

Year 2006

VCE

Specialist Mathematics

Examination 1

Suggested Solutions



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Solutions Specialist Mathematics Examination 1 2006

Question 1

a. $2xy - 9y^2 + 9 = 0$ taking $\frac{d}{dx}$ of each term $\frac{d}{dx}(2xy) - \frac{d}{dx}(9y^2) + \frac{d}{dx}(9) = 0$

using the product rule on the first term $2y + 2x\frac{dy}{dx} - 18y\frac{dy}{dx} = 0$

re-arranging to $2y = (18y - 2x)\frac{dy}{dx}$ so that $\frac{dy}{dx} = \frac{y}{9y - x}$

b. when $y = 1$ then $2x - 9 + 9 = 0$ so $x = 0$ the point is $(0, 1)$ $\left. \frac{dy}{dx} \right|_{(0,1)} = \frac{1}{9-0} = \frac{1}{9}$

Question 2

$\frac{dy}{dx} = x\sqrt{x^2 - 16}$; $x \geq 4$ given $y(5) = 13$ integrating

$y = \int x\sqrt{x^2 - 16} dx$ let $u = x^2 - 16$ $\frac{du}{dx} = 2x$ $x dx = \frac{1}{2} du$

$y = \frac{1}{2} \int u^{\frac{1}{2}} du = \frac{1}{3} u^{\frac{3}{2}} + c$

$y = \frac{1}{3} (x^2 - 16)^{\frac{3}{2}} + c$ to find c use $x = 5$ when $y = 13$

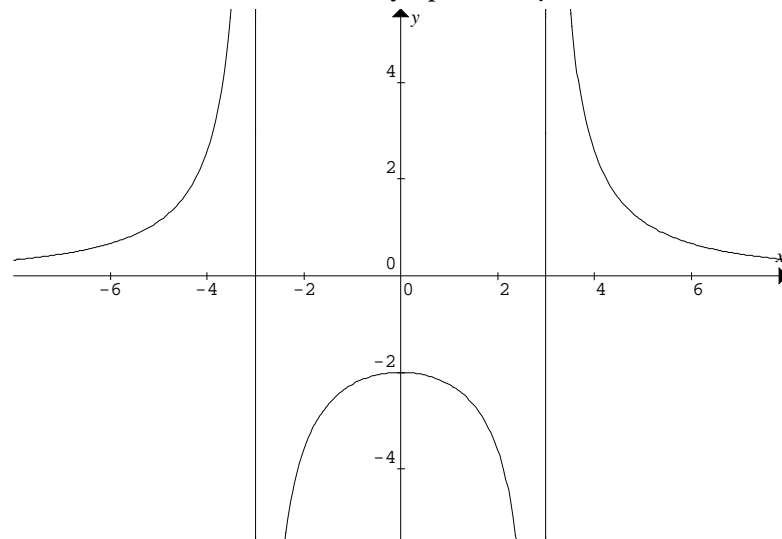
$13 = \frac{1}{3} (25 - 16)^{\frac{3}{2}} + c = 9 + c$ so that $c = 4$

$y = \frac{1}{3} (x^2 - 16)^{\frac{3}{2}} + 4$

Question 3

a. $y = \frac{36}{2x^2 - 18} = \frac{18}{x^2 - 9}$ there are vertical asymptotes when $x^2 - 9 = (x + 3)(x - 3) = 0$

at $x = \pm 3$, the graph crosses y-axis at $x = 0$ $y = -2$ $(0, -2)$, the graph does not cross the x-axis, since there is a horizontal asymptote at $y = 0$



b. Area = $\int_{-2}^2 \frac{36}{2x^2-18} dx = \int_0^2 \frac{36}{x^2-9} dx$ by symmetry

by partial fractions $\frac{36}{x^2-9} = \frac{A}{x+3} + \frac{B}{x-3} = \frac{A(x-3)+B(x+3)}{(x+3)(x-3)} = \frac{x(A+B)+3B-3A}{x^2-9}$

so $A+B=0 \Rightarrow A=-B$ and $3B-3A=36$ so $6B=36$ and $B=6$ and $A=-6$

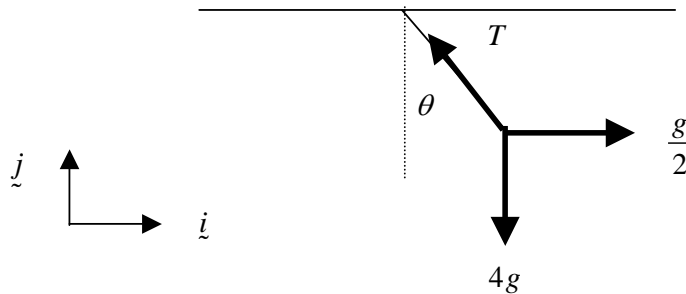
Area = $\int_0^2 \left(\frac{6}{x-3} - \frac{6}{x+3} \right) dx = \left[6 \log_e \left(\frac{|x-3|}{|x+3|} \right) \right]_0^2 = 6 \log_e \left| \frac{-1}{5} \right| - 6 \log_e \left| \frac{-3}{3} \right| = 6 \log_e \left(\frac{1}{5} \right)$

Area = $-6 \log_e(5)$ but the area is below the x -axis and area is negative.

so that the area is $6 \log_e(5)$

Question 4

a.



b. resolving horizontally $i \quad \frac{g}{2} - T \sin \theta = 0 \quad (1)$

resolving vertically $j \quad T \cos \theta - 4g = 0 \quad (2)$

$(1) \Rightarrow T \sin \theta = \frac{g}{2}$ and $(2) \Rightarrow T \cos \theta = 4g$ squaring and adding gives

$$T^2 (\sin^2(\theta) + \cos^2(\theta)) = \frac{g^2}{4} + 16g^2 = g^2 \left(16 + \frac{1}{4} \right) = \frac{65g^2}{4}$$

$$T = \frac{g\sqrt{65}}{2} \text{ so } a = 65 \text{ } b = 2$$

Question 5

a. using $\tan(2A) = \frac{2 \tan(A)}{1 - \tan^2(A)}$ let $A = \frac{\pi}{8}$ $\tan\left(\frac{\pi}{4}\right) = 1 = \frac{2 \tan\left(\frac{\pi}{8}\right)}{1 - \tan^2\left(\frac{\pi}{8}\right)}$

let $u = \tan\left(\frac{\pi}{8}\right)$ $1 = \frac{2u}{1-u^2}$ or $2u = 1-u^2$ so that $u^2 + 2u - 1 = 0$

solving the quadratic $\Delta = 4 - 4 = 8$ so $u = \frac{-2 \pm \sqrt{8}}{2} = -1 \pm \sqrt{2}$ but $u = \tan\left(\frac{\pi}{8}\right) > 0$

so take the positive $\tan\left(\frac{\pi}{8}\right) = \sqrt{2} - 1$

- b. If $y = \tan^{-1}(x-1) + a \tan\left(\frac{\pi}{8}\right)$ and $y > 0$.

Now the range of $y = \tan^{-1}(x)$ or $y = \tan^{-1}(x-1)$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$\text{so } a\left(\sqrt{2}-1\right) - \frac{\pi}{2} > 0 \quad \text{hence } a > \frac{\pi}{2(\sqrt{2}-1)}$$

$$\text{so the minimum value of } a = \frac{\pi}{2(\sqrt{2}-1)} = \frac{\pi(\sqrt{2}+1)}{2}$$

Question 6

- a. If $y = e^{-x}$ then $y^2 = e^{-2x}$ the volume $V = \pi \int_0^a y^2 dx$

$$\text{so } V = \pi \int_0^a e^{-2x} dx$$

$$\text{b. } V = \pi \left[-\frac{1}{2} e^{-2x} \right]_0^a = \pi \left[-\frac{1}{2} e^{-2a} + \frac{1}{2} \right] = \frac{\pi}{2} (1 - e^{-2a})$$

$$\text{c. If } V = \frac{5\pi}{8} = \frac{\pi}{2} (1 - e^{-2a}) \quad \text{then } 1 - e^{-2a} = \frac{5}{9} \quad e^{-2a} = \frac{4}{9}$$

$$e^{2a} = \frac{9}{4} \quad e^a = \frac{3}{2} \quad \text{since } a > 0 \quad a = \log_e \left(\frac{3}{2} \right)$$

Question 7

$$\mathbf{r}(t) = \sqrt{t-2} \mathbf{i} + 2t \mathbf{j} \quad \text{for } 2 \leq t \leq 6$$

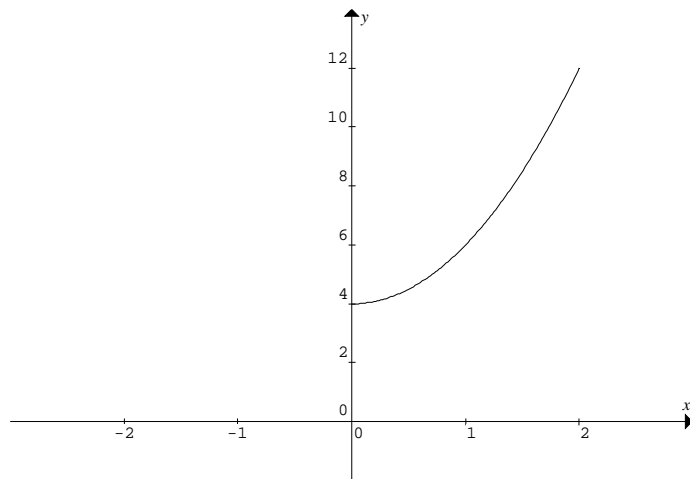
- a. the parametric equations are (1) $x = \sqrt{t-2}$ and (2) $y = 2t \Rightarrow t = \frac{y}{2}$ substitute into (1)

$$\text{so } x = \sqrt{\frac{y}{2} - 2} = \sqrt{\frac{y-4}{2}} \quad x^2 = \frac{y-4}{2} \quad 2x^2 = y-4$$

$$y = 4 + 2x^2 \quad \text{but only for } 0 \leq x \leq 2, \quad \text{the range is } 4 \leq y \leq 12$$

- b.

t	x	y
2	0	4
3	1	6
4	$\sqrt{2}$	8
5	$\sqrt{3}$	10
6	2	12



Question 8

$$\int \left(\frac{2+6x}{\sqrt{4-x^2}} \right) dx = \int \frac{2}{\sqrt{4-x^2}} + 6 \int \frac{x}{\sqrt{4-x^2}} dx$$

in the second integral let $u = 4 - x^2$ $\frac{du}{dx} = -2x$ $x dx = -\frac{1}{2} du$

$$\int \left(\frac{2+6x}{\sqrt{4-x^2}} \right) dx = 2 \sin^{-1} \left(\frac{x}{2} \right) - 3 \int u^{-\frac{1}{2}} du = -6u^{\frac{1}{2}}$$

$$\int \left(\frac{2+6x}{\sqrt{4-x^2}} \right) dx = 2 \sin^{-1} \left(\frac{x}{2} \right) - 6\sqrt{4-x^2} + C$$

Question 9

a. let $z = 1 + \sqrt{3}i$ $|z| = \sqrt{1+3} = \sqrt{4} = 2$ $\text{Arg}(z) = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

so $z = 2 \text{cis} \left(\frac{\pi}{3} \right)$

b. $z^2 + 2z - \sqrt{3}i = 0$ using the quadratic formula

$$\Delta = 4 + 4\sqrt{3}i = 4(1 + \sqrt{3}i)$$

$$z = \frac{-2 \pm \sqrt{\Delta}}{2} = \frac{-2 \pm 2\sqrt{1 + \sqrt{3}i}}{2}$$

$$= -1 \pm \sqrt{1 + \sqrt{3}i} \quad \text{to find the square root use a.}$$

Now let $w^2 = 1 + \sqrt{3}i = 2 \text{cis} \left(\frac{\pi}{3} + 2k\pi \right)$

$$w = \sqrt{2} \text{cis} \left(\frac{\pi}{6} + k\pi \right)$$

if $k = 0$ $w_1 = \sqrt{2} \text{cis} \left(\frac{\pi}{6} \right) = \sqrt{2} \left(\cos \left(\frac{\pi}{6} \right) + i \sin \left(\frac{\pi}{6} \right) \right) = \frac{\sqrt{6}}{2} + i \frac{\sqrt{2}}{2}$

if $k = 1$ $w_2 = \sqrt{2} \text{cis} \left(\frac{5\pi}{6} \right) = \sqrt{2} \left(\cos \left(\frac{5\pi}{6} \right) + i \sin \left(\frac{5\pi}{6} \right) \right) = \frac{-\sqrt{6}}{2} - i \frac{\sqrt{2}}{2} = -w_1$

so $\sqrt{1 + \sqrt{3}i} = \frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}i$

and $z = -1 \pm \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}i \right) = -\frac{(\sqrt{6}+2)}{2} - \frac{\sqrt{2}}{2}i$ and $\frac{\sqrt{6}-2}{2} + \frac{\sqrt{2}}{2}i$



Mathematics 2007

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