The Mathematical Association of Victoria

SPECIALIST MATHEMATICS

Trial written examination 1
(Facts, skills and applications)

2005

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: ____

PART I
MULTIPLE-CHOICE QUESTION BOOK

This examination has two parts: Part I (multiple-choice questions) and Part II (short-answer questions).
Part I consists of this question book and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of a separate question and answer book.
You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
30	30	30

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

These questions have been written and published to assist students in their preparations for the 2005 Specialist Mathematics Examination 1. The questions and associated answers and solutions do not necessarily reflect the views of the Victorian Curriculum and Assessment Authority. The Association gratefully acknowledges the permission of the Authority to reproduce the formula sheet.

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"Cliveden", 61 Blyth Street, Brunswick, 3056
Phone: (03) 9380 2399 Fax: (03) 9389 0399
E-mail: office@mav.vic.edu.au website: http://www.mav.vic.edu.au

Multiple-Choice Answer Sheet

Student's Name ____________________________

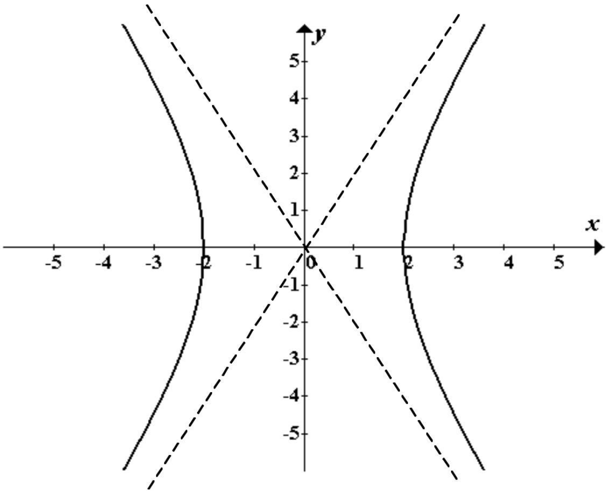
Circle the letter that corresponds to each answer.

1.	A	B	C	D	E
2.	A	B	C	D	E
3.	A	B	C	D	E
4.	A	B	C	D	E
5.	A	B	C	D	E
6.	A	B	C	D	E
7.	A	B	C	D	E
8.	A	B	C	D	E
9.	A	B	C	D	E
10.	A	B	C	D	E
11.	A	B	C	D	E
12.	A	B	C	D	E
13.	A	B	C	D	E
14.	A	B	C	D	E
15.	A	B	C	D	E
16.	A	B	C	D	E
17.	A	B	C	D	E
18.	A	B	C	D	E
19.	A	B	C	D	E
20.	A	B	C	D	E
21.	A	B	C	D	E
22.	A	B	C	D	E
23.	A	B	C	D	E
24.	A	B	C	D	E
25.	A	B	C	D	E
26.	A	B	C	D	E
27.	A	B	C	D	E
28.	A	B	C	D	E
29.	A	B	C	D	E
30.	A	B	C	D	E

Instructions for Part I

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where g = 9.8.

Question 1



The equation of the graph shown above could be

A. $4x^2 - y^2 = 16$

B. $x^2 - 4y^2 = 16$

C. $\frac{x^2}{2} - y^2 = 1$

D. $\frac{x^2}{4} - y^2 = 1$

E. $\frac{x^2}{4} - \frac{y^2}{8} = 1$

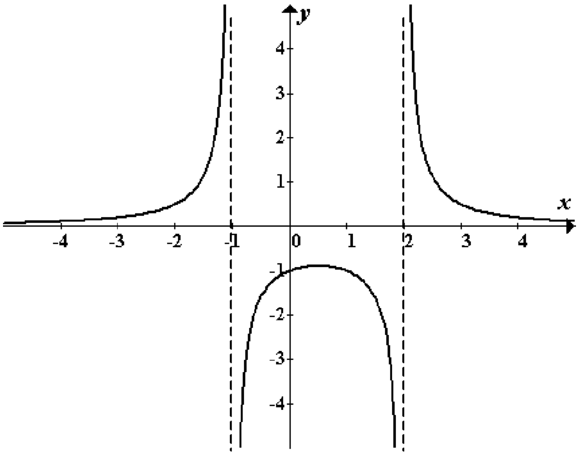
Question 2

The graph of $y = \frac{x^2 + 4}{4x}$ has

A. a minimum turning point at $x = -1$
B. a stationary point at $x = 1$
C. a maximum turning point at $x = -2$
D. no stationary points
E. a point of inflexion at $x = 0$

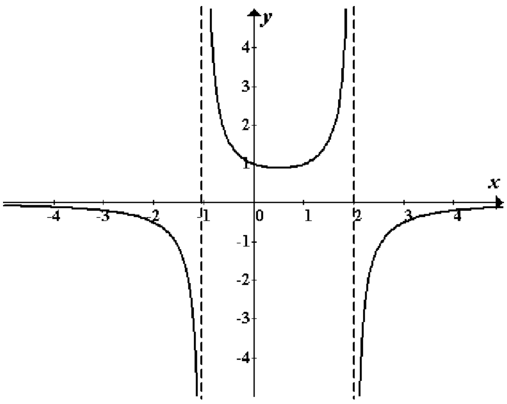
Question 3

The graph of $f(x)$ is shown below

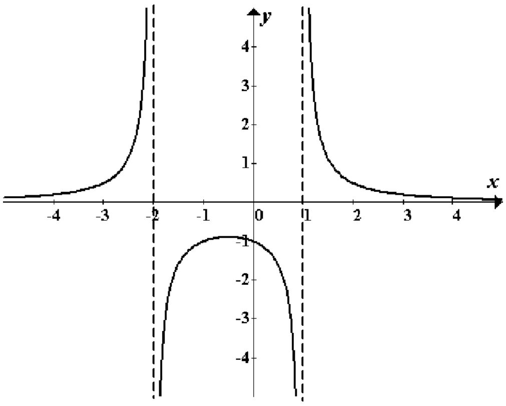


The graph of $\dfrac{1}{f(-x)}$ could be

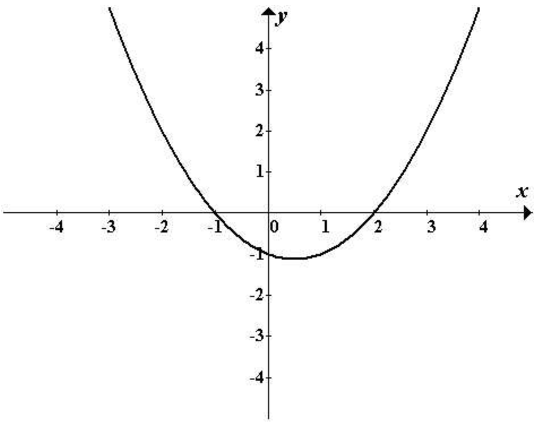
A.



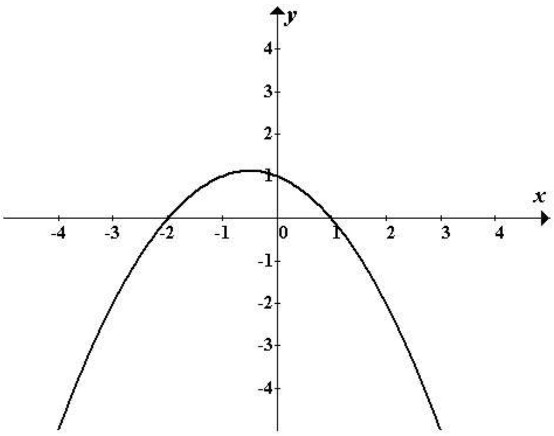
B.



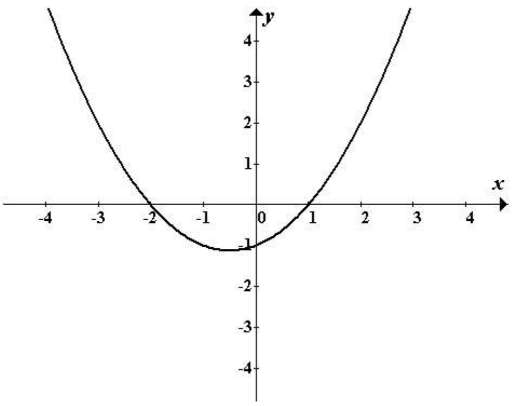
C.



D.



E.



Question 4

If $\tan x = a$, $x \in [\pi, \frac{3\pi}{2}]$ then $\cos x$ equals

A. $-\frac{1}{\sqrt{a^2 - x^2}}$

B. $-\frac{a}{\sqrt{a^2 + 1}}$

C. $-\frac{1}{\sqrt{a^2 + 1}}$

D. $\frac{1}{\sqrt{a^2 + 1}}$

E. $\frac{1}{a^2 + 1}$

Question 5

If $z = -\frac{\sqrt{3}}{2} - \frac{1}{2} i$ then $\operatorname{Arg} z$ is equal to

A. $-\frac{5\pi}{6}$

B. $-\frac{2\pi}{3}$

C. $\frac{\pi}{6}$

D. $\frac{5\pi}{6}$

E. $\frac{7\pi}{6}$

Question 6

The set of points in the complex plane defined by $\{z : |z - 2 + 3i| = 4\}$ is

A. a circle with centre (2, –3), radius 4
B. a circle with centre (2, –3), radius 2
C. a circle with centre (–2, 3), radius 4
D. a circle with centre (–2, 3), radius 2
E. the straight line with equation $y = 3 - x$

Question 7

If $z = c - di$, then the imaginary part of $\bar{z} - z$ is

A. 0
B. $2d\,i$
C. $2d$
D. $-2d$
E. $-2d\,i$

Question 8

If $z = 2 - i$ is one root of the equation $z^2 - 3z + 3 + i = 0$ then the other root is:

A. $2 + i$

B. 1

C. $-1 - i$

D. $1 - i$

E. $1 + i$

Question 9

A solution to the equation $\cot\left(3x - \frac{\pi}{2}\right) + \sqrt{3} = 0$ over $x \in [\pi\pi, 2\pi]$ is

A. $\frac{\pi}{9}$

B. $\frac{2\pi}{9}$

C. $\frac{11\pi}{9}$

D. $\frac{12\pi}{9}$

E. $\frac{13\pi}{9}$

Question 10

The expression cosec x $(\cos^2 x - \sin^2 x)$ is equivalent to

A. $\sin x \cos 2x$

B. $2 \cos x \cot 2x$

C. $\cot x - \sin x$

D. $\sec x \cos 2x$

E. $\cos x - \sin x$

Question 11

The range of the function $f(x) = \text{Tan}^{-1}(\text{Cos}\, x)$ is

A. R

B. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

C. $[-1, 1]$

D. $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

E. $[0, \pi]$

Question 12

If $y = \log_e (\sin 2x)$ then $\frac{d^2y}{dx^2}$ is

A. $-4 \operatorname{cosec}^2 2x$

B. $-4 (1 + \cos^2 2x)$

C. $2 \cot 2x$

D. $\frac{4}{\sin^2 2x}$

E. $-4 \tan 2x$

Question 13

An antiderivative of $\frac{1}{1 - 9x^2}$ is

A. $\frac{18x}{(1 - 9x^2)^2}$

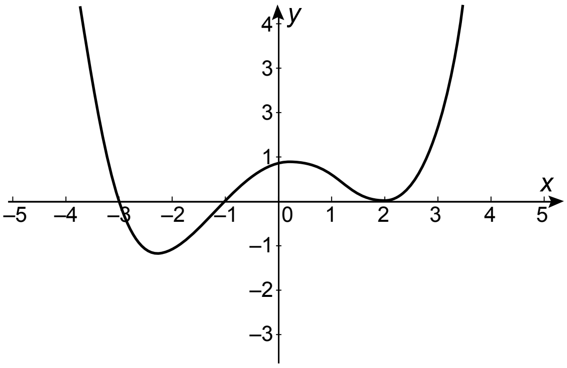
B. $\frac{1}{6} \log_e \left(\frac{1 + 3x}{1 - 3x} \right)$

C. $\frac{\log_e (1 - 9x^2)}{6}$

D. $\frac{1}{3} \operatorname{Sin}^{-1}(3x)$

E. $\frac{1}{9(1 - 9x)}$

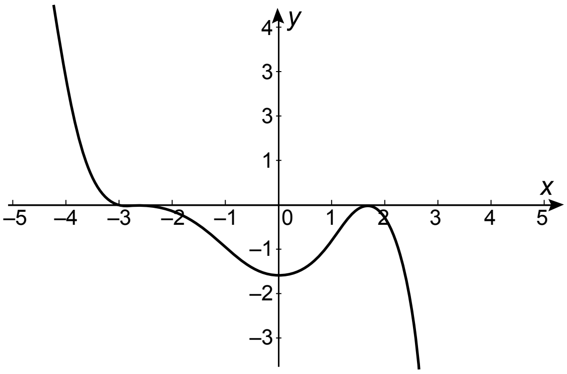
Question 14



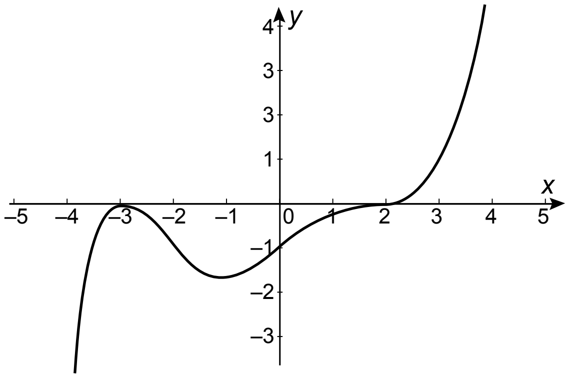
Part of a graph of a derivative function $f'(x)$ is shown above.

Which one of the graphs below could be the graph of the function $f(x)$?

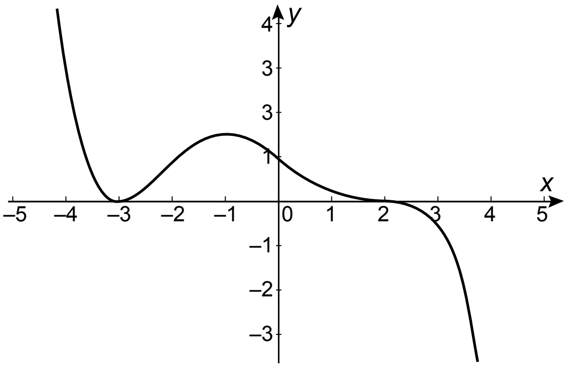
A.



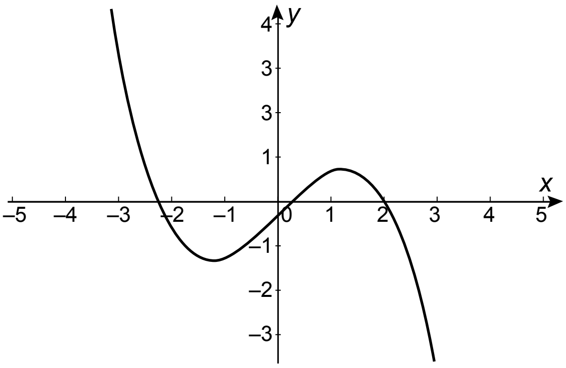
B.



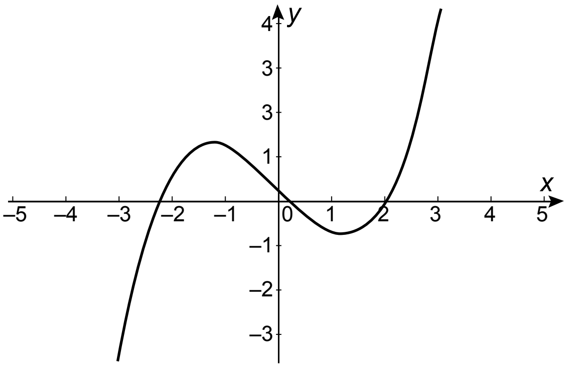
C.



D.



E.



Question 15

$\int_0^m \sin^2 x\,dx$ is equal o:

A. $\sin 2m$

B. $\frac{m}{2} + \frac{\sin 2m}{4}$

C. $\frac{m}{2} - \frac{\sin 2m}{4}$

D. $\frac{1}{3}\sin^3 m$

D. $\frac{1}{3}\sin^3 m \cos m$

Question 16

Euler's method, with step size 0.1, is used to approximate the solution to the differential equation

$$\frac{dy}{dx} = \frac{x^2}{x-1} \quad \text{with } y(2) = 4$$

When $x = 2.2$, the value for y, correct to four decimal places is

A. 4.0090
B. 4.0333
C. 4.4000
D. 4.8009
E. 4.8023

Question 17

Initially a tank contains 100 litres of pure water. A sugar solution with a concentration of 0.5 kg/litre is poured into the tank at a rate of 8 litres/minute. The mixture is kept uniform by stirring and flows out of the tank at a rate of 5 litres/minute.

Let Q kg be the amount of sugar dissolved in the tank after t minutes. $\frac{dQ}{dt}$ is equal to

A. $(100 + 3t)Q$

B. $4 - \frac{5Q}{100}$

C. $\frac{Q}{100 + 3t}$

D. $4 - \frac{5Q}{100 + 3t}$

E. $4 - \frac{5Q}{100 - 3t}$

Question 18

Let $\boldsymbol{a} = 2\boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{k}$ and $b = \boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k}$ The vector resolute of $\boldsymbol{a}$ in the direction of $\boldsymbol{b}$ is:

A. $\frac{3}{2}(\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k})$

B. $\frac{9}{\sqrt{6}}(\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k})$

C. $\frac{3}{\sqrt{2}}(\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k})$

D. $\frac{9}{14}(2\boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{k})$

E. $\frac{9}{\sqrt{14}}(2\boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{k})$

Question 19

If $\overrightarrow{OB} = 2\boldsymbol{i} + 3\boldsymbol{j} - 4\boldsymbol{k}$ and $\overrightarrow{AB} = \boldsymbol{i} - 3\boldsymbol{j} - 2\boldsymbol{k}$, then $|\overrightarrow{OA}|$ is equal to

A. 9

B. $\sqrt{33}$

C. $\sqrt{37}$

D. $\sqrt{41}$

E. $3\sqrt{5}$

Question 20

The velocity of a particle at time t, $t \geq 0$ is given by $\boldsymbol{r}'(t) = 2\sin(3t)\,\boldsymbol{i} - \boldsymbol{j}$.

The initial position of the particle is given by $\boldsymbol{r}\,(t) = -\frac{2}{3}\boldsymbol{i} + \boldsymbol{j}$. The position vector of the particle at time t is

A. $-\frac{2}{3}\cos(3t)\,\boldsymbol{i} - t\,\boldsymbol{j}$

B. $-\frac{2}{3}\cos(3t)\,\boldsymbol{i} + (1 - t)\boldsymbol{j}$

C. $\left(4 - \frac{2}{3}\cos(3t)\right)\boldsymbol{i} + \boldsymbol{j}$

D. $\left(4 - \frac{2}{3}\cos(3t)\right)\boldsymbol{i} + (1 - t)\boldsymbol{j}$

E. $\left(\frac{2}{3}\cos(3t) - \frac{4}{3}\right)\boldsymbol{i} + (1 - t)\,\boldsymbol{j}$

Question 21

The position vector of a particle at time t is given by $\boldsymbol{r}\,(t) = \sin(t)\boldsymbol{i} + \cos(2t)\,\boldsymbol{j}$, $0 \leq t \leq \pi$.
The Cartesian equation of the path of the particle is

A. $x^2 + \frac{y^2}{4} = 1, -1 \leq x \leq 1$

B. $x^2 + \frac{y^2}{4} = 1, 0 \leq x \leq 1$

C. $x^2 + 4y^2 = 1, -1 \leq x \leq 1$

D. $y = 1 - 2x^2, 0 \leq x \leq 1$

E. $y = 1 - 2x^2, -1 \leq x \leq 1$

Question 22

A particle moves in a straight line in such a way that its displacement, x metres, from a fixed origin at time t seconds is given by $x = 3.2t - 4\sin\left(\frac{t}{4}\right)$, $t \geq 0$. If the velocity of the particle at time t seconds is v metres per second, then the maximum value of v is

A. 19.2
B. 7.2
C. 4.2
D. 3.2
E. 0

Question 23

A particle initially moving at 12 m/s is subject to a constant retardation of 3 m/s^2. The distance, in metres, travelled before coming to rest will be

A. – 24
B. 4
C. 24
D. 36
E. 72

Question 24

The acceleration of a body moving in a straight line is given by $a = 4 - e^{2x}$, where $v = -2$ when $x = 0$. The velocity at any position is given by

A. $\sqrt{8x - e^{2x}}$
B. $-\sqrt{8x - e^{2x}}$
C. $4x - \frac{1}{2}e^{2x} - \frac{3}{2}$
D. $\sqrt{8x - e^{2x} + 5}$
E. $-\sqrt{8x - e^{2x} + 5}$

Question 25

A hot air balloon is 520 metres above the ground and rising vertically at a constant speed of 18 m/s. A stone is dropped over the side of the balloon, and strikes the ground t seconds later. Assuming air resistance is negligible, the value of t is

A. 1.8
B. 8.6
C. 10.5
D. 12.3
E. 100.7

Question 26

A body is in equilibrium under the action of three forces, $\boldsymbol{F}_1$, $\boldsymbol{F}_2$ and $\boldsymbol{F}_3$, where $\boldsymbol{F}_1 = 2\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{F}_2 = 2\boldsymbol{i} - 3\boldsymbol{j} - 2k$. The force $\boldsymbol{F}_3$ is

A. $4\boldsymbol{i} - 3k$

B. $-4\boldsymbol{i} + 3k$

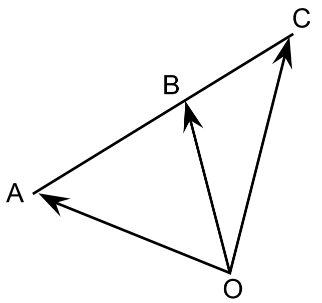
C. $6\boldsymbol{j} + k$

D. $-6\boldsymbol{j} - k$

E. $-4\boldsymbol{i} + k$

Question 27

Points A, B and C are collinear if:



A. $\overrightarrow{\boldsymbol{AB}} \cdot \overrightarrow{\boldsymbol{BC}} = 0$

B. $\overrightarrow{\boldsymbol{AB}} \cdot \overrightarrow{\boldsymbol{BC}} = 1$

C. $\overrightarrow{\boldsymbol{AB}} = k\,\overrightarrow{\boldsymbol{BC}}$

D. $\overrightarrow{\boldsymbol{AB}} = k\,\overrightarrow{\boldsymbol{OC}}$

E. $\overrightarrow{\boldsymbol{OA}} + \overrightarrow{\boldsymbol{AB}} + \overrightarrow{\boldsymbol{BC}} + \overrightarrow{\boldsymbol{CO}} = 0$

Question 28

A woman of mass 55 kg is standing in an elevator which is ascending with an acceleration of 3 m/s^2. The reaction force, in newtons, that the elevator floor exerts on the woman is closest to:

A. 165

B. 374

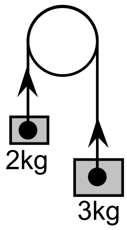
C. 539

D. 704

E. 715

Question 29

Two masses are connected by a light inelastic string which passes over a smooth pulley as shown. The acceleration due to gravity has magnitude g ms^{-2}. If a ms^{-2} is the magnitude of the common acceleration of each mass, then the tension in the string is:



A. $5g$

B. 0

C. $12g$

D. $\frac{5g}{2}$

E. $\frac{12g}{5}$

Question 30

A box of mass 3 kg is on a smooth plane inclined at an angle of 60° to the horizontal. This 3 kg mass is connected by a light string which passes over a smooth pulley to a 5 kg hanging vertically as shown. The magnitude of the acceleration of the 3 kg mass up the plane is

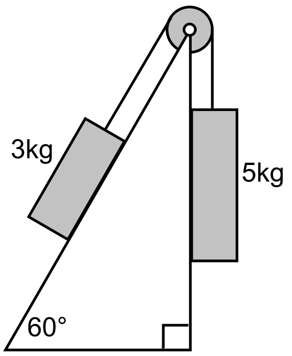
A. $\frac{7g}{16}$

B. $\frac{(3\sqrt{3} - 10)g}{4}$

C. $\frac{(10 - 3\sqrt{3})g}{16}$

D. $\frac{(3\sqrt{3} - 10)g}{16}$

E. $\frac{(10 - 3\sqrt{3})g}{6}$



END OF PART I MULTIPLE-CHOICE QUESTION BOOK

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$ $\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ $\sin(x - y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ $\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$ $\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

function	Sin^{-1}	Cos^{-1}	Tan^{-1}
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\operatorname{cis}\theta$

$|z|=\sqrt{x^2+y^2}=r$ $-\pi < \operatorname{Arg} z \le \pi$

$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$ $\frac{z_1}{z_2}=\frac{r_1}{r_2}\operatorname{cis}(\theta_1-\theta_2)$

$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$

$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$

$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$

$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$

$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$

$\int \frac{1}{x}\,dx = \log_e(x) + c$, for $x > 0$

$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$

$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$

$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$

$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$

$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$

$\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$

$\frac{d}{dx}\left(\text{Sin}^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$

$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \text{Sin}^{-1}\left(\frac{x}{a}\right) + c, a > 0$

$\frac{d}{dx}\left(\text{Cos}^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$

$\int \frac{-1}{\sqrt{a^2-x^2}}\,dx = \text{Cos}^{-1}\left(\frac{x}{a}\right) + c, a > 0$

$\frac{d}{dx}\left(\text{Tan}^{-1}(x)\right) = \frac{1}{1+x^2}$

$\int \frac{a}{a^2+x^2}\,dx = \text{Tan}^{-1}\left(\frac{x}{a}\right) + c$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

mid-point rule: $\int_a^b f(x)\,dx \approx (b-a)f\left(\frac{a+b}{2}\right)$

trapezoidal rule: $\int_a^b f(x)\,dx \approx \frac{1}{2}(b-a)\left(f(a) + f(b)\right)$

Euler's method: If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\quad s = ut + \frac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \frac{1}{2}(u+v)t$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \le \mu N$

END OF FORMULA SHEET