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# SPECIALIST MATHEMATICS

# WRITTEN TRIAL EXAMINATION 2

## 2006

Reading Time: 15 minutes
Writing time: 2 hours

**Instructions to students**

This exam consists of Section 1 and Section 2.
Section 1 consists of 22 multiple-choice questions and should be answered on the detachable answer sheet on page 32 of this exam. This section of the paper is worth 22 marks.
Section 2 consists of 5 extended-answer questions, all of which should be answered in the spaces provided. Section 2 begins on page 13 of this exam. This section of the paper is worth 58 marks.
There is a total of 80 marks available.
Where more than one mark is allocated to a question, appropriate working must be shown.
Unless otherwise stated, diagrams in this exam are not drawn to scale.
The acceleration due to gravity should be taken to have magnitude $g$ m/s$^2$ where $g = 9.8$
Students may bring one bound reference into the exam.
Students may bring a graphics or CAS calculator into the exam.
Formula sheets can be found on pages 29-31 of this exam.

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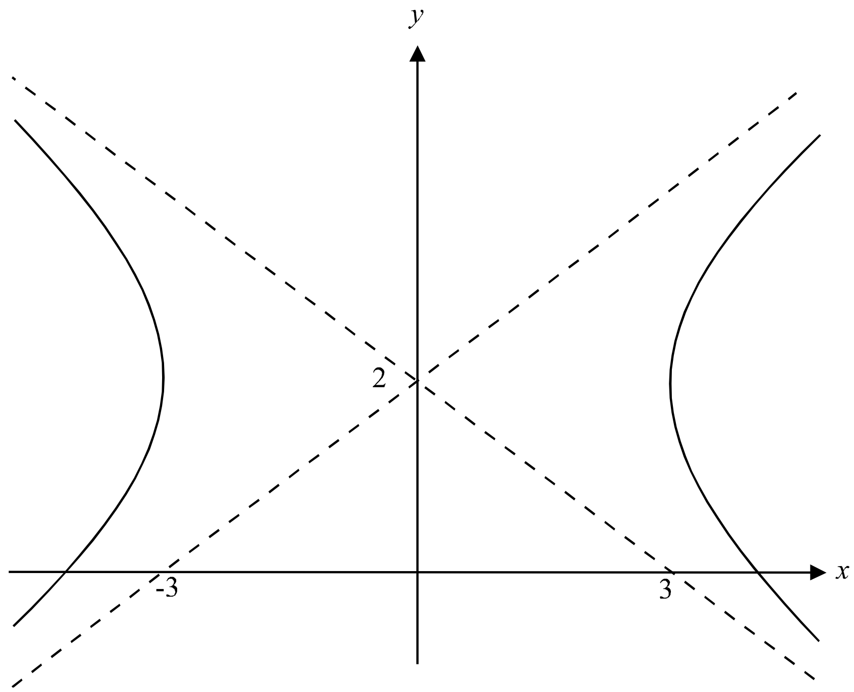
## Section 1

### Question 1




The graph shown above could have the equation

**A.** $\frac{x^2}{4} - \frac{(y-2)^2}{9} = 1$

**B.** $\frac{x^2}{9} - \frac{(y-2)^2}{16} = 1$

**C.** $\frac{x^2}{4} - \frac{(y-2)^2}{36} = 1$

**D.** $\frac{x^2}{16} - \frac{(y-2)^2}{9} = 1$

**E.** $\frac{x^2}{9} - \frac{(y-2)^2}{4} = 1$

**Question 2**

The graph of a rational function has one straight line asymptote and one other asymptote that is not a straight line.
Which one of the following could be the equation of this rational function?

**A.** $y = \dfrac{1}{(x-1)(x+1)}$

**B.** $y = \dfrac{x^2+1}{x}$

**C.** $y = \dfrac{x^3+1}{x^2}$

**D.** $y = \dfrac{x^4+1}{x^2}$

**E.** $y = x^2 + \dfrac{1}{x^2+1}$

**Question 3**

The graph of the function $f(x) = \sec(ax)$ where $a$ is a constant has asymptotes located at

**A.** $x = \ldots -2a\pi, -a\pi, 0, a\pi, 2a\pi \ldots$

**B.** $x = \ldots -\dfrac{3\pi}{2}, \dfrac{-\pi}{2}, \dfrac{\pi}{2}, \dfrac{3\pi}{2}, \ldots$

**C.** $x = \ldots -2\pi, -\pi, 0, \pi, 2\pi \ldots$

**D.** $x = \ldots \dfrac{-2\pi}{a}, \dfrac{-\pi}{a}, 0, \dfrac{\pi}{a}, \dfrac{2\pi}{a}, \ldots$

**E.** $x = \ldots \dfrac{-3\pi}{2a}, \dfrac{-\pi}{2a}, \dfrac{\pi}{2a}, \dfrac{3\pi}{2a}, \ldots$

**Question 4**

The expression $\sin(\theta)$ is **not** equal to

**A.** $\sqrt{\dfrac{1}{2}(1-\cos(2\theta))}$

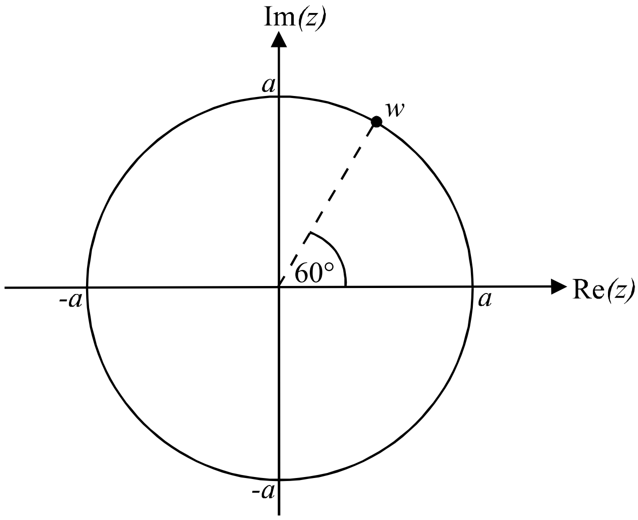
**B.** $\sqrt{1-\cos^2(\theta)}$

**C.** $2\sin\left(\dfrac{\theta}{2}\right)\cos\left(\dfrac{\theta}{2}\right)$

**D.** $\sin(3\theta)\cos(2\theta) + \cos(3\theta)\sin(2\theta)$

**E.** $\sin\left(\dfrac{\theta}{3}\right)\cos\left(\dfrac{2\theta}{3}\right) + \cos\left(\dfrac{\theta}{3}\right)\sin\left(\dfrac{2\theta}{3}\right)$

*The following information relates to Questions 5, 6 and 7.*




The Argand diagram above shows the complex number $w$ where $a \geq 1$.

**Question 5**

The complex number $w$ is given by

**A.** $1+\sqrt{3}i$

**B.** $\frac{a}{2}\left(1+\sqrt{3}i\right)$

**C.** $a\left(1+\sqrt{3}i\right)$

**D.** $a\left(\sqrt{3}+i\right)$

**E.** $\frac{a}{\sqrt{2}}(1+i)$

**Question 6**

The complex number represented by the expression $-i(\overline{w}+w)$ is given by

**A.** $a\text{cis}\left(-90^\circ\right)$

**B.** $a\text{cis}\left(90^\circ\right)$

**C.** $a\text{cis}\left(180^\circ\right)$

**D.** $2a\text{cis}\left(30^\circ\right)$

**E.** $2a\text{cis}\left(60^\circ\right)$

**Question 7**

Which one of the following statements is **not true** about the cube roots of *w*.

**A.** They lie on or within the circle with radius *a* units.
**B.** One of the cube roots of *w* is located at $\sqrt[3]{a}\,\text{cis}\left(-60^\circ\right)$.
**C.** The three cube roots are spaced 120° apart from one another.
**D.** The three cube roots lie on a circle with centre at $0+0i$
**E.** The three cube roots of *w* are all distinct, that is, not equal to each other.

**Question 8**

If $y=\sin^{-1}\left(\frac{x}{\sqrt{a}}\right)$, *a* is a constant, then $\frac{d^2y}{dx^2}$ is equal to

**A.** $\frac{1}{\sqrt{a-x^2}}$

**B.** $\frac{x}{\sqrt{a-x^2}}$

**C.** $\frac{x}{\left(a-x^2\right)^{\frac{3}{2}}}$

**D.** $-\frac{1}{2\left(a-x^2\right)^{\frac{3}{2}}}$

**E.** $\frac{x}{\left(a^2-x^2\right)^{\frac{3}{2}}}$

**Question 9**

The gradient to the curve $xy^2=1$ at the point where $x=4$ in the fourth quadrant is

**A.** $\frac{1}{16}$
**B.** $\frac{1}{4}$
**C.** $\frac{1}{2}$
**D.** 2
**E.** 4

**Question 10**

With a suitable substitution $\int_0^{\pi} (\sin(x)\cos(x))^3 \, dx$ can be expressed as

**A.** $-\int_{-1}^{1} \left(u^3 - u^5\right) du$

**B.** $\int_{-1}^{1} \left(u^3 - u^5\right) du$

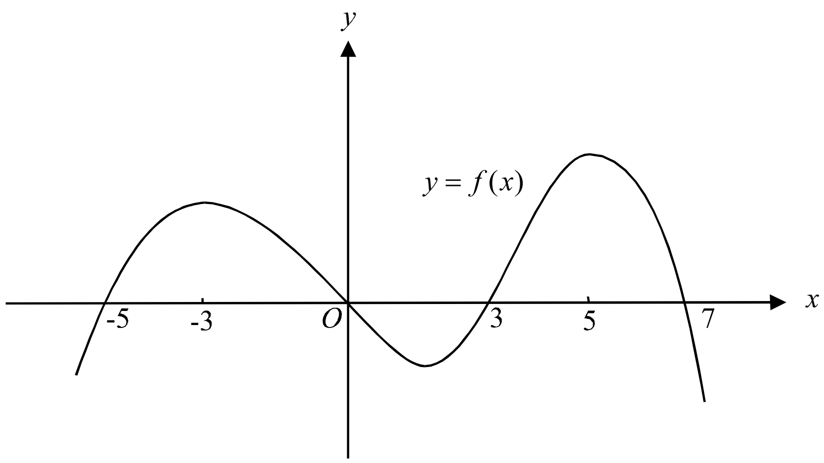
**C.** $\int_{1}^{-1} \left(u^3 - u^5\right) du$

**D.** $\int_{0}^{1} \left(u^3 - u^5\right) du$

**E.** $\int_{0}^{\pi} \left(u^3 - u^5\right) du$

**Question 11**

The graph of the function $y = f(x)$ is shown below.




Let $F(x)$ be the antiderivative of $f(x)$.
Which one of the following statements about the graph of $y = F(x)$ is **not** true?

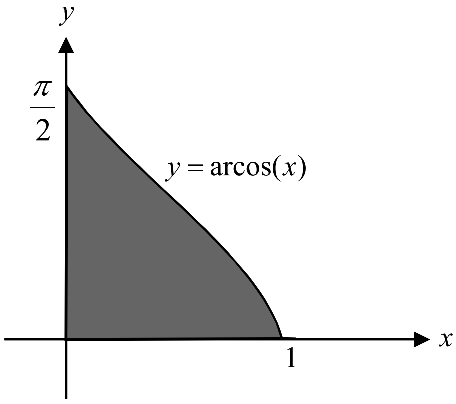
**A.** It has no stationary points of inflection.
**B.** It has a positive gradient for $x \in (-5, 0) \cup (3, 7)$.
**C.** It has a minimum turning point at $x = 0$.
**D.** It has a point of inflection at $x = 5$.
**E.** It is decreasing for $x \in (0, 3)$.

**Question 12**

The graph of the function

$$g:[0,1]\to R, \text{ where } g(x)=\arccos(x)$$

is shown below.




The shaded region is bounded by the graph of $y=g(x)$ and the positive $x$ and $y$-axes.
This region is rotated around the $y$-axis to form a solid of revolution.
The volume of this solid in cubic units is given by

**A.** $\dfrac{\pi}{2}\displaystyle\int_0^{\frac{\pi}{2}}(\cos(2y)+1)\,dy$

**B.** $\pi\displaystyle\int_0^{\frac{\pi}{2}}(\cos(2y)+1)\,dy$

**C.** $\pi\displaystyle\int_0^{\frac{\pi}{2}}\cos(y)\,dy$

**D.** $\pi\displaystyle\int_0^{1}\arccos(x^2)\,dx$

**E.** $\pi\displaystyle\int_0^{1}(\arccos(x))^2\,dx$

**Question 13**

The vectors $\underset{\sim}{a}, \underset{\sim}{b}$ and $\underset{\sim}{c}$ are all non-zero vectors. If $\underset{\sim}{a}$ is perpendicular to $\underset{\sim}{c}$, which one of the following statements must be true?

**A.** $\underset{\sim}{a} \bullet \underset{\sim}{a} = \underset{\sim}{c} \bullet \underset{\sim}{c}$

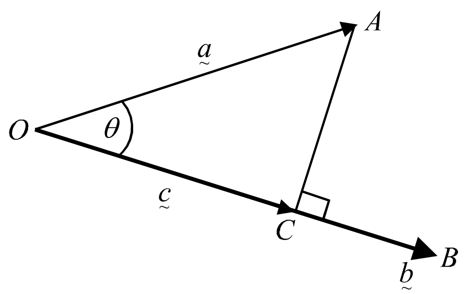
**B.** $\underset{\sim}{a} \bullet \underset{\sim}{b} = \underset{\sim}{b} \bullet \underset{\sim}{c}$

**C.** $\left(\underset{\sim}{a} \bullet \underset{\sim}{c}\right) \underset{\sim}{b} = \underset{\sim}{b}$

**D.** $\underset{\sim}{a} \bullet \left(\underset{\sim}{a} + \underset{\sim}{c}\right) = \left|\underset{\sim}{a}\right|^2$

**E.** $\left(\underset{\sim}{a} + \underset{\sim}{c}\right) \bullet \left(\underset{\sim}{a} - \underset{\sim}{c}\right) = \left|\underset{\sim}{a}\right| - \left|\underset{\sim}{c}\right|$

**Question 14**




In the diagram above, $\underset{\sim}{a} = \overrightarrow{OA}$, $\underset{\sim}{b} = \overrightarrow{OB}$ and $\underset{\sim}{c} = \overrightarrow{OC}$.

Also, $\underset{\sim}{c} = k\underset{\sim}{b}$ where $k$ is a constant.

An expression for $\underset{\sim}{c}$ is

**A.** $k\underset{\sim}{a} \bullet \underset{\sim}{b}$

**B.** $k\underset{\sim}{a} \bullet \hat{\underset{\sim}{b}}$

**C.** $\underset{\sim}{a} \bullet \hat{\underset{\sim}{b}}\cos(\theta)$

**D.** $\left(\underset{\sim}{a} \bullet \hat{\underset{\sim}{b}}\right) \hat{\underset{\sim}{b}}$

**E.** $\underset{\sim}{a} - \left(\underset{\sim}{a} \bullet \hat{\underset{\sim}{b}}\right) \hat{\underset{\sim}{b}}$

**Question 15**

A pulling force of $200N$ is applied at an angle of 30° to the horizontal in order to pull a body of mass 30 kg across a rough horizontal surface. The coefficient of friction between the body and the surface is 0.5. The acceleration in $\text{m/s}^2$ is given by

**A.** $\dfrac{10\sqrt{3}}{3}$

**B.** $\dfrac{8.5-5\sqrt{3}}{3}$

**C.** $\dfrac{10\sqrt{3}+3.5}{3}$

**D.** $\dfrac{10-1.5g-5\sqrt{3}}{3}$

**E.** $\dfrac{10\sqrt{3}-1.5g+5}{3}$

**Question 16**

A particle has three concurrent coplanar forces $\underset{\sim}{P}, \underset{\sim}{Q}$ and $\underset{\sim}{R}$ acting on it. The particle remains in equilibrium. If $|\underset{\sim}{P}| = 1$ newton and $\underset{\sim}{P}$ acts in the west direction and $|\underset{\sim}{R}| = \sqrt{3}$ newtons and acts in the south direction, then the magnitude and direction of $Q$ are given respectively by

**A.** 1 newton, $N30°E$
**B.** 1 newton, $N60°E$
**C.** 2 newtons, $N30°E$
**D.** 2 newtons, $N60°E$
**E.** $\sqrt{3}$ newtons, $N45°E$

**Question 17**

A particle of mass 3kg moves so that the horizontal component of its velocity has magnitude 6 m/s and the vertical component of its velocity has magnitude of 8 m/s.
The magnitude of the particles momentum, in kg m/s is

**A.** 10
**B.** 12
**C.** 16
**D.** 30
**E.** 144

**Question 18**

Consider the differential equation $\frac{dy}{dx} = a + by$ where $a$ and $b$ are constants and $y = 0$ when $x = 0$. If $c$ is a constant of integration, then which one of the following statements is **not** true?

**A.** $x = \int \frac{1}{a+by}\,dy$

**B.** $c = \log_e |a|^{-\frac{1}{b}}$

**C.** $x = \frac{1}{b}\log_e \left|\frac{a+by}{a}\right|$

**D.** $e^{bx} = 1 + \frac{by}{a}$

**E.** $y = \frac{a}{b}\left(e^{bx} + 1\right)$

**Question 19**

The differential equation $\frac{dy}{dx} = \frac{1}{x+1}$, with initial conditions $x = 0$ when $y = 0$, is solved using Euler's method with a step size 0.1.
The approximation for $y$ when $x = 0.2$ is given by

**A.** 0

**B.** $\frac{1}{10}$

**C.** $\frac{2}{10}$

**D.** $\frac{3}{11}$

**E.** $\frac{21}{110}$

**Question 20**

Water is added to a tank at the rate of 2 L/s.
The tank had originally contained 300L of water in which 45g of sugar had been dissolved. The mixture in the tank is continually stirred and the solution is drained from the tank at the rate of 5 L/s.
Let $S$ be the amount of sugar present in the tank at time $t$ seconds.
Which one of the following differential equations correctly describes this situation?

**A.** $-\frac{S}{60}$

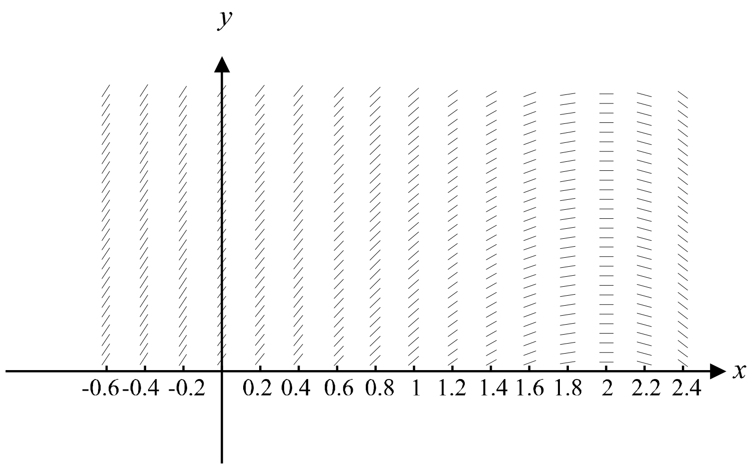
**B.** $S(2-5t)$

**C.** $\frac{-5S}{3(100-t)}$

**D.** $\frac{5S}{3(100-t)}$

**E.** $2-\frac{5S}{3(100-t)}$

**Question 21**




A certain first order differential equation has the direction (slope) field shown above.
Which one of the following functions could be a solution to this differential equation?

**A.** $y=\sqrt{x}$

**B.** $y=-x(x-4)$

**C.** $y=\log_e(x)$

**D.** $y=2x-x^2$

**E.** $y=1-e^{-x}$

**Question 22**

A particle moves in a straight line with velocity $v$ m/s.
At time $t$ seconds, $t \geq 0$, the displacement from a fixed point on the straight line is $x$ metres.

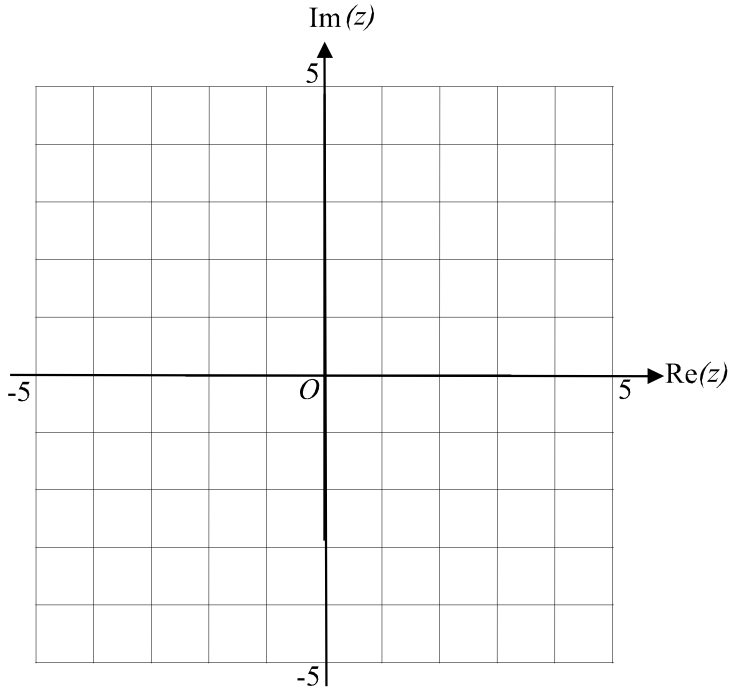
If $v^2 = -2\int 5\,dx$ then the particle is moving with

**A.** decreasing acceleration and decreasing velocity
**B.** decreasing acceleration and constant velocity
**C.** constant acceleration and decreasing velocity
**D.** constant acceleration and increasing velocity
**E.** increasing acceleration and increasing velocity

## Section 2

### Question 1

Let $u = 2 + 3i$ and $v = 1 + \frac{3}{2}i$.




**a.** Plot the complex numbers $u$ and $v$ on the Argand diagram above, labelling them as $U$ and $V$ respectively.

1 mark

**b.** Let $T$ be defined by $T = \{z : |z - u| = 2,\ z \in C\}$.
If $w = bi$, find the value $b$ such that $w \in T$.

1 mark

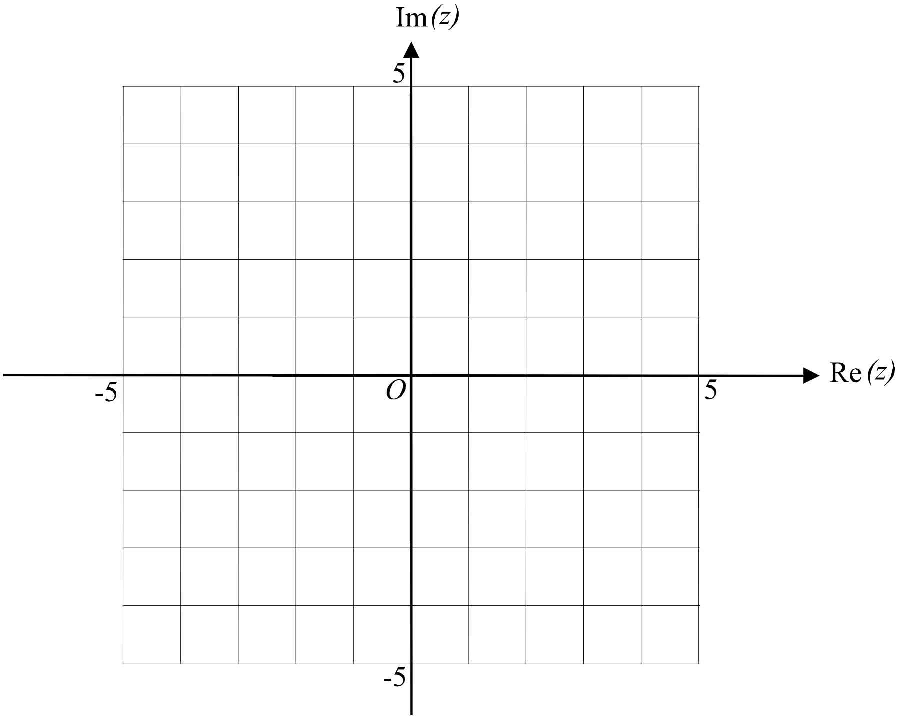
**c.** Let $R$ be defined by $R = \{z : |z - u| \le 2,\ z \in C\}$.

**i.** Sketch $R$ on the Argand diagram used in part **a**.

**ii.** Confirm algebraically that $v \in R$.

1 + 2 = 3 marks

**d.** **i.** On the Argand diagram below, sketch $S$ where $S = R \cap \{z : |z - w| = |z - u|\}$ and where $w \in T$.




**ii.** If $p = x + yi$ and $p \in S$, find the maximum and minimum values of $y$.

1 + 2 = 3 marks

**e.** Use a vector method to show that the points $U$, $V$ and $O$ are collinear.

3 marks
Total 11 marks

**Question 2**

The position vectors of two particles $A$ and $B$ are given respectively by

$$\underset{\sim}{a} = \sqrt{2}\sin(t)\underset{\sim}{i} + \sqrt{2}\cos(t)\underset{\sim}{j} \qquad \text{and} \qquad \underset{\sim}{b} = \sqrt{2}\cos(t)\underset{\sim}{i} + \underset{\sim}{j}$$

where $t = 0$ seconds corresponds to the start of the motion of each particle and $t \geq 0$.

**a.** **i.** Find the Cartesian equation of the path of particle $A$.

**ii.** Describe the movement of particle $A$ including its starting point and its subsequent movement in the $\underset{\sim}{i} - \underset{\sim}{j}$ plane.

1 + 2 = 3 marks

**b.** **i.** State the Cartesian equation of the path of particle $B$.

**ii.** Write down the domain of the Cartesian equation that describes the path of particle $B$.

1 + 1 = 2 marks

**c.** Evaluate $\underset{\sim}{a} \bullet \underset{\sim}{b}$ and hence find the exact time when the two particles are first at right angles to one another.

4 marks

**d.** Show that particle $A$ moves with constant speed.

2 marks

**e.** **i.** Verify that the particles will collide.

**ii.** How far does particle $A$ travel from its starting position before it collides with particle $B$?

$2 + 1 = 3$ marks
Total 14 marks

**Question 3**

Consider the function $g$ where

$$g(x) = \log_e\left(x^2 + 2x + 1\right)$$

**a.** Write down the maximal domain of $g$.

1 mark

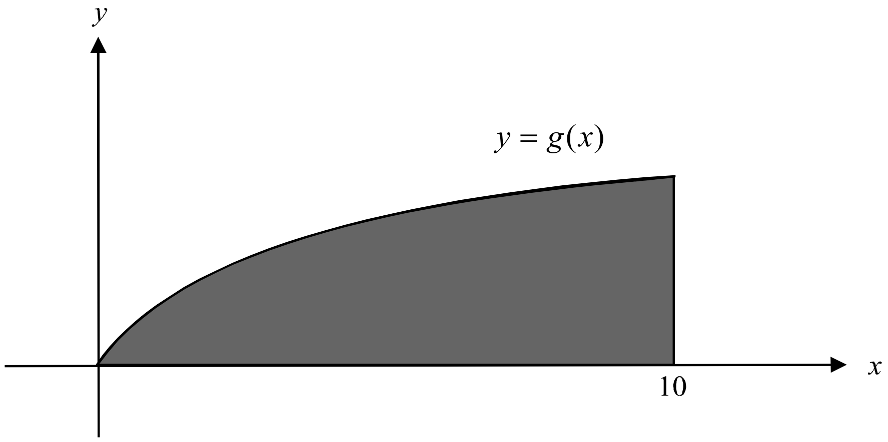
**b.** **i.** Show that $g^{-1}$, the inverse function of $g$, is given by

$$y = -1 \pm \sqrt{e^x}\ .$$

**ii.** State the range of $g^{-1}(x)$.

$1 + 1 = 2$ marks

A mould used in the manufacture of a standard wine glass is produced by rotating, about the $x$-axis, the region enclosed by the curve $y = g(x)$, the $x$-axis and the line $x = 10$. The unit of measurement is the centimetre.




**c.** Find the exact diameter, in cm, of the rim of the standard wine glass.

1 mark

**d.** **i.** Write a definite integral that gives the volume of the mould used in the manufacture of a standard wine glass.

**ii.** If a standard drink is 100 ml $\left(100\text{cm}^3\right)$ find how many standard drinks (correct to 1 decimal place) the standard wine glass can hold?

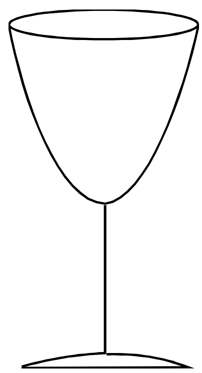
2 + 2 = 4 marks

The function $f(x)=\log_e(x^2+ax+1)$, where $a$ is an integer constant and $a>0$, is used to create moulds for non-standard wine glasses, in the same way as the function $g$ was used to create the mould for the standard wine glass.

**e.** Find the largest value of $a$ for which the rim of the non-standard wine glass (that still has a depth of 10 cm) has a diameter less than 10cm.

2 marks

**f.** Explain why at no point on the side of a non-standard wine glass can the slope of the glass be vertical.

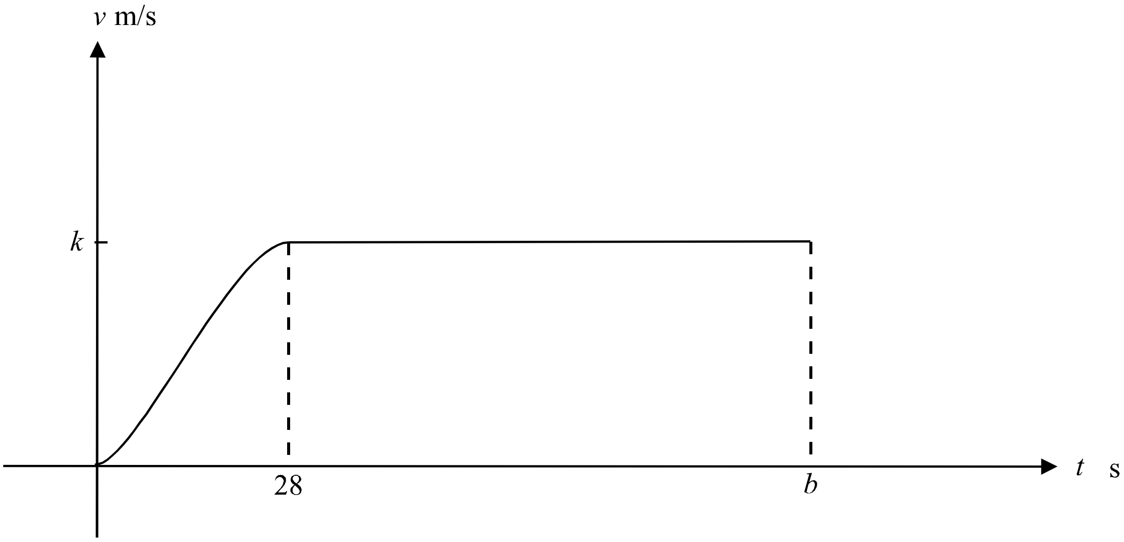


2 marks

Total 12 marks

**Question 4**

A stationary train pulls out of a station and travels in a straight line until it stops at the next station.
The velocity-time graph showing the motion of the train for $0 \le t \le b$ is shown below.




The velocity, in metres per second, of the train between the stations is given by

$$v(t) = \begin{cases} \dfrac{-t^3}{480} + \dfrac{28t^2}{320}, & \text{for } 0 \le t < 28 \\ k, & \text{for } 28 \le t \le b \\ g(t), & \text{for } b < t \le c \end{cases}$$

where $k$ is a constant.

**a.** Find the maximum speed of the train for $0 \le t \le b$.

1 mark

**b.** Given that the train maintains this maximum speed over a distance of 2.058 km, show that $b = 118$.

1 mark

**c.** **i.** What is the average acceleration of the train between $t = 10$ and $t = 50$ seconds? Express your answer in $\text{m/s}^2$ correct to 2 decimal places.

**ii.** What is the instantaneous acceleration of the train at $t = 10$ seconds?

$1 + 1 = 2$ marks

The deceleration phase of the train's trip is described by the function

$$g(t) = \frac{-23 \cdot 6}{\pi} \tan^{-1}(t - 130) + 11 \cdot 691, \qquad 118 < t \le c$$

where $t = c$ represents the time when the train stops at the second station.

**d.** Find $c$. Express your answer correct to two decimal places.

1 mark

**e.** Find an expression for the distance between the two stations involving two definite integrals.

2 marks

**f.** Describe the motion of the train as it approaches the second station.

1 mark

**g.** Given that $g'(t) = \dfrac{-23 \cdot 6}{\pi\left(1 + (t - 130)^2\right)}$ explain why the velocity of the train is never constant for $b \le t \le c$.

1 mark

A second train travels between the same two stations on another occasion. Its motion is similar to the first train in all ways except that it mistakenly expresses through the second station.
Realizing the mistake, the driver then has to reverse back to the second station so that passengers can alight.

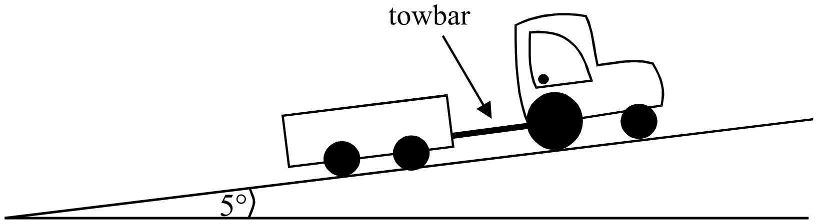
**h.** Draw a possible velocity-time graph showing the motion of this second train.

Indicate on your graph

- the area that approximately represents the distance between the two stations
- the point where the train begins to reverse.

3 marks
Total 12 marks

**Question 5**




A truck of mass 650kg is towing an empty trailer of mass 500kg along a straight track that is inclined at an angle of 5° to the horizontal. The tractor and trailer are connected by a tow bar. The tractor and trailer are moving along the track with an acceleration of $0 \cdot 15\text{m/s}^2$.

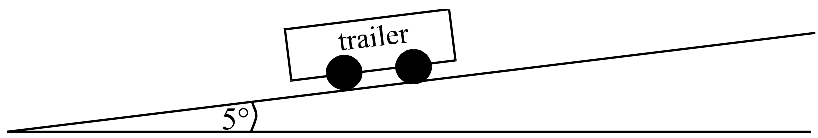
**a.** If the tension in the towbar is 2 000*N* show that the coefficient of friction between the trailer and the track is 0.307 correct to three decimal places.

4 marks

**b.** The truck and trailer continue along the track. When the speed of the truck is 4m/s, the driver notices a tree over the track 150m ahead. He brakes and the truck decelerates at a constant rate until it stops at the point where the tree is across the track. How long does it take for the truck to stop after the driver notices the tree?

2 marks

The driver unhooks the trailer from the truck, and leaves the trailer stationary on the track.
The trailer is then filled with timber from the tree that is being cut up.
The coefficient of friction between the trailer and the track is still 0.307.




**c.** **i.** Show that the trailer will not start to move down the track despite being loaded with timber.

**ii.** Find the minimum value of the coefficient of friction that would prevent the trailer from rolling down the track. Express your answer correct to 3 decimal places.

$2 + 1 = 3$ marks
Total 9 marks

# END OF EXAM

# Specialist Mathematics Formulas

## Mensuration

| | |
|---|---|
| area of a trapezium: | $\frac{1}{2}(a+b)h$ |
| curved surface area of a cylinder: | $2\pi rh$ |
| volume of a cylinder: | $\pi r^2 h$ |
| volume of a cone: | $\frac{1}{3}\pi r^2 h$ |
| volume of a pyramid: | $\frac{1}{3}Ah$ |
| volume of a sphere: | $\frac{4}{3}\pi r^3$ |
| area of a triangle: | $\frac{1}{2}bc\sin A$ |
| sine rule: | $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ |
| cosine rule: | $c^2=a^2+b^2-2ab\cos C$ |

## Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

## Circular (trigonometric) functions

$\cos^2(x)+\sin^2(x)=1$

$1+\tan^2(x)=\sec^2(x)$ $\qquad$ $\cot^2(x)+1=\operatorname{cosec}^2(x)$

$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$ $\qquad$ $\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$

$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$ $\qquad$ $\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$

$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\qquad$ $\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$

$\sin(2x)=2\sin(x)\cos(x)$ $\qquad$ $\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

| function | $\sin^{-1}$ | $\cos^{-1}$ | $\tan^{-1}$ |
|---|---|---|---|
| domain | $[-1, 1]$ | $[-1, 1]$ | $R$ |
| range | $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ | $[0,\pi]$ | $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ |

## Algebra (Complex numbers)

$z=x+yi=r(\cos\theta+i\sin\theta)=r\operatorname{cis}\theta$

$|z|=\sqrt{x^2+y^2}=r$ $\qquad$ $-\pi<\operatorname{Arg} z\le\pi$

$z_1z_2=r_1r_2\operatorname{cis}(\theta_1+\theta_2)$ $\qquad$ $\frac{z_1}{z_2}=\frac{r_1}{r_2}\operatorname{cis}(\theta_1-\theta_2)$

$z^n=r^n\operatorname{cis}(n\theta)$ (de Moivre's theorem)

**Calculus**

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$

$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$

$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$

$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$

$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$

$\int \frac{1}{x}\,dx = \log_e|x| + c$

$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$

$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$

$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$

$\int \cos\,(ax)\,dx = \frac{1}{a}\sin(ax) + c$

$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$

$\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$

$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$

$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$

$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$

$\int \frac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$

$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$

$\int \frac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Euler's method: If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$,
then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\quad s = ut + \frac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \frac{1}{2}(u+v)t$

**Vectors in two and three dimensions**

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$ $\qquad\qquad$ $\underset{\sim}{r}_1 . \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1x_2 + y_1y_2 + z_1z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

**Mechanics**

| | |
|---|---|
| momentum: | $\underset{\sim}{p} = m\underset{\sim}{v}$ |
| equation of motion: | $\underset{\sim}{R} = m\underset{\sim}{a}$ |
| friction: | $F \le \mu N$ |

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# SPECIALIST MATHEMATICS

# TRIAL EXAMINATION 2

## MULTIPLE- CHOICE ANSWER SHEET

*STUDENT NAME:*..........................................................................................

## INSTRUCTIONS

Fill in the letter that corresponds to your choice. Example: A ▬ C D E

The answer selected is B. Only one answer should be selected.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | A | B | C | D | E | 12. | A | B | C | D | E |
| 2. | A | B | C | D | E | 13. | A | B | C | D | E |
| 3. | A | B | C | D | E | 14. | A | B | C | D | E |
| 4. | A | B | C | D | E | 15. | A | B | C | D | E |
| 5. | A | B | C | D | E | 16. | A | B | C | D | E |
| 6. | A | B | C | D | E | 17. | A | B | C | D | E |
| 7. | A | B | C | D | E | 18. | A | B | C | D | E |
| 8. | A | B | C | D | E | 19. | A | B | C | D | E |
| 9. | A | B | C | D | E | 20. | A | B | C | D | E |
| 10. | A | B | C | D | E | 21. | A | B | C | D | E |
| 11. | A | B | C | D | E | 22. | A | B | C | D | E |