THE
HEFFERNAN
GROUP

P.O. Box 1180
Surrey Hills North VIC 3127
ABN 20 607 374 020
Phone 9836 5021
Fax 9836 5025

Students Name:………………………………………

SPECIALIST MATHEMATICS

WRITTEN TRIAL EXAMINATION 1

2006

Reading Time: 15 minutes
Writing time: 1hour

Instructions to students

This exam consists of 9 questions.
All questions should be answered.
There is a total of 40 marks available.
The marks allocated to each of the nine questions are indicated throughout.
Where more than one mark is allocated to a question, appropriate working must be shown.
Unless otherwise indicated, diagrams in this exam are not drawn to scale.
Students may not bring any notes or calculators into the exam.
The acceleration due to gravity should be taken to have magnitude g m/s^2 where $g = 9.8$
Formula sheets can be found on pages 12-14 of this exam.

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Question 1

A particle of mass 2 kg moves in a straight line.

The velocity v m/s, of the particle at time t seconds, $t \geq 0$ is given by

$$v = \sin(3t) - \frac{t}{2}.$$

a. Find the acceleration of the particle expressed as a function of *t*.

1 mark

b. Find the maximum resultant force acting on the particle during its motion.

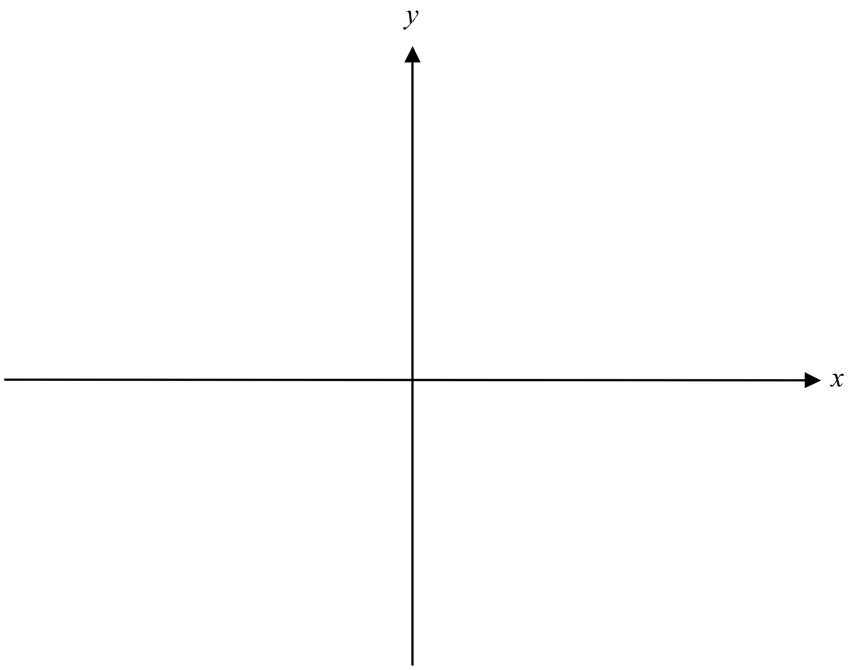
2 marks

Question 2

Consider the relation given by

$$\frac{(x-1)^2}{4}+\frac{y^2}{9}=1\,.$$

a. Sketch the graph of the relation on the set of axes below. The y-intercepts are not required.



2 marks

b. Find an expression for $\frac{dy}{dx}$ in terms of x and y.

3 marks

c. Hence show that if $y > 0$, then $\frac{dy}{dx} > 0$ for $x \in (-1, 1)$.

2 marks

Question 3

a. Show that $\frac{d}{dx}\left(\arctan\left(e^{2x}\right)\right)=\frac{2e^{2x}}{1+e^{4x}}$.

1 mark

b. **Hence** find the exact value of $\int_0^{\log_e 5} \frac{e^{2x}}{1+e^{4x}}\,dx$.

3 marks

Question 4

a. Given that $z=\sqrt{3}i$ is a solution to the equation

$$z^4-2z^3+5z^2-6z+a=0,$$

show that $a=6$.

1 mark

b. Hence find all the solutions to the equation

$$z^4-2z^3+5z^2-6z+6=0 \text{ for } z\in C.$$

3 marks

Question 5

a. Find $\int \left(\frac{\sec(2x)}{\tan(2x)}\right)^2 \, dx$.

1 mark

b. Evaluate $\int_0^1 \frac{x}{\sqrt{2-x}} \, dx$.

3 marks

Question 6

Let $\underset{\sim}{u} = \underset{\sim}{i} + \sqrt{2}\,\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{v} = \underset{\sim}{i} + a\,\underset{\sim}{j} - \underset{\sim}{k}$ where $a \in R$.

The angle between $\underset{\sim}{u}$ and $\underset{\sim}{v}$ is $\frac{\pi}{3}$.

Find the value of a.

4 marks

Question 7

Solve the differential equation

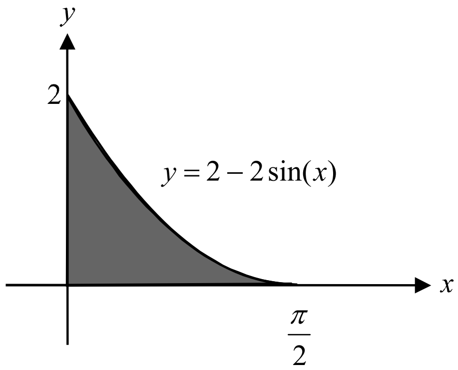
$$\frac{dy}{dx} = \frac{x^2 + 7}{x^2 + 4}$$

given that $y(0) = 0$.

3 marks

Question 8

The region enclosed by the graph of $y = 2 - 2\sin(x)$ and the positive x and y-axes is shaded in the diagram below.



This shaded region is rotated about the x-axis to form a solid of revolution.
Find the volume of this solid of revolution.

4 marks

Question 9

a. **i.** Find the coordinates of the stationary point of the graph of the function

$$y = \frac{1}{x^2 - 2x - 3}.$$

ii. Find the nature of this stationary point.

2 + 1 = 3 marks

b. Express $\dfrac{1}{x^2 - 2x - 3}$ in partial fraction form.

2 marks

c. Hence find the area enclosed by the curve with equation $y = \dfrac{1}{x^2 - 2x - 3}$, the x-axis and the lines $x = 0$ and $x = 2$.

2 marks

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$ $\qquad$ $\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ $\qquad$ $\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ $\qquad$ $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$ $\qquad$ $\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$ $\qquad$ $\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\operatorname{cis}\theta$

$|z| = \sqrt{x^2 + y^2} = r$ $\qquad$ $-\pi < \operatorname{Arg} z \le \pi$

$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$ $\qquad$ $\frac{z_1}{z_2} = \frac{r_1}{r_2}\operatorname{cis}(\theta_1 - \theta_2)$

$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$

$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$

$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$

$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$

$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$

$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$

$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$

$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$

$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$

$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$

$\int \frac{1}{x}\,dx = \log_e|x| + c$

$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$

$\int \cos\,(ax)\,dx = \frac{1}{a}\sin(ax) + c$

$\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$

$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$

$\int \frac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$

$\int \frac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Euler's method: If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$,

then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\quad s = ut + \frac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \frac{1}{2}(u+v)t$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$ $\qquad \underset{\sim}{r_1} . \underset{\sim}{r_2} = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \le \mu N$