The Mathematical Association of Victoria

SPECIALIST MATHEMATICS

Trial written examination 1

2006

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: ....

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

These questions have been written and published to assist students in their preparations for the 2006 Specialist Mathematics Examination 1. The questions and associated answers and solutions do not necessarily reflect the views of the Victorian Curriculum and Assessment Authority. The Association gratefully acknowledges the permission of the Authority to reproduce the formula sheet.

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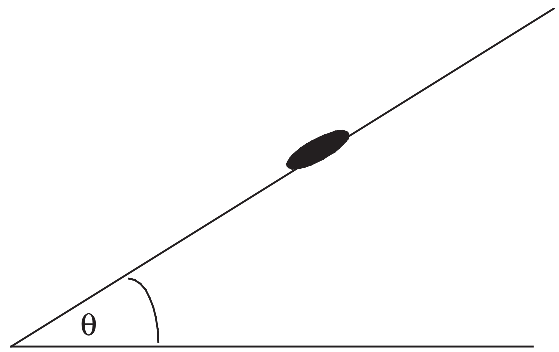
Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

a. A coin of mass m kg is just prevented from slipping down a book when it is inclined at an angle of θ to the horizontal.

i. On the diagram below mark in all the forces acting on the coin.



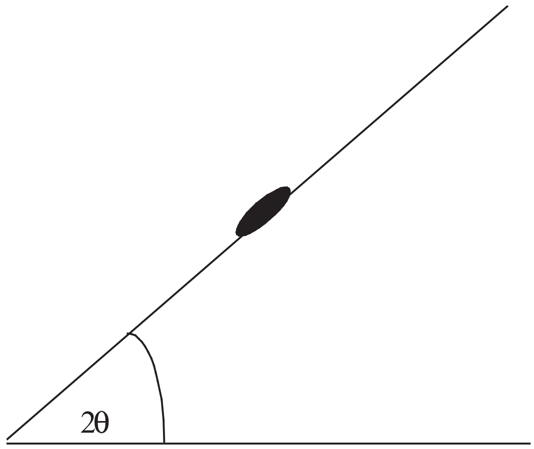
1 mark

ii. Show that the co-efficient of friction between the book and the coin is given by $\mu = \tan(\theta)$

1 mark

b. The book is now raised so that it is inclined at an angle of 2θ to the horizontal, and a force of T newtons acts on the coin, up and parallel to the book. The coin is just on the point of moving up the book.

i. On the diagram below mark in all the forces acting on the coin.



1 mark

ii. Show that $T = \dfrac{mg\sin(3\theta)}{\cos(\theta)}$

3 marks

Question 2

$y = \cos\left(x^2\right)$ is a solution of the differential equation $x\dfrac{d^2y}{dx^2} + a\dfrac{dy}{dx} + bx^3y = 0$ where $a, b \in R$.
Find the values of a and b.

4 marks

Question 3

Consider the relation $2x^2 + 12x + y^2 - 8y + 22 = 0$

Find an expression for $\dfrac{dy}{dx}$ in terms of both x and y. Hence find the value of x for which the tangent to the curve is parallel to the x-axis.

3 marks

Question 4

a. Given $P(z) = z^4 + pz^2 - 8$, where p is a real constant. If $P(2i) = 0$ show that $p = 2$

1 mark

b. Find all the roots of $z^4 + 2z^2 - 8 = 0$

2 marks

PART I - continued
TURN OVER

Question 5

a. Show that $\dfrac{d}{dx}\left(\tan^{-1}\left(\dfrac{3x^2}{4}\right)\right) = \dfrac{24x}{9x^4 + 16}$

2 marks

b. Hence find the exact value of $\displaystyle\int_0^{\frac{2\sqrt{3}}{3}} \frac{x}{9x^4 + 16}\,dx$.

2 marks

Question 6

a. If $f(x) = \frac{x}{\sqrt{2x-3}}$ then gradient function $f'(x)$ can be represented as $\frac{ax+b}{\sqrt{(2x-3)^3}}$.

Find the exact values of a and b.

3 marks

b. Find, using calculus, the exact area A bounded by the curve $y = \frac{x}{\sqrt{2x-3}}$, the x axis and the lines $x = 2$ and $x = 6$.

3 marks

Question 7

A particle moves so that its position vector is given by $\underset{\sim}{r}(t) = (3 + 4\cos(2t))\underset{\sim}{i} + (-2 + 3\sin(2t))\underset{\sim}{j}$ for $t \geq 0$

a. Find the Cartesian equation of the path.

2 marks

b. Determine the speed of the particle and find the maximum and minimum speeds.

3 marks

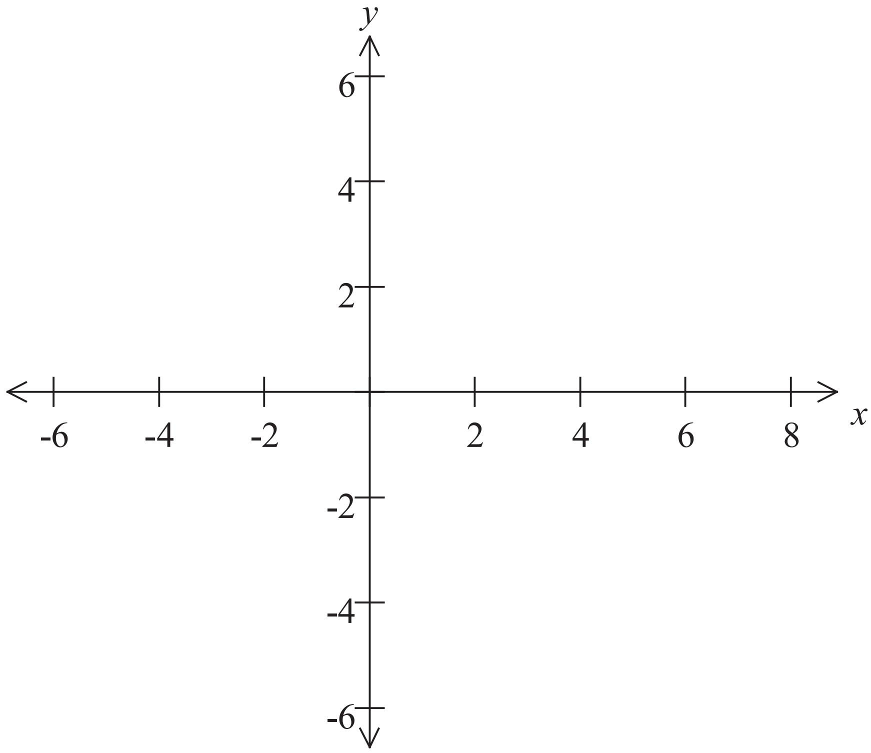
Question 8

Consider the ellipse with the equation $\frac{(x-c)^2}{16}+\frac{(y+2)^2}{9}=1$, where c is a real constant.
If the domain is $[-1, 7]$

a. Show that $c = 3$

1 mark

b. Sketch the graph of the ellipse on the following set of axes.

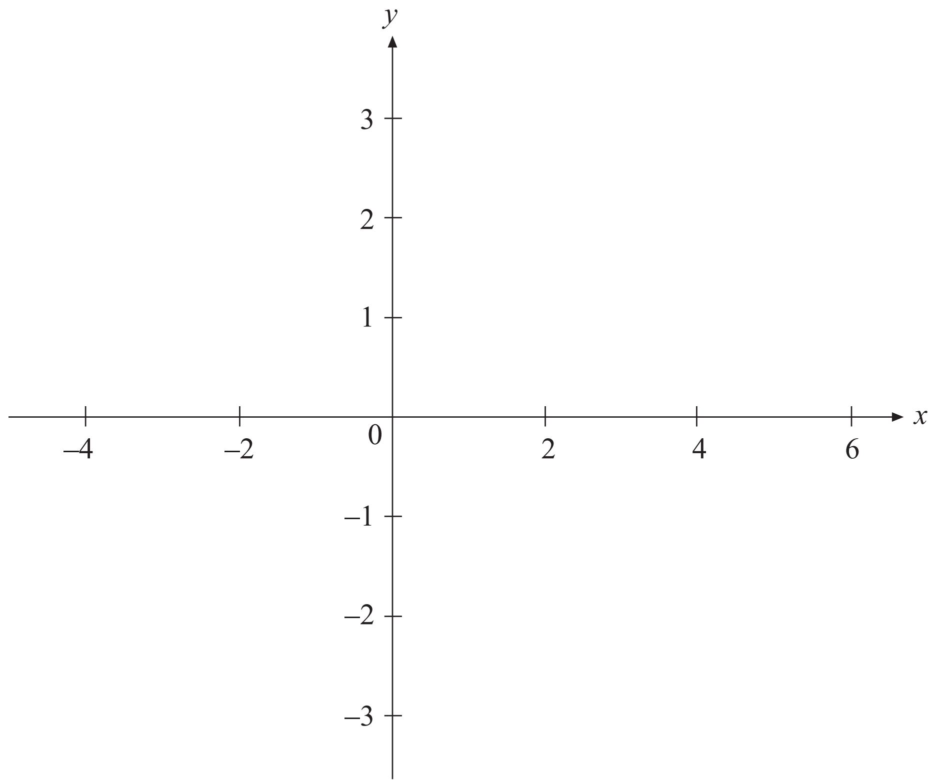


2 marks

PART I - continued
TURN OVER

Question 9

a. Sketch the graph of $y = \dfrac{12}{12 + 4x - x^2}$ on the axes below, clearly indicating the equations of all asymptotes, and the coordinates of any stationary points and axial intercepts.



3 marks

b. The area bounded by the curve $y = \dfrac{12}{12 + 4x - x^2}$, the co-ordinate axes and the line $x = 3$, can be expressed in the form $\log_e\left(\sqrt{p}\right)$.

Find the exact value of p.

3 marks

END OF PART I QUESTION AND ANSWER BOOK

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

Version 3 – March 2006

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$ $\cot^2(x) + 1 = \text{cosec}^2(x)$

$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ $\sin(x - y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ $\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$ $\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\,\text{cis}\,\theta$

$|z|=\sqrt{x^2+y^2}=r$ $-\pi < \text{Arg}\, z \le \pi$

$z_1z_2 = r_1r_2\,\text{cis}(\theta_1 + \theta_2)$ $\frac{z_1}{z_2}=\frac{r_1}{r_2}\text{cis}(\theta_1-\theta_2)$

$z^n = r^n\,\text{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e |x| + c$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$$

$$\int \sin(ax)\, dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$$

$$\int \cos(ax)\, dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$$

$$\int \sec^2(ax)\, dx = \frac{1}{a}\tan(ax) + c$$

$$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$$

$$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$$

$$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Euler's method: If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\quad s = ut + \frac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \frac{1}{2}(u+v)t$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1x_2 + y_1y_2 + z_1z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \le \mu N$

END OF FORMULA SHEET