Trial Examination 2006

VCE Specialist Mathematics Units 3 & 4

Written Examination 1

Suggested Solutions

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Question 1

a. $2 - 3i$ — A1

b. $(z - 2 - 3i)(z - 2 + 3i)$ — M1

$= (z - 2)^2 - (3i)^2$

$= z^2 - 4z + 13$ — A1

c. By division or inspection

$z = 2$ — A1

Question 2

a. $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= 3\underset{\sim}{q} - 2\underset{\sim}{p}$ — A1

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$

$= 9\underset{\sim}{q} - 6\underset{\sim}{p}$

$= 3\overrightarrow{AB}$ — A1

So $\overrightarrow{AB}$ is parallel to $\overrightarrow{AC}$ and A is the point in common. — M1

$\therefore$ A, B and C are collinear.

b. $\overrightarrow{BC} = 6\underset{\sim}{q} - 4\underset{\sim}{p}$

$= 2\overrightarrow{AB}$

$\therefore$ $|\overrightarrow{AB}|:|\overrightarrow{BC}| = 1 : 2$ — A1

Question 3

Use of $\cos(2x) = 2\cos^2(x) - 1$ — M1

$\text{LHS} = \cos^2(2x) - \sin^2(x) = (2\cos^2(x) - 1)^2 - \sin^2(x)$ — A1

$= 4\cos^4(x) - 4\cos^2(x) + 1 - (1 - \cos^2(x))$

$= 4\cos^4(x) - 3\cos^2(x)$ — A1

$= \cos^2(x)(4\cos^2(x) - 3)$

$= \cos^2(x)(2\cos(x) - \sqrt{3})(2\cos(x) + \sqrt{3}) = \text{RHS}$ — A1

Question 4

$f(x) = \arctan(2x)$

$= \arctan\left(\frac{x}{1/2}\right)$

$f'(x) = \frac{1/2}{1/4 + x^2}$ — M1

$= \frac{2}{4x^2 + 1}$ — A1

$f''(x) = -\frac{16x}{(4x^2 + 1)^2}$ — A1

Question 5

$\frac{dT}{dt} = k(T - 20) \qquad (k < 0)$ A1

$\frac{dt}{dT} = \frac{1}{k(T - 20)}$

$t = \frac{1}{k}\log_e(T - 20) + c \quad$ (since $T - 20 > 0$, $|T - 20|$ is not required) A1

When $t = 0$, $T = 80$

$0 = \frac{1}{k}\log_e(60) + c$

$c = -\frac{1}{k}\log_e(60)$ M1

$t = \frac{1}{k}\log_e\left(\frac{T - 20}{60}\right)$

When $t = 10$, $T = 65$

$10 = \frac{1}{k}\log_e\left(\frac{45}{60}\right)$

$k = \frac{1}{10}\log_e\left(\frac{3}{4}\right)$ A1

$t = \frac{10}{\log_e\left(\frac{3}{4}\right)}\log_e\left(\frac{T - 20}{60}\right)$

The time taken to cool to 50°C is $\frac{10\log_e\left(\frac{1}{2}\right)}{\log_e\left(\frac{3}{4}\right)}$ minutes. A1

Question 6

$\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{1}{\sqrt{1 - 4x^2}}dx = \frac{1}{2}\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{1}{\sqrt{\frac{1}{4} - x^2}}dx$ M1

$= \frac{1}{2}\left[\sin^{-1}(2x)\right]_{\frac{1}{4}}^{\frac{1}{2}}$ A1

$= \frac{1}{2}\left(\sin^{-1}(1) - \sin^{-1}\left(\frac{1}{2}\right)\right)$

$= \frac{1}{2}\left(\frac{\pi}{2} - \frac{\pi}{6}\right)$ A1

$= \frac{\pi}{6}$

Hence $k = \frac{1}{6}$ A1

Question 7

$\frac{dy}{dx} = \sin(x)\cos^2(x)$

$y = \int (\sin(x)\cos^2(x))dx$

Let $u = \cos(x)$ and so $\frac{du}{dx} = -\sin(x)$. M1

$y = -\int u^2 du$ A1

$y = -\frac{u^3}{3} + c$

$y = -\frac{\cos^3(x)}{3} + c$ A1

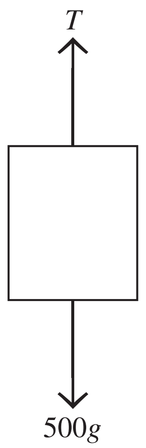
Apply the condition to find the value of c.

$-\frac{4}{3} = -\frac{1}{3} + c$ and so $c = -1$

Hence $y = -\left(\frac{\cos^3(x)}{3} + 1\right)$. A1

Question 8

a.



where T = tension in the cable

A1

b. $\Sigma F = ma \Rightarrow 500 \times g - T = 500 \times 1.8$

$$T = 500(g - 1.8)$$
$$= 500 \times 8$$
$$= 4000 \text{ N}$$ A1

c.

where N = normal reaction

A1

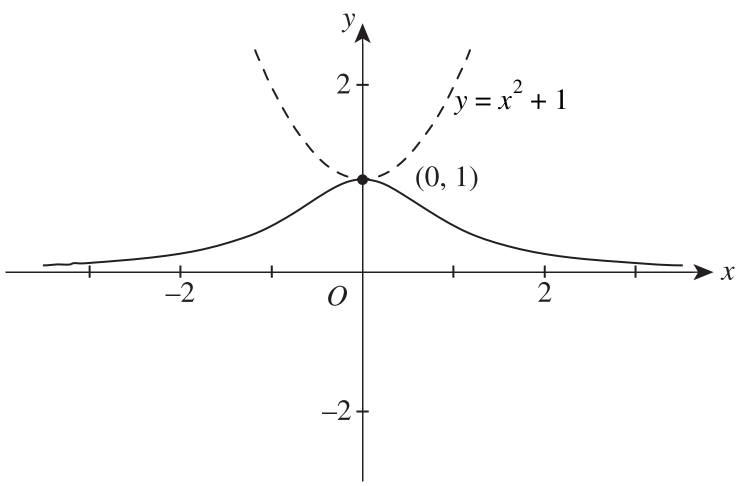
d. $\Sigma F = ma \Rightarrow 80 \times g - N = 80 \times 1.8$

$N = 80 \times 8$

$= 640$ N A1

Question 9

a.



Maximum turning point labelled correctly and horizontal asymptote at $y = 0$. A1

b. $b^2 - 4c < 0$ or $b^2 < 4c$ A1

c. $f(x) = (x^2 + bx + c)^{-1}$

$f'(x) = \dfrac{-(2x+b)}{(x^2+bx+c)^2}$ M1

$f'(x) = 0$ when $2x + b = 0$

Since $x = -3$, $-6 + b = 0$ and so $b = 6$. A1

d. There is an asymptote at $x = 0$,

so $x^2 + 6x + c$ has x as a factor.

$\therefore\ c = 0$ A1

e. $f(x) = \dfrac{1}{x^2 + 6x}$

$= \dfrac{1}{x(x+6)}$

$= \dfrac{1}{6}\left(\dfrac{1}{x} - \dfrac{1}{x+6}\right)$ A1

So $A = -\dfrac{1}{6}\displaystyle\int_{-2}^{-1}\left(\dfrac{1}{x} - \dfrac{1}{x+6}\right)dx$

$= -\dfrac{1}{6}\left[\log_e\left|\dfrac{x}{x+6}\right|\right]_{-2}^{-1}$ M1

$= -\dfrac{1}{6}\left(\log_e\left(\dfrac{1}{5}\right) - \log_e\left(\dfrac{1}{2}\right)\right)$

$= -\dfrac{1}{6}\log_e\left(\dfrac{2}{5}\right)$ $\left(= \dfrac{1}{6}\log_e\left(\dfrac{5}{2}\right)\right)$ A1