $\dfrac{\pi a^2}{2p}$

C. $\dfrac{4\pi a^2}{p^2}$

D. $\dfrac{\pi^2 a^2}{p}$

E. $\dfrac{\pi^2 a^2}{2p}$

Question 13

The exact equation of the normal to the curve $y = \text{Sin}^{-1}\left(\frac{x}{2}\right)$, at the point where $x = 1$, is

A. $y = 0.577x - 0.054$

B. $y = \frac{\sqrt{3}x}{3} - \frac{\sqrt{3}}{3} + \frac{\pi}{6}$

C. $y = -\sqrt{3}\,x + \sqrt{3} + \frac{\pi}{6}$

D. $y = -\sqrt{3}\,x + \sqrt{3} + \frac{\pi}{3}$

E. $y = \frac{\sqrt{3}\,x}{3} + \frac{\pi - \sqrt{3}}{3}$

Question 14

Using an appropriate substitution, $\int_0^2 \frac{x\left(2x^2+3\right)}{\sqrt{2x^2+1}}\,dx$ is equal to

A. $\frac{1}{4}\int_0^2 \frac{u+2}{\sqrt{u}}\,du$

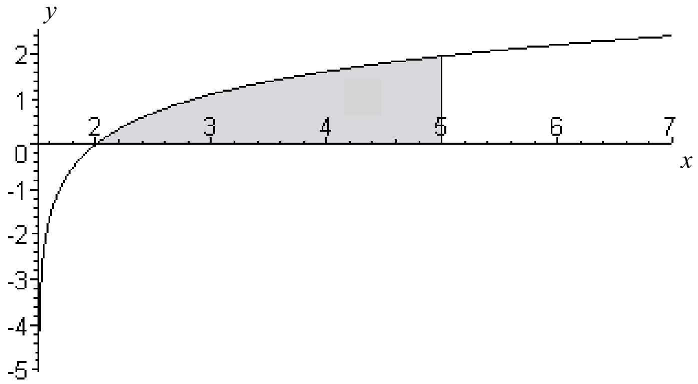
B. $\frac{1}{4}\int_0^2 \frac{u}{\sqrt{u-2}}\,du$

C. $\frac{1}{4}\int_1^9 \frac{u+2}{\sqrt{u}}\,du$

D. $\frac{1}{4}\int_1^9 \frac{u+2\sqrt{u-1}}{\sqrt{2u}}\,du$

E. $4\int_1^9 \frac{u+2}{\sqrt{u}}\,du$

Question 15



The shaded region in the diagram above is the area bounded by the graph of $y = \log_e(2x-3)$, the x-axis, and the line with the equation $x = 5$. The mid-point rule with three equal subintervals is used to estimate the area of the shaded region.

The value obtained is $\log_e(a)$, where a is equal to

A. 48

B. 15

C. 105

D. 24

E. 8

Question 16

Euler's method, with a step size of 0.25, is used to solve the differential equation

$\frac{dy}{dx} = \log_e(2x-3)$, with initial condition $y = 1$ when $x = 2$.

The approximation obtained for y when $x = 3$ is given by

A. $\frac{3}{2}\log_e(3) - 1$

B. $1 + \frac{1}{4}\log_e\left(\frac{3}{2}\right)$

C. $1 + \frac{1}{4}\log_e(3)$

D. $1 + \frac{1}{4}\log_e\left(\frac{15}{2}\right)$

E. $3\log_e(3)$

Question 17

If $f'(x) = 6\cos^2\left(\frac{x}{4}\right)\sin\left(\frac{x}{4}\right)$ and $f(0) = 0$, then $f(x)$ is equal to

A. $3\cos^3\left(\frac{x}{4}\right) - 3$

B. $8\cos^3\left(\frac{x}{4}\right) - 8$

C. $8 - 8\cos^3\left(\frac{x}{4}\right)$

D. $12 - 12\cos^3\left(\frac{x}{4}\right)$

E. $\frac{1}{2} - \frac{1}{2}\cos^3\left(\frac{x}{4}\right)$

Question 18

The diagram, show the two functions : $y = 4\cos^2(2x)$ and $y = 4\sin^2(2x)$.

The curves intersect at $x = -\frac{3\pi}{8}, -\frac{\pi}{8}, \frac{\pi}{8}$ and $\frac{3\pi}{8}$. The **total** shaded area is equal to

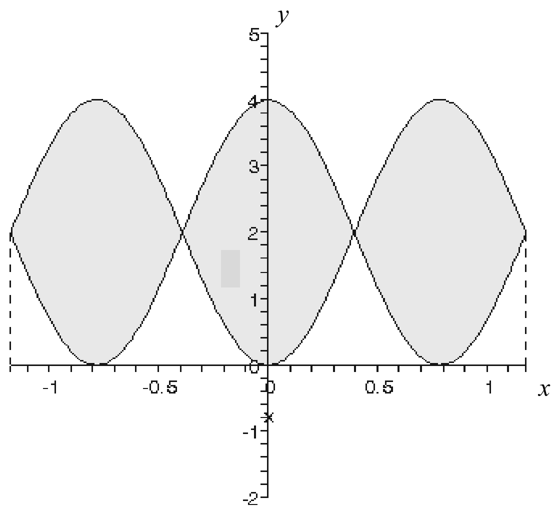
A. $4\int_{-\frac{3\pi}{8}}^{\frac{3\pi}{8}} \cos(4x)\,dx$

B. $24\int_{0}^{\frac{\pi}{8}} \cos(4x)\,dx$

C. $8\int_{0}^{\frac{3\pi}{8}} \cos(4x)\,dx$

D. $3\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \cos(4x)\,dx$

E. $12\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \cos(4x)\,dx$



Question 19

A vector of magnitude 26 in the opposite direction to $4\underset{\sim}{i} - 3\underset{\sim}{j} + 12\underset{\sim}{k}$ is

A. $26\left(4\underset{\sim}{i} - 3\underset{\sim}{j} + 12\underset{\sim}{k}\right)$

B. $26\left(-4\underset{\sim}{i} + 3\underset{\sim}{j} - 12\underset{\sim}{k}\right)$

C. $2\left(4\underset{\sim}{i} - 3\underset{\sim}{j} + 12\underset{\sim}{k}\right)$

D. $2\left(-4\underset{\sim}{i} + 3\underset{\sim}{j} - 12\underset{\sim}{k}\right)$

E. $\frac{1}{\sqrt{26}}\left(4\underset{\sim}{i} - 3\underset{\sim}{j} + 12\underset{\sim}{k}\right)$

Question 20

The scalar resolute of $\underset{\sim}{p}$ in the direction of $-\underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k}$ is -3. The vector resolute of $\underset{\sim}{p}$ perpendicular $-\underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k}$ is $2\underset{\sim}{i} - \underset{\sim}{k}$. The vector $\underset{\sim}{p}$ is

A. $\underset{\sim}{i} + 2\underset{\sim}{j} - 3\underset{\sim}{k}$

B. $3\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$

C. $-3\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$

D. $5\underset{\sim}{i} - 6\underset{\sim}{j} + 5\underset{\sim}{k}$

E. $-\underset{\sim}{i} + 6\underset{\sim}{j} - 7\underset{\sim}{k}$

Question 21

A particle moves so that its position vector at a time *t* is given by

$\underset{\sim}{r}(t) = 2\text{Cos}^{-1}(3t)\underset{\sim}{i} + 6\sin(2t)\underset{\sim}{j} + 3\underset{\sim}{k}$

The initial direction of the motion of the particle is

A. $2\underset{\sim}{i} + 3\underset{\sim}{k}$

B. $-6\underset{\sim}{i} - 12\underset{\sim}{j}$

C. $6\underset{\sim}{i} + 12\underset{\sim}{j}$

D. $-6\underset{\sim}{i} + 12\underset{\sim}{j}$

E. $-2\underset{\sim}{i} + 12\underset{\sim}{j}$

Question 22

OABC is a square with $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OC} = \underset{\sim}{c}$

Which of the following statements is **false** ?

A. $\underset{\sim}{a}.\underset{\sim}{c} = 0$

B. $|\underset{\sim}{a}| = |\underset{\sim}{c}|$

C. $|\underset{\sim}{a}|^2 + |\underset{\sim}{c}|^2 = |\underset{\sim}{a} + \underset{\sim}{c}|^2$

D. $\dfrac{\underset{\sim}{a}.(\underset{\sim}{a} + \underset{\sim}{c})}{|\underset{\sim}{a}||\underset{\sim}{a} + \underset{\sim}{c}|} = \dfrac{\sqrt{2}}{2}$

E. Since $\underset{\sim}{a}.(\underset{\sim}{a} + \underset{\sim}{c}) = \underset{\sim}{c}.(\underset{\sim}{a} + \underset{\sim}{c})$ it follows that $\underset{\sim}{a} = \underset{\sim}{c}$

Question 23

A ship leaves a port and travels a distance of s_1 kilometres on a bearing Nθ_1 °W and a second ship leaves the same port at the same time and travels a distance of s_2 kilometres on a bearing S θ_2 °W. If $\underset{\sim}{i}$ and $\underset{\sim}{j}$ are unit vectors of one kilometre in the east and north directions respectively, then the displacement of the second ship from the first is given by

A. $(s_1 \sin\theta_1 - s_2 \sin\theta_2)\underset{\sim}{i} - (s_1 \cos\theta_1 + s_2 \cos\theta_2)\underset{\sim}{j}$

B. $(-s_1 \sin\theta_1 + s_2 \sin\theta_2)\underset{\sim}{i} + (s_1 \cos\theta_1 + s_2 \cos\theta_2)\underset{\sim}{j}$

C. $(s_1 \cos\theta_1 - s_2 \cos\theta_2)\underset{\sim}{i} - (s_1 \sin\theta_1 + s_2 \sin\theta_2)\underset{\sim}{j}$

D. $(-s_1 \cos\theta_1 + s_2 \cos\theta_2)\underset{\sim}{i} + (s_1 \sin\theta_1 - s_2 \sin\theta_2)\underset{\sim}{j}$

E. $(s_1 \sin\theta_1 + s_2 \sin\theta_2)\underset{\sim}{i} + (s_1 \cos\theta_1 + s_2 \cos\theta_2)\underset{\sim}{j}$

Question 24

A particle moves so that its position vector at a time *t* is given by

$\underset{\sim}{r}(t) = a\cos(\omega t)\underset{\sim}{i} + a\sin(\omega t)\underset{\sim}{j} \quad t \geq 0$ where ω and a are both positive real constants.

Which of the following statements is **false** ?

A. The particle moves on a circle.

B. The speed of the particle is constant.

C. The acceleration vector is in the opposite direction to the position vector.

D. The velocity vector is perpendicular to the position vector.

E. The acceleration vector is constant.

Question 25

A suitcase of mass 12 kilograms rests on a rough, level ground. The suitcase is pulled with a force of magnitude P newtons acting at an angle of $30°$ to the horizontal. The suitcase is just on the point of sliding along the ground. If the coefficient of friction between the suitcase and the plane is 0.25, then P is closest to

A. 58.8

B. 29.67

C. 39.67

D. 3.03

E. 33.95

Question 26

A ball is projected vertically upwards with a speed of U m/s from the top of a second storey balcony, at a height H m above the ground. The ball rises and hits the ground after a time of T s, with a speed double the initial speed of projection. Which of the following is true?

A. $T = \frac{3U}{g}$ and $H + UT - \frac{1}{2}gT^2 = 0$

B. $T = \frac{3U}{g}$ and $H - UT - \frac{1}{2}gT^2 = 0$

C. $T = \frac{U}{g}$ and $H + UT - \frac{1}{2}gT^2 = 0$

D. $T = \frac{U}{g}$ and $H - UT - \frac{1}{2}gT^2 = 0$

E. $T = \frac{U}{g}$ and $\frac{1}{2}gT^2 + UT + H = 0$

Question 27

A particle moves so that its position at a time t is given by $x = 3\log_e\left(1 + 2e^{-3t}\right) - 6t$

The initial velocity of the particle is given by

A. -6

B. -8

C. -9

D. -12

E. -15

Question 28

The velocity vector of a particle at time t, $t \geq 0$, is given by $\dot{\underset{\sim}{r}}(t) = 2\sin(2t)\underset{\sim}{i} + 2\cos(2t)\underset{\sim}{j}$

The initial position of the particle is given by $\underset{\sim}{r}(0) = \underset{\sim}{i} + 2\underset{\sim}{j}$

Which of the following statements is correct?

A. The particle moves on a circle with centre at $(2,2)$ and radius 1

B. The particle moves on a circle with centre at $(2,0)$ and radius 1

C. The particle moves on a circle with centre at $(0,-2)$ and radius 1

D. The particle moves on a circle with centre at $(2,-2)$ and radius 1

E. The particle moves on a circle with centre at the origin and radius 1

Question 29

A block of mass m kg is lying on a smooth horizontal table and is joined by a light inextensible string to a another block of mass of $2m$ kg hanging vertically. This string passes over a smooth pulley at the edge of the table. When the system is released from rest, the acceleration of the blocks in m/s^2 is given by

A. $\dfrac{2}{3}$

B. $\dfrac{2g}{3}$

C. 1

D. $2g$

E. g

Question 30

A particle moves in a straight line. When its displacement from a fixed origin is x m, its velocity is v m/s and its acceleration is a m/s^2, at a time t s.
Given that $a = 2x$, and that, initially, when $t = 0$, $x = \sqrt{2}$ and $v = 2$, which of the following statements is true?

A. $v = 2 + 2xt$

B. $v^2 = 4 + 4x^2$

C. $x = \left(\frac{2+v}{2}\right)t$

D. $x = 2t + xt^2$

E. $x = \sqrt{2}\, e^{\sqrt{2}\, t}$

Instructions for Part II

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
Where an exact answer is required for a question, appropriate working must be shown.
Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$

Question 1

$y = Ax\sin(3x)$ is a solution of the differential equation $\frac{d^2y}{dx^2} + 9y = 12\cos(3x)$.

Find the value of A.

2 marks

Question 2

a. Show that for $0 < x < 1$, $\frac{d}{dx}\left(\text{Cos}^{-1}\left(x^2\right)\right) = \frac{-2x}{\sqrt{1-x^4}}$

2 marks

b. **Hence,** find using calculus, the exact value of $\int_0^{\frac{1}{\sqrt{2}}} \frac{x}{\sqrt{1-x^4}}$

2 marks

Question 3

Let $\underset{\sim}{a} = -5\underset{\sim}{i} + 2\underset{\sim}{j} + z\underset{\sim}{k}$. Find the value of the scalar z if

a. the length of the vector $\underset{\sim}{a}$ is 6

2 marks

b. the vector $\underset{\sim}{a}$ makes an angle of $\text{Cos}^{-1}\left(-\frac{\sqrt{7}}{6}\right)$ with the z-axis.

2 marks

Question 4

Let $u = 3 - ki$ and $v = k - 2i$, where $k \in R$.

a. Find the value of k if $\frac{u}{v}$ is a real number.

2 marks

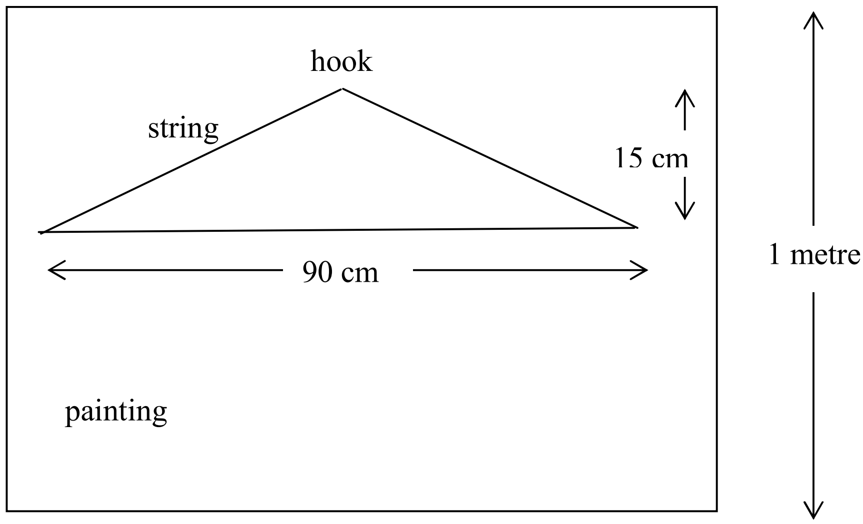
Question 4 (continued)

b. Let $S = \{\, z : |z - u| = |z - v| \,\}$ and $T = \{\, z : 6\operatorname{Re}(z) + 8\operatorname{Im}(z) + 5 = 0 \,\}$
Find the value of k if $S = T$.

2 marks

Question 5

A painting with a mass of 5 kg is to be hung on a wall. It is attached to a hook on the wall by a string attached to two fixed points on the same horizontal level at distance of 90 cm apart at the back of the painting. The hook is centred 15 cm above the horizontal level as shown in the diagram below.

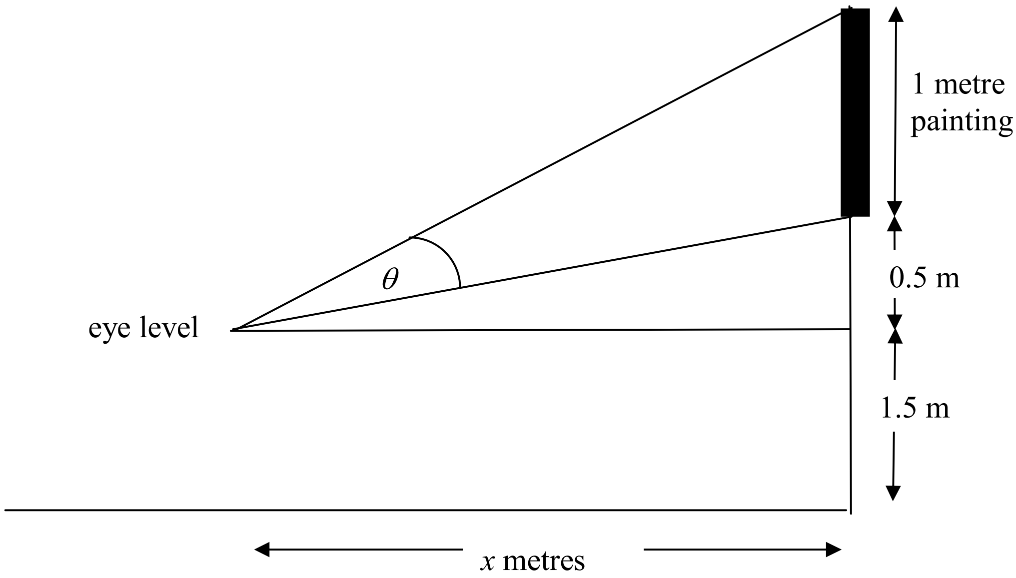


a. Find the tensions in the string in newtons, correct to two decimal places.

2 marks

Question 5 (continued)

The painting is to be hung at an art gallery. The painting is one metre high and is hanging so that the bottom of the painting is 50 cm above eye level, as shown in the diagram below.
The viewing angle is θ when a person, 1.5 metres tall, is standing x metres from the base of the wall.



b. Show that $\tan\theta = \dfrac{4x}{3+4x^2}$.

1 mark

Question 5 (continued)

c. Hence, find using calculus, how far from the wall a person should stand in order to maximise the viewing angle. Find the maximum viewing angle giving your answer in degrees.

3 marks

End of 2005 Specialist Mathematics Trial Examination 1
Question and Answer Book

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