Student Name: ______________________________

Specialist Mathematics

Written examination 1

2005 Trial Examination

Reading Time: 15 minutes
Writing Time: 1 Hour and 30 minutes

QUESTION BOOK

Structure of Book

Part	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	30	30	30
2	6	6	20
			Total 50

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, and rulers, up to 4 pages (2 A4 sheets) of pre written notes and an approved graphics calculator and/or scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question book of 18 pages.
- Answer sheet for multiple choice questions.
- Formula Sheet.

Instructions
- Print your name in the space provided on the top of this page.
- All written responses must be in English.

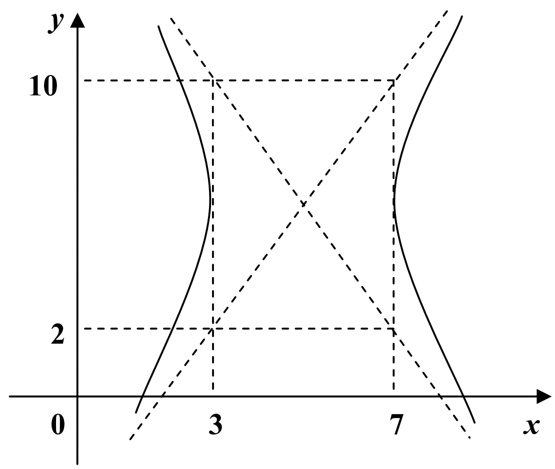
Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

Specific Instructions for Part I

A correct answer scores 1, an incorrect answer scores 0. Marks are not deducted for incorrect answers. If more than 1 answer is completed for any question, no mark will be given.

Question 1

The equation of the hyperbola at right is:



A. $\frac{(x-5)^2}{4}-\frac{(y-6)^2}{16}=1$

B. $\frac{(x-6)^2}{4}-\frac{(y-5)^2}{16}=1$

C. $\frac{(x-5)^2}{16}-\frac{(y-6)^2}{4}=1$

D. $\frac{(x-5)^2}{4}-\frac{(y-6)^2}{9}=1$

E. $\frac{(x-5)^2}{9}-\frac{(y-6)^2}{16}=1$

Question 2

If $\tan x=-\frac{1}{2}, \quad \frac{\pi}{2}\leq x\leq \pi$, then $\sec x$ is equal to:

A. $\frac{\sqrt{5}}{2}$

B. $-\frac{\sqrt{5}}{2}$

C. $\frac{2\sqrt{5}}{5}$

D. $-\frac{2\sqrt{5}}{5}$

E. $-\sqrt{5}$

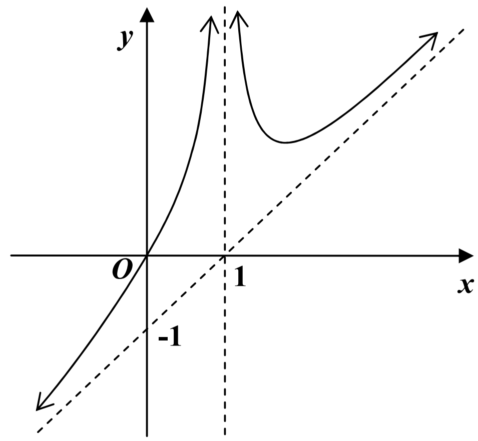
Question 3

The number of solutions of $2\sin^2(2x)=3, \quad 0\leq x\leq 2\pi$, is:

A. 0
B. 2
C. 4
D. 6
E. 8

Question 4

The equation of the curve at right is:



A. $y = \dfrac{x^2 - 3x + 3}{(x-1)^2}$

B. $y = \dfrac{x(x^2 + 3x + 3)}{(x-1)^2}$

C. $y = \dfrac{x(x^2 - 3x + 3)}{(x-1)^2}$

D. $y = \dfrac{x(x^2 - 3x + 3)}{(x+1)^2}$

E. $y = \dfrac{x(x^2 - 3x + 3)}{(x-1)}$

Question 5

If $f(x) = Cos^{-1}\left(\dfrac{1}{x}\right)$ and $x \neq 0$, then $f'(\sqrt{5})$ is:

A. $-\dfrac{1}{2}$

B. 2

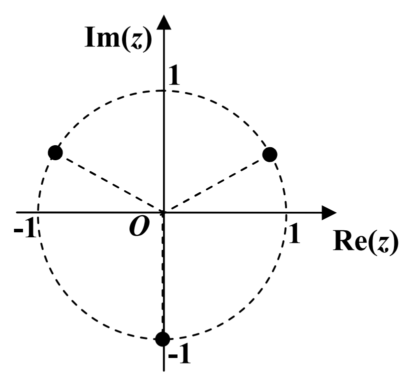
C. -2

D. $\dfrac{1}{2\sqrt{5}}$

E. $\dfrac{1}{\sqrt{5}}$

Question 6

The points on the Argand diagram at right are the roots of:



A. $P(z) = z^3 + i$

B. $P(z) = z^3 - 1$

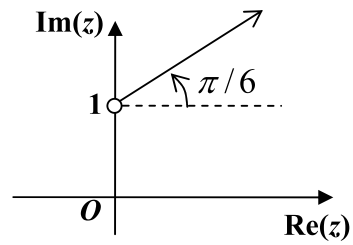
C. $P(z) = z^3 + 1$

D. $P(z) = z^3 - i$

E. $P(z) = z^2 - i$

Question 7

The graph of the Argand diagram at right is specified by:



A. $\text{Im}(z) = \text{Re}(z) + 1$

B. $Arg(z - 1) = \frac{\pi}{6}$

C. $Arg(z + i) = \frac{\pi}{6}$

D. $Arg(z - i) = \frac{\pi}{6}$

E. $Arg(z + 1) = \frac{\pi}{6}$

Question 8

Which one of the following is a polar form of $12 - 5i$?

A. $13\,cis\,\theta, \theta = Sin^{-1}\left(\frac{-5}{13}\right)$

B. $13\,cis\,\theta, \theta = Sin^{-1}\left(\frac{5}{13}\right)$

C. $13\,cis\,\theta, \theta = Tan^{-1}\left(\frac{5}{12}\right)$

D. $12\,cis\,\theta, \theta = Tan^{-1}\left(\frac{-5}{12}\right)$

E. $17\,cis\,\theta, \theta = Sin^{-1}\left(\frac{-5}{13}\right)$

Question 9

If *a* and *b* are real constant and $z = i$ is a root of the polynomial $P(z) = z^3 + az^2 + bz - 1$, then:

A. $a = 1$ & $b = 1$
B. $a = -1$ & $b = 1$
C. $a = 1$ & $b = -1$
D. $a = -1$ & $b = -1$
E. $a = 1$ & $b = 2$

Question 10

Using a suitable substitution, $\int_{e}^{e^2}\left(\frac{\ln x}{x}\right)dx$ can be expressed as:

A. $\int_{1}^{2} u\,du$

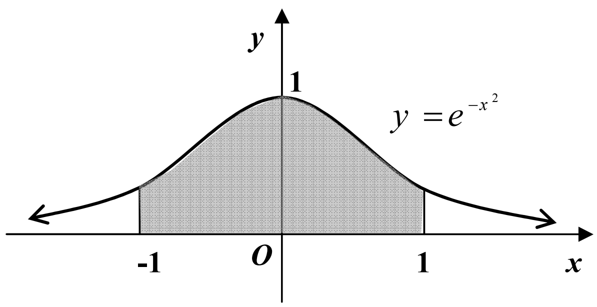
B. $\int_{e}^{e^2} u\,du$

C. $\int_{1}^{2}\left(\frac{1}{u}\right)du$

D. $\int_{1}^{2} \ln u\,du$

E. $\int_{0}^{\ln 2} u\,du$

Question 11



The shaded region in the diagram above is bounded by the graph of $y = e^{-x^2}$, the x-axis and the two lines $x = \pm 1$. Using the trapezium rule with four equal intervals, the shaded region is approximated by:

A. $\frac{1}{2}\left(1 + e^{-1/4} + e^{-1}\right)$

B. $\frac{1}{2}\left(1 + 2e^{-1/2} + e^{-1}\right)$

C. $\frac{1}{2}\left(1 + 2e^{-1/4} + e^{-1}\right)$

D. $\frac{1}{2}\left(1 + 2e^{1/4} + e\right)$

E. $\frac{1}{2}\left(1 + 2e^{1/2} + e\right)$

Question 12

The integral $\int 8\sin^4 x\,dx$ can be simplified to:

A. $\int [3+4\cos(2x)-\cos(4x)]dx$

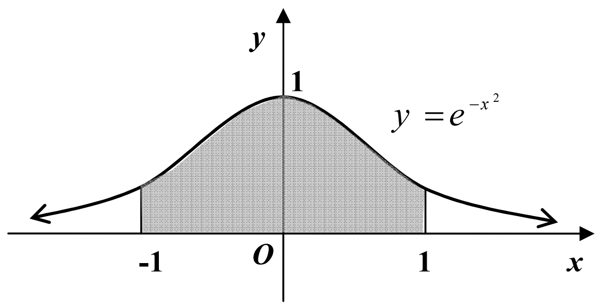
B. $\int [3-4\sin(2x)+\sin(4x)]dx$

C. $\int [4-3\cos(2x)-\cos(4x)]dx$

D. $\int [3-4\cos(2x)+\sin(4x)]dx$

E. $\int [3-4\cos(2x)+\cos(4x)]dx$

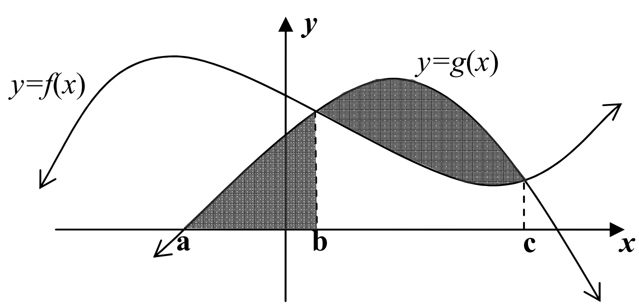
Question 13



The shaded area in units squared, correct to four decimal places, is:

A. 1.3211
B. 1.3239
C. 1.3553
D. 1.4936
E. 1.5320

Question 14



The total area of the shaded region is given by

A. $\int_a^b f(x)dx - \int_b^c [g(x)-f(x)]dx$

B. $\int_a^b f(x)dx + \int_b^c [g(x)-f(x)]dx$

C. $\int_a^b [f(x)-g(x)]dx + \int_b^c [g(x)-f(x)]dx$

D. $\int_a^b g(x)dx + \int_b^c [g(x)-f(x)]dx$

E. $\int_a^c [g(x)-f(x)]dx$

Question 15

If $f'(x) = 4\cos^2 x$ and $f(\pi) = 0$, then:

A. $f(x) = \cos(2x) + 2x - 1$
B. $f(x) = \sin(2x) - 2x + 2\pi$
C. $f(x) = \sin(2x) + 2x - 2\pi$
D. $f(x) = \sin(2x)$
E. $f(x) = \cos(2x) - 2x + 2\pi - 2$

Question 16:

The function $y = \sin(2x)$ is a solution of the differential equation:

A. $y'' + 2y' + 4y = \cos(2x)$
B. $y'' + 2y' + 4y = 2\cos(2x)$
C. $y'' - 2y' + 4y = 4\cos(2x)$
D. $y'' + 2y' + 4y = 4\cos(2x)$
E. $y'' + 2y' + 4y = -4\cos(2x)$

Question 17

Using Euler's method with step size of 0.5 and initial condition $y = 0$ at $x = 0$, the solution of the differential equation $\frac{dy}{dx} = e^{x^2}$, when $x = 2$, correct to four decimal places, is:

A. 2.5012
B. 3.5123
C. 4.1420
D. 5.9018
E. 7.2450

Question 18

If $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are nonzero vectors such that $\underset{\sim}{a} \cdot \underset{\sim}{b} = \underset{\sim}{a} \cdot \underset{\sim}{c}$, then:

A. $\underset{\sim}{b} = \underset{\sim}{c}$ only.
B. $\underset{\sim}{a}$ is perpendicular to $\underset{\sim}{b} - \underset{\sim}{c}$ only.
C. the vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are linearly dependent.
D. either $\underset{\sim}{b} = \underset{\sim}{c}$ or $\underset{\sim}{a}$ is perpendicular to $\underset{\sim}{b} - \underset{\sim}{c}$.
E. the vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are linearly independent.

Question 19

The two vectors $\underset{\sim}{i} + m\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{i} + m\,\underset{\sim}{j} + n\,\underset{\sim}{k}$ are perpendicular if:

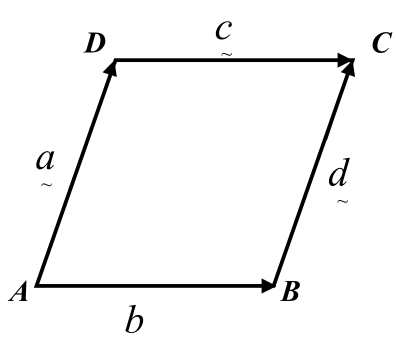
A. $m = 1, n = 1$
B. $m = 1, n = -1$
C. $m = \pm 1, n = 2$
D. $m = 1, n = \pm 2$
E. $m = \pm 1, n = -2$

Question 20

The vector resolute of the vector $\underset{\sim}{i} - 3\underset{\sim}{j} - 4\underset{\sim}{k}$ perpendicular to the vector $\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}$ is:

A. $\frac{1}{3}(8\underset{\sim}{i} - 4\underset{\sim}{j} - 2\underset{\sim}{k})$
B. $-\frac{1}{3}(8\underset{\sim}{i} - 4\underset{\sim}{j} - 2\underset{\sim}{k})$
C. $\frac{1}{6}(8\underset{\sim}{i} + 5\underset{\sim}{j} - 11\underset{\sim}{k})$
D. $-\frac{1}{7}(8\underset{\sim}{i} + 5\underset{\sim}{j} - 11\underset{\sim}{k})$
A. $\frac{1}{6}(8\underset{\sim}{i} + 5\underset{\sim}{j} + 11\underset{\sim}{k})$

Question 21



In the quadrilateral ***ABCD***, $\overrightarrow{AD} = \underset{\sim}{a}$, $\overrightarrow{AB} = \underset{\sim}{b}$, $\overrightarrow{DC} = \underset{\sim}{c}$, and $\overrightarrow{BC} = \underset{\sim}{d}$ as shown above. To prove that ***ABCD*** is a rhombus, it is enough to show that:

A. $\left(\underset{\sim}{a} - \underset{\sim}{b}\right) \cdot \left(\underset{\sim}{a} + \underset{\sim}{c}\right) = 0$
B. $\underset{\sim}{a} = \underset{\sim}{d}$
C. $\underset{\sim}{a} = \underset{\sim}{d}$ and $\underset{\sim}{b} = \underset{\sim}{c}$
D. $|\underset{\sim}{a}| = |\underset{\sim}{b}|$
E. $\underset{\sim}{a} = \underset{\sim}{d}$ and $|\underset{\sim}{a}| = |\underset{\sim}{b}|$

Question 22

The position vector of a particle at time t is $\underset{\sim}{r} = t\underset{\sim}{i} - 2t^2\underset{\sim}{j} + e^{2t}\underset{\sim}{k},\ t \geq 0$. The initial direction of the motion is:

A. $\frac{1}{\sqrt{3}}\left(\underset{\sim}{i} + 2\underset{\sim}{j}\right)$

B. $\frac{1}{\sqrt{3}}\left(\underset{\sim}{i} + 2\underset{\sim}{k}\right)$

C. $\frac{1}{\sqrt{2}}\left(\underset{\sim}{j} + \underset{\sim}{k}\right)$

D. $\underset{\sim}{k}$

E. $\frac{1}{\sqrt{3}}\left(\underset{\sim}{j} + 2\underset{\sim}{k}\right)$

Question 23

The position vector of a particle at time t is $\underset{\sim}{r} = (t+1)\underset{\sim}{i} + (1-t^2)\underset{\sim}{j},\ t \geq 0$. The equation of the path is:

A. $y = x(2-x),\ x \in R$

B. $y = x(2+x),\ x \geq 1$

C. $y = x(x-2),\ x \geq 1$

D. $y = x(2-x),\ x \geq 1$

E. $y = x(2-x),\ x \geq 0$

Question 24

The position vector of a particle at time t is $\underset{\sim}{r} = 3\sin(2t)\underset{\sim}{i} + 2\cos(2t)\underset{\sim}{j},\ t \geq 0$. The maximum speed of the particle is:

A. 3

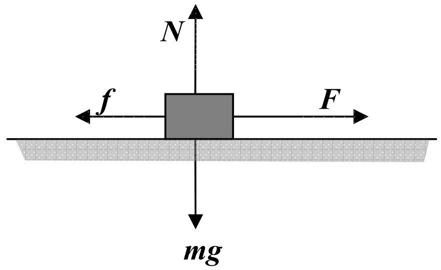
B. $\sqrt{13}$

C. 4

D. $\sqrt{20}$

E. 6

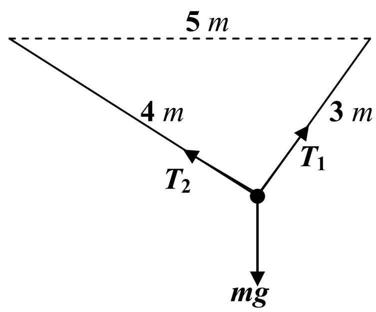
Question 25



A body of mass m kg is pulled along a rough, horizontal ground by a horizontal force F newtons as shown above. If N is the normal reaction, f is the force of friction and μ is the coefficient of friction, which one of the following statements is true?

A. $N = mg, f = \mu N < F$
B. $N = mg, f < \mu N = F$
C. $N > mg, f = \mu N < F$
D. $N = mg, f < \mu N < F$
E. $N < mg, f = \mu N = F$

Question 26



A particle of mass m kg is supported by two strings of lengths 3 m and 4 m as shown above. The other ends of the two strings are fixed at two points 5 m apart on the same horizontal level. If tensions in the strings are T_1 and T_2 newtons, which one of the following statements is true?

A. $\frac{T_1}{3} = \frac{T_2}{4} = \frac{mg}{5}$
B. $\frac{T_1}{3} = \frac{T_2}{5} = \frac{mg}{4}$
C. $\frac{T_1}{4} = \frac{T_2}{3} = \frac{mg}{5}$
D. $\frac{T_1}{4} = \frac{T_2}{5} = \frac{mg}{3}$
E. $\frac{T_1}{5} = \frac{T_2}{3} = \frac{mg}{4}$

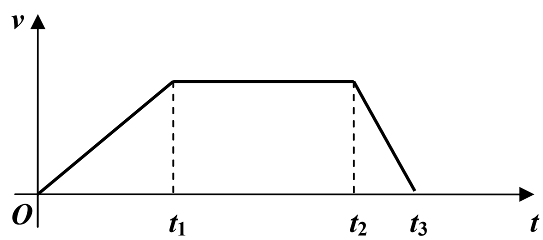
Question 27

A particle moves in a straight line such that the acceleration at any time t is $a(t) = \sqrt{3}\sin t + \cos t$.
If $v = 0$ initially, the exact **maximum speed** is:

A. $2-\sqrt{3}$
B. $\sqrt{2}+3$
C. $3-\sqrt{2}$
D. $\sqrt{3}/2$
E. $2+\sqrt{3}$

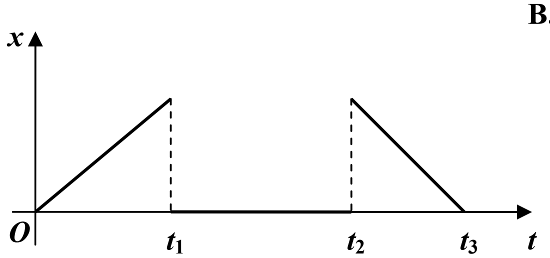
Question 28

The following is the velocity-time graph of a racing car over a short course.

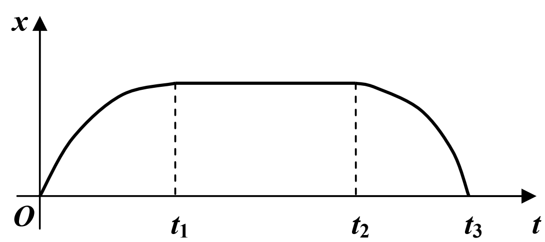


Which one of the following could be the displacement-time graph of the car's motion?

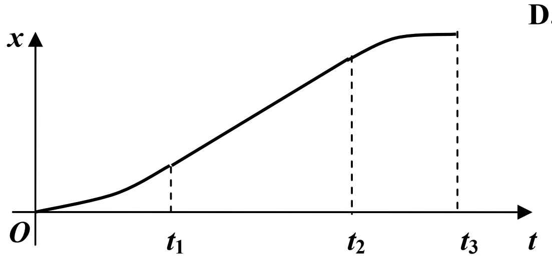
A.



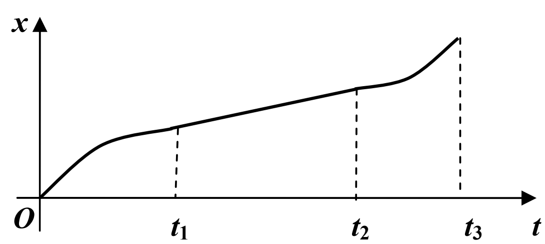
B.



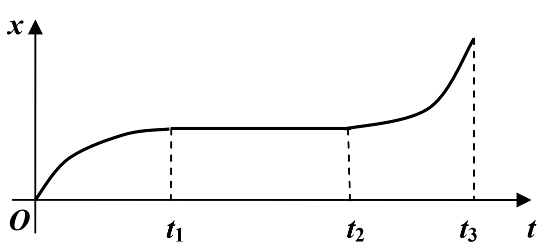
C.



D.



E.

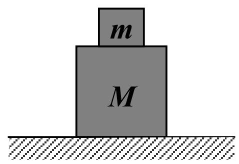


Question 29

A particle moves in a straight line such that the velocity is given by $v = 2x$, where x is the displacement at time t. If initially $x = 1$, then the acceleration is:

A. $a = e^{t}$
B. $a = e^{2t}$
C. $a = 2e^{2t}$
D. $a = 4e^{2t}$
E. $a = 2e^{4t}$

Question 30



A small mass of m kg sits on top of a larger mass of M kg on level ground as shown above. The two masses are at rest. Let R_1 and R_2 be the reaction forces acting on m and M respectively and P be the pressure on M due to m. The force diagrams are shown below.

According to Newton's **third** law:

A. $R_1 = P$
B. $R_1 = mg$
C. $R_2 = Mg + P$
D. $R_2 = P$
E. $R_2 = Mg$

END OF PART I

Specific Instructions for Part II

A decimal approximation will not be accepted if the question specifically asks for an exact answer. In questions worth more than one mark, appropriate working must be shown. The diagrams are not drawn to scale. Marks are given as specified for each question.

Question 1

If $y = x\sin x$ is a solution to the differential equation $\frac{d^2y}{dx^2} + my = n\cos x$, find the values of the real constants m and n.

3 marks

Question 2

On the axes below, sketch the graph of the hyperbola $(y-2)^2 - \frac{(x-3)}{4} = 1$. Give the equations of any asymptotes and the coordinates of the centre and vertices.

4 marks

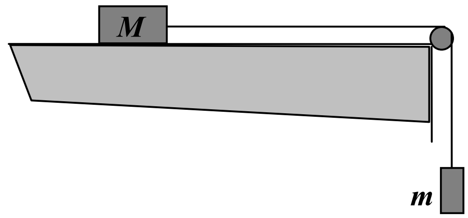
Question 3

Use calculus to find the **exact** value of $\int_0^{\pi} 8\sin^4 x\, dx$.

3 marks

Question 4

Two masses m and M kg are attached by a light inextensible string which passes overt a smooth pulley. The mass M is pulled along a smooth surface by the mass m which moves vertically down as shown below.



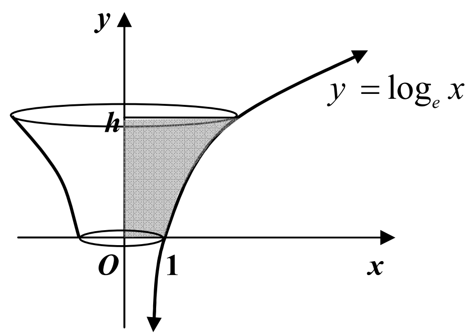
a. Find the acceleration (a) of the system of the two masses in terms of m, M and g.

2 marks

b. Find the tension in the string in terms of m, M and g.

1 mark

Question 5



The area, enclosed by the curve $y = \log_e x$, the line $y = h$ and the axes, is rotated about the y-axis to form a solid of revolution. Express the volume of this solid of revolution in terms of h.

3 marks

Question 6

Find, in Cartesian form, the roots of the complex equation $z^4 + 16 = 0$.

4 marks

END OF PART II

END OF EXAMINATION 1