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SPECIALIST MATHEMATICS

WRITTEN TRIAL EXAMINATION 1

2007

Reading Time: 15 minutes
Writing time: 1 hour

Instructions to students

This exam consists of 9 questions.
All questions should be answered.
There is a total of 40 marks available.
The marks allocated to each of the nine questions are indicated throughout.
Students may not bring any notes or calculators into the exam.
Where more than one mark is allocated to a question, appropriate working must be shown.
Where an exact answer is required, a decimal approximation will not be accepted.
Unless otherwise indicated, diagrams in this exam are not drawn to scale.
The acceleration due to gravity should be taken to have magnitude g m/s^2 where $g = 9.8$
Formula sheets can be found on pages 10-12 of this exam.

Question 1

a. Show that for $x > \frac{1}{2}$, $\frac{d}{dx}\left(\arctan\left(\sqrt{2x-1}\right)\right) = \frac{1}{2x\sqrt{2x-1}}$.

2 marks

b. Hence find the exact value of $\int_1^2 \frac{1}{x\sqrt{2x-1}}\,dx$.

2 marks

Question 2

Consider the relation given by $2x^2y+y^2-5x=3$.

a. Find $\frac{dy}{dx}$ in terms of *x* and *y*.

2 marks

b. Hence evaluate $\frac{dy}{dx}$ when $y=0$.

2 marks

Question 3

Solve the differential equation $\frac{dy}{dx}=(x-2)\sqrt{x-1}$ given that $y(1)=0$.

3 marks

Question 4

Solve the following equations over C

a. $z^2 - z + 2.5 = 0$

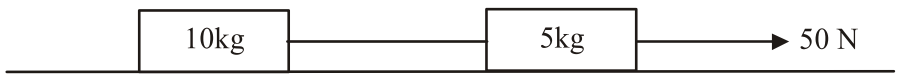
2 marks

b. $z^3 + z^2 + 3z - 5 = 0$

3 marks

Question 5

Two containers of mass 5kg and 10kg are connected by a light rope.
The containers are pulled along a rough horizontal surface by a force of 50*N* acting horizontally as shown in the diagram below.
The coefficient of friction between each of the containers and the surface is the same. The containers are moving with an acceleration of $2\text{m}/\text{s}^2$.



a. Show that the coefficient of friction equals $\frac{4}{3g}$.

3 marks

b. Hence find the tension in the rope connecting the containers.

1 mark

Question 6

The area enclosed by the graph of $y = \dfrac{1}{x^2 - 4x}$, the x-axis and the lines $x = 1$ and $x = 3$ is given by $a\log_e(b)$ square units where a and b are positive constants.
Find the values of a and b.

5 marks

Question 7

The region enclosed by the graph of the continuous function $y = \dfrac{1}{\sqrt{4 + x^2}}$, the x-axis, the y-axis and the line $x = 2$ is rotated about the x-axis to form a solid of revolution.
Find the volume of this solid.

3 marks

Question 8

Evaluate $\int_0^3 \frac{4(x-1)}{\sqrt{9-x^2}}\,dx$.

4 marks

Question 9

The velocity of a particle at time t seconds is given by $\underset{\sim}{v}(t) = \left(2\sin^2(t) - 1\right)\underset{\sim}{i} - \sin(2t)\underset{\sim}{j}\quad t \geq 0$.

a. Show that the speed of the particle is constant.

2 marks

b. Show that the position vector of the particle is given by

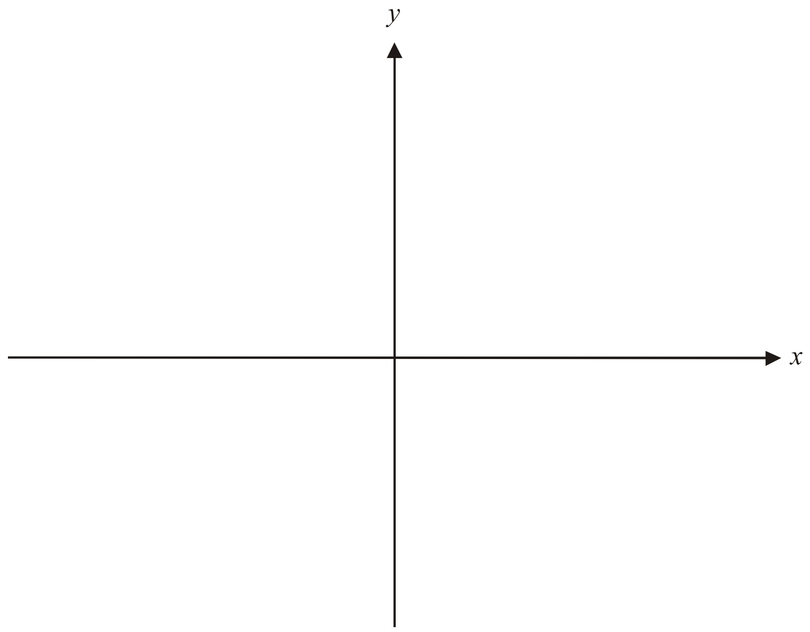
$$\underset{\sim}{r} = -\frac{1}{2}\sin(2t)\underset{\sim}{i} + \frac{1}{2}\cos(2t)\underset{\sim}{j},\quad t \geq 0,\quad \text{given that } \underset{\sim}{r}(0) = \frac{1}{2}\underset{\sim}{j}.$$

2 marks

c. Find the Cartesian equation of the path of the particle.

1 mark

d. **i.** Sketch the path of the particle on the set of axes below.



ii. Describe the path of the particle including it's starting point and the subsequent direction of movement.

1 + 2 = 3 marks

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$ $\qquad$ $\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ $\qquad$ $\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ $\qquad$ $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$ $\qquad$ $\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$ $\qquad$ $\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\operatorname{cis}\theta$

$|z| = \sqrt{x^2 + y^2} = r$ $\qquad$ $-\pi < \operatorname{Arg} z \le \pi$

$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$ $\qquad$ $\frac{z_1}{z_2} = \frac{r_1}{r_2}\operatorname{cis}(\theta_1 - \theta_2)$

$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$ $$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$ $$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$$ $$\int \frac{1}{x}\,dx = \log_e|x| + c$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$$ $$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$$ $$\int \cos\,(ax)\,dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$$ $$\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$$

$$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$$ $$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$$

$$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$$ $$\int \frac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$$

$$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$$ $$\int \frac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Euler's method: If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$,
then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\quad s = ut + \frac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \frac{1}{2}(u+v)t$

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The VCAA publish an exam issue supplement to the VCAA bulletin.

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\underset{\sim}{r}_1 . \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1x_2 + y_1y_2 + z_1z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \le \mu N$