# Year 2006

# VCE

# Specialist Mathematics

# Examination 1

# Suggested Solutions

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# Solutions Specialist Mathematics Examination 1 2006

## Question 1

**a.** $2xy - 9y^2 + 9 = 0$ taking $\frac{d}{dx}$ of each term $\frac{d}{dx}(2xy) - \frac{d}{dx}(9y^2) + \frac{d}{dx}(9) = 0$

using the product rule on the first term $2y + 2x\frac{dy}{dx} - 18y\frac{dy}{dx} = 0$

re-arranging to $2y = (18y - 2x)\frac{dy}{dx}$ so that $\frac{dy}{dx} = \frac{y}{9y - x}$

**b.** when $y = 1$ then $2x - 9 + 9 = 0$ so $x = 0$ the point is $(0,1)$ $\left.\frac{dy}{dx}\right|_{(0,1)} = \frac{1}{9-0} = \frac{1}{9}$

## Question 2

$\frac{dy}{dx} = x\sqrt{x^2 - 16}$ ; $x \ge 4$ given $y(5) = 13$ integrating

$y = \int x\sqrt{x^2 - 16}\ dx$ let $u = x^2 - 16$ $\frac{du}{dx} = 2x$ $x\,dx = \frac{1}{2}\,du$

$y = \frac{1}{2}\int u^{\frac{1}{2}}\ du = \frac{1}{3}u^{\frac{3}{2}} + c$

$y = \frac{1}{3}(x^2 - 16)^{\frac{3}{2}} + c$ to find $c$ use $x = 5$ when $y = 13$

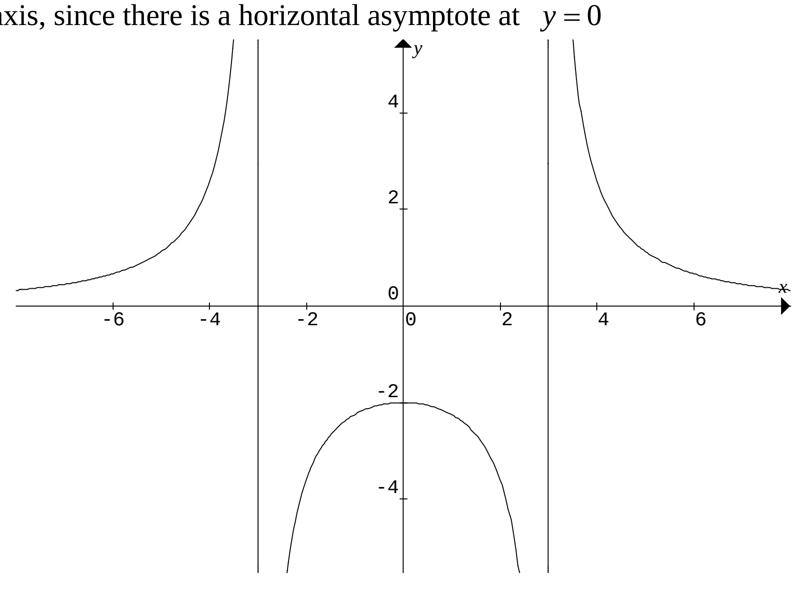
$13 = \frac{1}{3}(25 - 16)^{\frac{3}{2}} + c = 9 + c$ so that $c = 4$

$y = \frac{1}{3}(x^2 - 16)^{\frac{3}{2}} + 4$

## Question 3

**a.** $y = \frac{36}{2x^2 - 18} = \frac{18}{x^2 - 9}$ there are vertical asymptotes when $x^2 - 9 = (x+3)(x-3) = 0$

at $x = \pm 3$ , the graph crosses *y*-axis at $x = 0$ $y = -2$ $(0,-2)$ , the graph does not cross the *x*-axis, since there is a horizontal asymptote at $y = 0$



**b.** Area $= \displaystyle\int_{-2}^{2} \frac{36}{2x^2-18}\,dx = \int_{0}^{2} \frac{36}{x^2-9}\,dx$ by symmetry

by partial fractions $\dfrac{36}{x^2-9} = \dfrac{A}{x+3} + \dfrac{B}{x-3} = \dfrac{A(x-3)+B(x+3)}{(x+3)(x-3)} = \dfrac{x(A+B)+3B-3A}{x^2-9}$

so $A+B=0 \quad \Rightarrow A=-B$ and $3B-3A=36$ so $6B=36$ and $B=6$ and $A=-6$

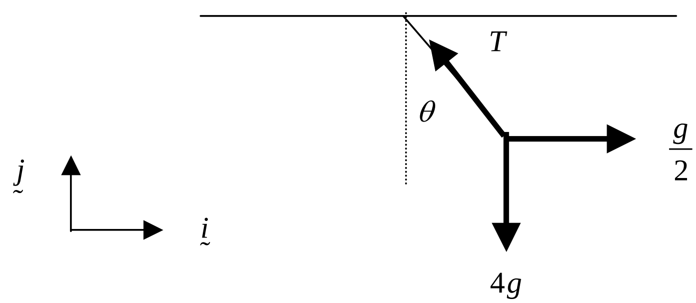
Area $= \displaystyle\int_0^2 \left(\frac{6}{x-3} - \frac{6}{x+3}\right) dx = \left[6\log_e\left(\left|\frac{x-3}{x+3}\right|\right)\right]_0^2 = 6\log_e\left|\frac{-1}{5}\right| - 6\log_e\left|\frac{-3}{3}\right| = 6\log_e\left(\frac{1}{5}\right)$

Area $= -6\log_e(5)$ but the area is below the $x$-axis and area is negative.

so that the area is $6\log_e(5)$

**Question 4**

**a.**




**b.** resolving horizontally $\underset{\sim}{i}$ $\quad \dfrac{g}{2} - T\sin\theta = 0 \quad (1)$

resolving vertically $\underset{\sim}{j}$ $\quad T\cos\theta - 4g = 0 \quad (2)$

$(1) \Rightarrow T\sin\theta = \dfrac{g}{2}$ and $(2) \Rightarrow T\cos\theta = 4g$ squaring and adding gives

$T^2\left(\sin^2(\theta)+\cos^2(\theta)\right) = \dfrac{g^2}{4} + 16g^2 = g^2\left(16+\dfrac{1}{4}\right) = \dfrac{65g^2}{4}$

$T = \dfrac{g\sqrt{65}}{2}$ so $a=65 \quad b=2$

**Question 5**

**a.** using $\tan(2A) = \dfrac{2\tan(A)}{1-\tan^2(A)}$ let $A = \dfrac{\pi}{8}$ $\quad \tan\left(\dfrac{\pi}{4}\right) = 1 = \dfrac{2\tan\left(\frac{\pi}{8}\right)}{1-\tan^2\left(\frac{\pi}{8}\right)}$

let $u = \tan\left(\dfrac{\pi}{8}\right)$ $\quad 1 = \dfrac{2u}{1-u^2}$ or $2u = 1-u^2$ so that $u^2+2u-1=0$

solving the quadratic $\Delta = 4 - -4 = 8$ so $u = \dfrac{-2\pm\sqrt{8}}{2} = -1\pm\sqrt{2}$ but $u = \tan\left(\dfrac{\pi}{8}\right) > 0$

so take the positive $\tan\left(\dfrac{\pi}{8}\right) = \sqrt{2}-1$

**b.** If $y = \tan^{-1}(x-1) + a\tan\left(\frac{\pi}{8}\right)$ and $y > 0$.

Now the range of $y = \tan^{-1}(x)$ or $y = \tan^{-1}(x-1)$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

so $a\left(\sqrt{2}-1\right) - \frac{\pi}{2} > 0$ hence $a > \frac{\pi}{2\left(\sqrt{2}-1\right)}$

so the minimum value of $a = \frac{\pi}{2\left(\sqrt{2}-1\right)} = \frac{\pi\left(\sqrt{2}+1\right)}{2}$

**Question 6**

**a.** If $y = e^{-x}$ then $y^2 = e^{-2x}$ the volume $V = \pi \int_0^a y^2 \, dx$

so $V = \pi \int_0^a e^{-2x} \, dx$

**b.** $V = \pi\left[-\frac{1}{2}e^{-2x}\right]_0^a = \pi\left[-\frac{1}{2}e^{-2a} + \frac{1}{2}\right] = \frac{\pi}{2}\left(1 - e^{-2a}\right)$

**c.** If $V = \frac{5\pi}{8} = \frac{\pi}{2}\left(1 - e^{-2a}\right)$ then $1 - e^{-2a} = \frac{5}{9}$ $e^{-2a} = \frac{4}{9}$

$e^{2a} = \frac{9}{4}$ $e^{a} = \frac{3}{2}$ since $a > 0$ $a = \log_e\left(\frac{3}{2}\right)$

**Question 7**

$\underset{\sim}{r}(t) = \sqrt{t-2}\,\underset{\sim}{i} + 2t\,\underset{\sim}{j}$ for $2 \le t \le 6$

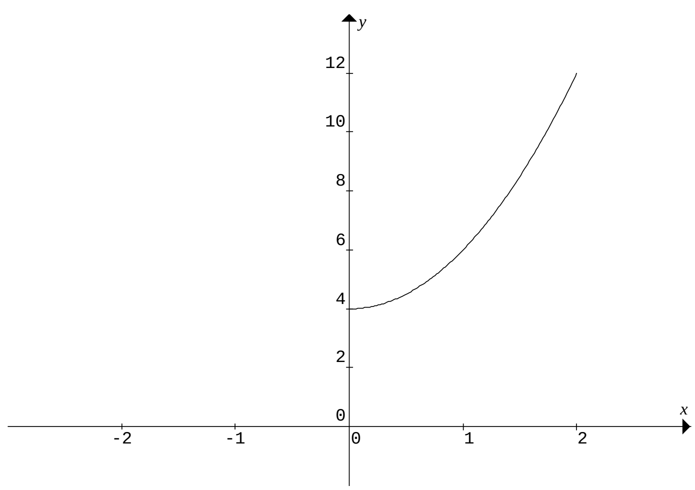
**a.** the parametric equations are $(1)$ $x = \sqrt{t-2}$ and $(2)$ $y = 2t \Rightarrow t = \frac{y}{2}$ substitute into $(1)$

so $x = \sqrt{\frac{y}{2} - 2} = \sqrt{\frac{y-4}{2}}$ $x^2 = \frac{y-4}{2}$ $2x^2 = y - 4$

$y = 4 + 2x^2$ but only for $0 \le x \le 2$, the range is $4 \le y \le 12$

**b.**

| $t$ | $x$ | $y$ |
|---|---|---|
| 2 | 0 | 4 |
| 3 | 1 | 6 |
| 4 | $\sqrt{2}$ | 8 |
| 5 | $\sqrt{3}$ | 10 |
| 6 | 2 | 12 |



**Question 8**

$$\int\left(\frac{2+6x}{\sqrt{4-x^2}}\right)dx = \int\frac{2}{\sqrt{4-x^2}}+6\int\frac{x}{\sqrt{4-x^2}}dx$$

in the second integral let $u=4-x^2$ $\frac{du}{dx}=-2x$ $x\,dx=-\frac{1}{2}du$

$$\int\left(\frac{2+6x}{\sqrt{4-x^2}}\right)dx = 2\sin^{-1}\left(\frac{x}{2}\right)-3\int u^{-\frac{1}{2}}du=-6u^{\frac{1}{2}}$$

$$\int\left(\frac{2+6x}{\sqrt{4-x^2}}\right)dx = 2\sin^{-1}\left(\frac{x}{2}\right)-6\sqrt{4-x^2}+C$$

**Question 9**

**a.** let $z=1+\sqrt{3}\,i$ $|z|=\sqrt{1+3}=\sqrt{4}=2$ $Arg(z)=\tan^{-1}\left(\sqrt{3}\right)=\frac{\pi}{3}$

so $z=2\,\text{cis}\left(\frac{\pi}{3}\right)$

**b.** $z^2+2z-\sqrt{3}\,i=0$ using the quadratic formula

$\Delta=4+4\sqrt{3}\ i=4\left(1+\sqrt{3}i\right)$

$$z=\frac{-2\pm\sqrt{\Delta}}{2}=\frac{-2\pm2\sqrt{1+\sqrt{3}\,i}}{2}$$

$=-1\pm\sqrt{1+\sqrt{3}\,i}$ to find the square root use **a.**

Now let $w^2=1+\sqrt{3}\,i=2\,\text{cis}\left(\frac{\pi}{3}+2k\pi\right)$

$$w=\sqrt{2}\,\text{cis}\left(\frac{\pi}{6}+k\pi\right)$$

if $k=0$ $w_1=\sqrt{2}\,\text{cis}\left(\frac{\pi}{6}\right)=\sqrt{2}\left(\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)\right)=\frac{\sqrt{6}}{2}+i\frac{\sqrt{2}}{2}$

if $k=1$ $w_2=\sqrt{2}\,\text{cis}\left(\frac{5\pi}{6}\right)=\sqrt{2}\left(\cos\left(\frac{5\pi}{6}\right)+i\sin\left(\frac{5\pi}{6}\right)\right)=\frac{-\sqrt{6}}{2}-i\frac{\sqrt{2}}{2}=-w_1$

so $\sqrt{1+\sqrt{3}\ i}=\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}i$

and $z=-1\pm\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}i\right)=-\frac{\left(\sqrt{6}+2\right)}{2}-\frac{\sqrt{2}}{2}\,i$ and $\frac{\sqrt{6}-2}{2}+\frac{\sqrt{2}}{2}\,i$

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# Mathematics 2007


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