Insight Publications

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Trial Exam Paper

2006

SPECIALIST MATHEMATICS

Written examination 1

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes
Writing time: 1 hour

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring notes of any kind, sheets of paper or white out liquid/tape into the examination.
- Calculators are not permitted in this examination.

Materials provided

- The question and answer book of 9 pages with a separate sheet of miscellaneous formulas.
- Working space is provided throughout this book.

Instructions

- Write your **name** in the box provided.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

Instructions

Answer **all** questions in the spaces provided.
A decimal approximation **will not** be accepted if an exact answer is required.
In questions qhere more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, diagrams in this book **are not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$

Question 1

Consider the function $f(x) = \dfrac{1}{2x^2 - x - 3}$

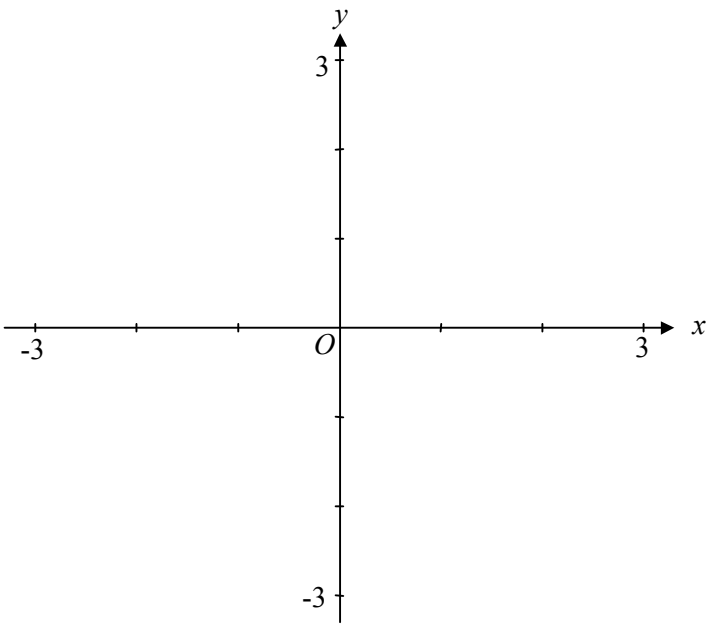
a. Determine the equations of the asymptotes of f.

2 marks

b. Find the coordinates of any intercepts and stationary points of f.

3 marks

c. Sketch f on the axes below labeling all key features.



1 mark

Question 2

a. Show that $\dfrac{1}{\cos^4 x - \sin^4 x} = \sec(2x)$

2 marks

b. Hence find the exact values of x for which $\dfrac{1}{\cos^4 x - \sin^4 x} = 2$, $x \in [0, 2\pi]$

2 marks

Question 3

Find the fourth roots of $16i$ in exact polar form.

4 marks

Question 4

Determine the rate of change of y with respect to x on the curve $y = x - 5xy^2$ at the point where $y = 1$.

4 marks

Question 5

a. **i.** Give the domain over which $\frac{d}{dx}\left(\arcsin(2x) + 2x\sqrt{1-4x^2}\right)$ is defined.

ii. Show that $\frac{d}{dx}\left(\arcsin(2x) + 2x\sqrt{1-4x^2}\right) = 4\sqrt{1-4x^2}$

1 + 2 = 3 marks

b. Hence, find the exact area enclosed by the curve $4x^2 + y^2 = 1$.

3 marks

Question 6

Find constants *m* and *n* such that $y = \dfrac{\log_e |x|}{x}$, $x \neq 0$ is a solution of the differential equation

$x^2 \dfrac{d^2y}{dx^2} + mx\dfrac{dy}{dx} + ny = 0$

4 marks

Question 7

Given $\frac{dy}{dx} = x\sqrt{1+x^2}$ and $y = 1$ when $x = 0$.

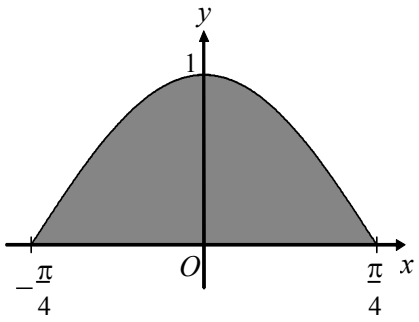
Find the value of y when $x = \sqrt{3}$.

4 marks

TURN OVER

Question 8

The graph below shows the region bounded by the curve $y = \cos(2x)$ and the x-axis.



Find the exact value of the volume of the solid of revolution formed when this region is rotated around the x-axis.

4 marks

Question 9

A particle moves in such a way that its position vector at time t seconds is given by

$\underset{\sim}{r} = 2t\,\underset{\sim}{i} + \cos(2\pi t)\,\underset{\sim}{j} + \sin(2\pi t)\,\underset{\sim}{k}$

a. Find the constant speed at which the particle is moving.

2 marks

b. Show that the velocity and acceleration are always perpendicular.

2 marks

END OF QUESTION AND ANSWER BOOK