VCAA Specialist Mathematics Exam 1 Solutions 2005
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Part I

1	2	3	4	5	6	7	8	9	10
E	**D**	**C**	**D**	**E**	**D**	**A**	**C**	**B**	**B**

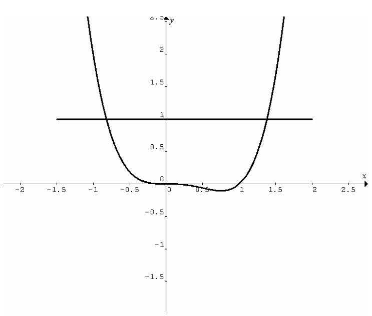
11	12	13	14	15	16	17	18	19	20
A	**C**	**A**	**A**	**D**	**E**	**D**	**A**	**B**	**C**

21	22	23	24	25	26	27	28	29	30
B	**B**	**C**	**E**	**A**	**?**	**E**	**AC**	**C**	**B**

Q1 Max (or min) occurs at the vertical axis of symmetry $x=-3$, where $\frac{(y-4)^2}{6}=3$. $\therefore\ y-4=\pm\sqrt{18}$ or $y=4\pm 3\sqrt{2}$.
$\therefore$Max value $=4+3\sqrt{2}$. E

Q2 No vertical asymptotes $\rightarrow$ no linear factors $\rightarrow \Delta<0$.
$\therefore\ m^2-4(1)(-n)<0$, i.e. $m^2<-4n$. D

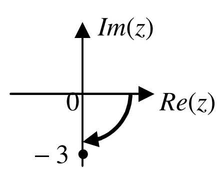
Q3 Since $\operatorname{cosec}^2(x)-\cot^2(x)=1$, $\therefore\ x^4-x^3=1$.
Graph $y=x^4-x^3$ and $y=1$. Only two intersections. C



Q4 At $x=\frac{\pi}{3}$, $y=0$. Only D satisfies this requirement. Use $\cot\theta=\frac{\cos\theta}{\sin\theta}$, not $\cot\theta=\frac{1}{\tan\theta}$, to evaluate. D

Q5 Use the chain rule. Let $u=\sqrt{3x}$, $\therefore\ y=Tan^{-1}(u)$.
$\frac{du}{dx}=\frac{1}{2\sqrt{3x}}\times 3=\frac{3}{2\sqrt{3x}}$, $\frac{dy}{du}=\frac{1}{1+u^2}=\frac{1}{1+3x}$.
$\therefore\ \frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}=\frac{3}{2\sqrt{3}\sqrt{x}(1+3x)}=\frac{\sqrt{3}}{2\sqrt{x}(1+3x)}$. E

Q6 $z=\frac{(3-6i)(2-i)}{(2+i)(2-i)}=\frac{-15i}{5}=-3i$.
$\therefore\ |z|=3$, $Arg(z)=-\frac{\pi}{2}$. D

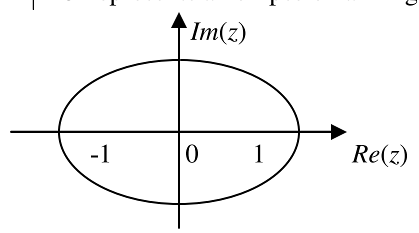


Q7 $\left[7cis\left(\frac{\pi}{4}\right)\right][acis(b)]=42cis\left(\frac{\pi}{20}\right)$,
$\therefore\ 7acis\left(\frac{\pi}{4}+b\right)=42cis\left(\frac{\pi}{20}\right)$. Hence $7a=42$, $a=6$;
$\frac{\pi}{4}+b=\frac{\pi}{20}$, $b=-\frac{\pi}{5}$. A

Q8 $\Delta=(4i)^2-4(1+i)(-2(1-i))=-16+8(1+i)(1-i)=0$ C

Q9 $z^{\frac{1}{4}}=\sqrt{2}cis\left(\frac{\pi}{16}\right)$, $z=\left(\sqrt{2}cis\left(\frac{\pi}{16}\right)\right)^4=\left(\sqrt{2}\right)^4 cis\left(4\times\frac{\pi}{16}\right)$,
i.e. $z=4cis\left(\frac{\pi}{4}\right)$. Hence $z^{-1}=4^{-1}cis\left(-\frac{\pi}{4}\right)=\frac{1}{4}cis\left(-\frac{\pi}{4}\right)$. B

Q10 $|z-1|+|z+1|=3$ represents an ellipse on an Argand diagram.



B

Q11 $\int\frac{6}{\sqrt{1-4x^2}}dx=6\int\frac{1}{\sqrt{1-(2x)^2}}dx=\frac{6}{2}Sin^{-1}(2x)+C$
$=3Sin^{-1}(2x)+C$. A

Q12 Change $\sin^2(2x)$ to $1-\cos^2(2x)$, and let $u=\cos(2x)$, then $\frac{du}{dx}=-2\sin(2x)$ or $\sin(2x)=-\frac{1}{2}\frac{du}{dx}$.
When $x=\frac{\pi}{2}$, $u=\cos\left(2\times\frac{\pi}{2}\right)=-1$.
When $x=\pi$, $u=\cos(2\pi)=1$.
$\int_{\frac{\pi}{2}}^{\pi}\sin^2(2x)\sin(2x)dx=\int_{\frac{\pi}{2}}^{\pi}\left(1-\cos^2(2x)\right)\sin(2x)dx$
$=-\frac{1}{2}\int_{-1}^{1}\left(1-u^2\right)\frac{du}{dx}dx=-\frac{1}{2}\int_{-1}^{1}\left(1-u^2\right)du$. C

Q13 $\int_0^2\pi R^2dx-\int_0^2\pi r^2dx=\pi\int_0^2\left(\frac{5}{x^2+1}\right)^2dx-\pi\int_0^2 1^2dx$
$=\pi\int_0^2\left(\left(\frac{5}{x^2+1}\right)^2-1\right)dx$ A

Q14 Graphics calculator: Graph $y=\frac{x+3}{2\sin(x)}$ and calc $\int dx$ from 4 to 5 to obtain -4.014 A

Q15 Linear substitution: $u = 3 - x$, $x = 3 - u$, $\frac{du}{dx} = -1$ or $-\frac{du}{dx} = 1$.

$$\int \left(x\sqrt{3-x}\right)dx = \int -(3-u)\sqrt{u}\,\frac{du}{dx}\,dx = \int \left(-3u^{\frac{1}{2}} + u^{\frac{3}{2}}\right)du$$

$$= -\frac{3u^{\frac{3}{2}}}{\frac{3}{2}} + \frac{u^{\frac{5}{2}}}{\frac{5}{2}} + C = -2(3-x)^{\frac{3}{2}} + \frac{2}{5}(3-x)^{\frac{5}{2}} + C$$ D

Q16 Substitution: $u = 2\tan(2x)$, $\frac{du}{dx} = 4\sec^2(2x)$ or $\frac{1}{4}\frac{du}{dx} = \sec^2(2x)$. When $x = 0$, $u = 2\tan(0) = 0$. When $x = \frac{\pi}{8}$, $u = 2\tan\left(2 \times \frac{\pi}{8}\right) = 2$.

$$\int_0^{\frac{\pi}{8}} \sec^2(2x)e^{2\tan(2x)}dx = \int_0^2 \frac{1}{4}e^u \frac{du}{dx}dx = \int_0^2 \frac{1}{4}e^u du$$

$$= \left[\frac{1}{4}e^u\right]_0^2 = \frac{1}{4}e^2 - \frac{1}{4}e^0 = \frac{1}{4}\left(e^2 - 1\right).$$ E

Q17 $y_{new} \approx y_{old} + hy'_{old}$ where $y' = e^{-x}$ and $h = 0.1$.

$x = 2,\quad y = 1$

$x = 2.1,\quad y = 1 + 0.1e^{-2} = 1.01353$

$x = 2.2,\quad y = 1.01353 + 0.1e^{-2.1} = 1.0258$ D

Q18 $A = \pi r^2$, $\frac{dA}{dr} = 2\pi r$. The rate of change of A is related to the rate of change of r by $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$. $\therefore 10 = 2\pi r \frac{dr}{dt}$, $\therefore \frac{dr}{dt} = \frac{5}{\pi r}$. A

Q19 $\frac{dy}{dx} = y^2 + 1$, $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}} = \frac{1}{y^2+1}$, $x = \int \frac{1}{1+y^2}dy$,

$x = Tan^{-1}(y) + C$. At $x = 0$, $y = 1$. $\therefore 0 = Tan^{-1}(1) + C$,

$C = -\frac{\pi}{4}$. Hence $Tan^{-1}(y) = x + \frac{\pi}{4}$ or $y = Tan\left(x + \frac{\pi}{4}\right)$. B

Q20 $v = \frac{2}{\sqrt{1-x^2}}$, $\therefore v^2 = \frac{4}{1-x^2}$. $a = \frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{d}{dx}\left(\frac{2}{1-x^2}\right)$

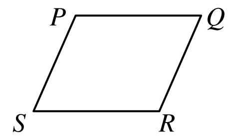
$= -\frac{2}{\left(1-x^2\right)^2} \times {}^{-}2x = \frac{4x}{\left(1-x^2\right)^2}$. C

Q21 $\frac{dv}{dt} = \frac{3}{v^2-9}$, $\frac{dt}{dv} = \frac{1}{\frac{dv}{dt}} = \frac{v^2-9}{3}$. $\therefore t = \int \frac{v^2-9}{3}dv$.

From $v = 2$ (initial) to $v = 1$ (final), $\Delta t = \int_2^1 \frac{v^2-9}{3}dv$.

Check: Δt has a positive value. B

Q22 $\overrightarrow{PQ} = \underset{\sim}{q} - \underset{\sim}{p} = \underset{\sim}{i} + y\underset{\sim}{j} + 3\underset{\sim}{k}$

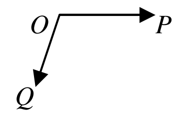


$\overrightarrow{SR} = \underset{\sim}{r} - \underset{\sim}{s} = (5-y)\underset{\sim}{i} + 2x\underset{\sim}{j} + 3\underset{\sim}{k}$

$PQRS$ is a parallelogram, $\therefore \overrightarrow{PQ} = \overrightarrow{SR}$.

$\therefore 5 - y = 1$, i.e. $y = 4$ and $2x = y$, i.e. $x = 2$. B

Q23 $\overrightarrow{OP} = 2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$ and $\overrightarrow{OQ} = -4\underset{\sim}{i} - 3\underset{\sim}{k}$.



$\cos\angle POQ = \frac{\overrightarrow{OP} \bullet \overrightarrow{OQ}}{\left|\overrightarrow{OP}\right|\left|\overrightarrow{OQ}\right|} = \frac{-5}{3 \times 5} = -\frac{1}{3}$. C

Q24 Since $\sin^2 t + \cos^2 t = 1$, $\therefore$ **either** $(x+1)^2 = \sin^2 t$ and $y^2 = \cos^2 t$ **or** $(x+1)^2 = \cos^2 t$ and $y^2 = \sin^2 t$.

The possibilities are:

$x + 1 = -\sin t$ and $y = \cos t$

$x + 1 = \sin t$ and $y = \cos t$

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$x + 1 = \cos t$ and $y = \sin t$

$x + 1 = -\cos t$ and $y = -\sin t$

$x + 1 = \cos t$ and $y = -\sin t$. Only the second possibility leads to choice E. E

Q25 $\underset{\sim}{v} = \dot{\underset{\sim}{r}} = 6t\underset{\sim}{i} + 5\underset{\sim}{j}$. A

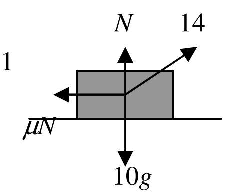
Q26 Note: Since the particle moves in a straight line (given information), $\therefore$ the direction of its velocity vector must be constant until it moves backwards (if it does). None of the choices meets this requirement.

Q27 $R = ma = 5(20 - 10\cos(2t))$, max R occurs when $\cos(2t) = -1$, $\therefore R_{\max} = 5(30) = 150$ E

Q28 There are two possibilities:

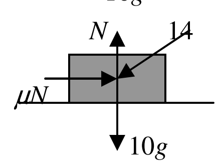
Case 1. $N + 14\sin 30^o = 10g = 98$, $\therefore N = 91$

$14\cos 30^o = \mu N = 91\mu$, $\therefore \mu = \frac{\sqrt{3}}{13}$.



Case 2. $N = 10g + 14\sin 30^o$, $\therefore N = 105$

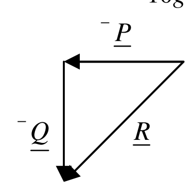
$14\cos 30^o = \mu N = 105\mu$, $\therefore \mu = \frac{\sqrt{3}}{15}$.



A, C

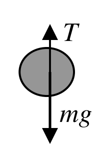
Q29 $\underset{\sim}{P} + \underset{\sim}{Q} + \underset{\sim}{R} = \underset{\sim}{0}$, $\therefore \underset{\sim}{R} = {}^{-}\underset{\sim}{P} + {}^{-}\underset{\sim}{Q}$

$\therefore \underset{\sim}{R} = 5\sqrt{2}$ SW C



Q30 $a = \frac{R}{m} = \frac{mg - T}{m} = g - \frac{T}{m} = 9.8 - \frac{1000}{200} = 4.8$

B



Part II

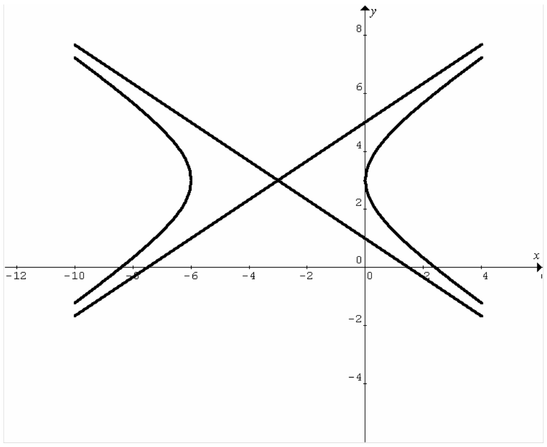
Q1a Equation of asymptote $y-k=\frac{b}{a}(x-h)$,

$y-3=\frac{2}{3}(x-c)$,

$y-3=\frac{2}{3}x-\frac{2c}{3}$. Given $y=\frac{2}{3}x+5$, i.e. $y-3=\frac{2}{3}x+2$.

$\therefore -\frac{2c}{3}=2$ or $c=-3$.

Q1b



Q2 $y=e^{2x}\cos(x)$, $\frac{dy}{dx}=2e^{2x}\cos(x)-e^{2x}\sin(x)$,

$\frac{d^2y}{dx^2}=2\left(2e^{2x}\cos(x)-e^{2x}\sin(x)\right)-\left(2e^{2x}\sin(x)+e^{2x}\cos(x)\right)$

$=3e^{2x}\cos(x)-4e^{2x}\sin(x)$.

$\therefore 3e^{2x}\cos(x)-4e^{2x}\sin(x)+k\left(2e^{2x}\cos(x)-e^{2x}\sin(x)\right)+e^{2x}\cos(x)$

$=-2e^{2x}\sin(x)$.

$\therefore 3+2k+1=0$ and $-4-k=-2$, $\therefore k=-2$.

Q3a The two resolutes are perpendicular,

$\therefore \left(3\underset{\sim}{i}-2\underset{\sim}{j}+\underset{\sim}{k}\right)\bullet\left(2\underset{\sim}{i}+x\underset{\sim}{j}+2\underset{\sim}{k}\right)=0$, $\therefore 6-2x+2=0$, $x=4$.

Q3b $\underset{\sim}{u}=\left(3\underset{\sim}{i}-2\underset{\sim}{j}+\underset{\sim}{k}\right)+\left(2\underset{\sim}{i}+4\underset{\sim}{j}+2\underset{\sim}{k}\right)=5\underset{\sim}{i}+2\underset{\sim}{j}+3\underset{\sim}{k}$.

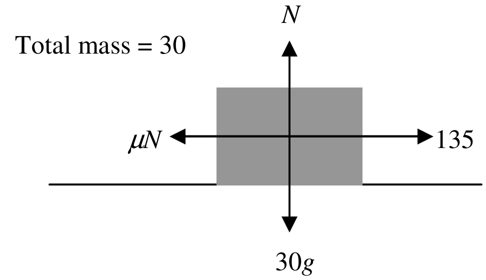
Q4a When $t=12$ (not 8), $v=8\tan\left(\frac{\pi}{4}\right)=8$.

Q4b Equate displacements (area under each graph) of A and B.
Let T (T>12) be the time B passes A.

$\int_4^{12}(t-4)\tan\left(\frac{\pi}{48}t\right)dt+8(T-12)=6T$. Use graphics calculator to evaluate the definite integral $=22.89$.

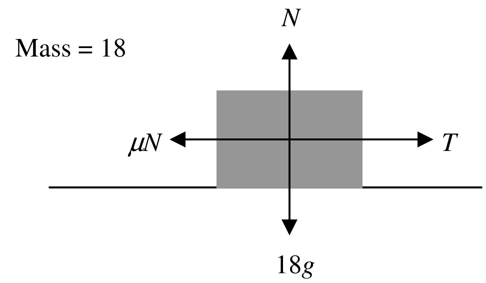
$\therefore 22.89+8(T-12)=6T$, $\therefore T=36.6$ s.

Q5a



$N=30g$, $R=ma$, $\therefore 135-\mu(30g)=30(0.5)$, $\therefore \mu=0.41$.

Q5b



Let T be the tension in the rope.

$N=18g$, $R=ma$, $\therefore T-0.41(18g)=18(0.5)$,

$\therefore T=81.3$ newtons.

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