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Specialist Mathematics

2007

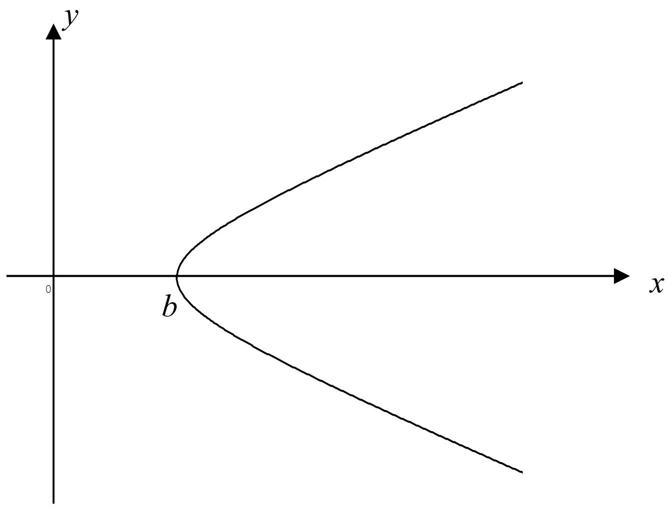
Trial Examination 2

SECTION 1 Multiple-choice questions

Instructions for Section 1

Answer **all** questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1



Given $a, b \in R^+$, the best equation for the above graph is

A. $y = \pm\sqrt{\dfrac{x-b}{a}}$

B. $y = \pm\sqrt{\dfrac{b-x}{a}}$

C. $y = \pm\dfrac{\sqrt{x-b}}{a}$

D. $y = \pm\dfrac{a}{b}\sqrt{x^2 - b^2}$

E. $y = \pm\dfrac{a}{b}\sqrt{b^2 - x^2}$

Question 2 $\frac{1}{1+\sin x}$ is identical to

A. $\sec^2 x + \cos ec^2 x \tan x$

B. $\tan^2 x + \sec x \cos ec^2 x$

C. $\tan^2 x + \sec^2 x \cos ecx$

D. $\sec^2 x - \cos ecx \tan^2 x$

E. $\sec^2 x - \cos ec^2 x \tan x$

Question 3 $f(x) = \cos\left[\sin^{-1}(ax)\right]$ where $a > 1$, has an inverse function if

A. $x \in \left[-\frac{1}{a}, \frac{1}{a}\right]$

B. $x \in [0, a]$

C. $x \in \left[-\frac{1}{a}, 0\right]$

D. $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

E. $x \in \left[0, \frac{\pi}{2}\right]$

Question 4 The graph of $f(x) = x - a - \frac{1}{(x-a)^2}$ for $x > a$ has

A. two stationary points and two asymptotes.

B. one stationary point and two asymptotes.

C. two asymptotes and no stationary points.

D. only one stationary point and one asymptote.

E. one asymptote only and no stationary points.

Question 5 $\sqrt{-1}$ equals

A. i only B. $-i$ only C. $-i$ or i D. $-i^2$ only E. i^2 only

Question 6 Given $a \in R^-$, $b \in R^+$ and $z = a - bi$, $|z|$ equals

A. $|a+b|$ B. $|a+b|$ C. $\sqrt{a^2 - b^2}$ D. $\sqrt{(a-b)^2}$ E. $\sqrt{a^2 + b^2}$

Question 7 The factors of $z^3 - iz^2 - a^2 z + ia^2$, where $a \in R$, are

A. $-a$, a, i

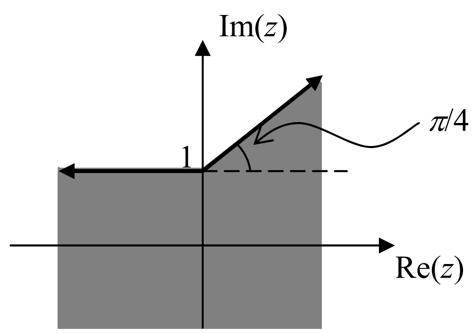
B. $-a$, a, $-i$

C. -1, 1, $-ia^2$

D. $z-1$, $z+1$, $z+ia^2$

E. $z-a$, $z+a$, $z-i$

Question 8 Consider the shaded region in the complex plane shown in the following Argand diagram.



The shaded region including the two rays can be represented by

A. $\left\{z : Arg(z-i) \le \frac{\pi}{4}\right\} \cup \{z : Arg(z-i) = \pi\}$

B. $\left\{z : Arg(z-i) \le \frac{\pi}{4}\right\}$

C. $\left\{z : Arg(z+i) \le \frac{\pi}{4}\right\} \cap \{z : Arg(z+i) = \pi\}$

D. $\left\{z : Arg(z+i) \le \frac{\pi}{4}\right\}$

E. $\{z : \text{Im}(z) - \text{Re}(z) \le 1\}$

Question 9 Function $f(x)$ is continuous and $f'(x)$ is defined over interval (p,q). If $f''(a)=0$ where $a \in (p,q)$, then $(a, f(a))$ must be

A. a point of inflection.

B. a local maximum point.

C. a local minimum point.

D. a turning point.

E. a turning point or a point of inflection.

Question 10 A spherical balloon is inflated at 0.25 m^3s^{-1}. The rate of increase in its surface area A when $A = \pi \text{ m}^2$ is

A. $\sqrt{\pi}$ m^2s^{-1} B. 1 m^2s^{-1} C. $\frac{\sqrt{\pi}}{4}$ m^2s^{-1} D. $\frac{1}{4}$ m^2s^{-1} E. $\frac{1}{\sqrt{\pi}}$ m^2s^{-1}

Question 11 Given $f(x) = \frac{-\sqrt{a}}{\sqrt{a-x}}$ where $a \in R^+$, an antiderivative of $f(x)$ is

A. $2\sqrt{a^2 - ax}$ B. $\sqrt{a}\sin^{-1}\left(\frac{x}{\sqrt{a}}\right)$ C. $\sqrt{a}\cos^{-1}\left(\frac{x}{\sqrt{a}}\right)$ D. $\sqrt{a}\sin^{-1}\left(\frac{x}{a}\right)$ E. $\sqrt{a}\cos^{-1}\left(\frac{x}{a}\right)$

Question 12 The integral $\int \frac{4x}{1+4x^2}dx$ equals

A. $\log_e\left(1+4x^2\right)+c$

B. $\log_e\sqrt{1+4x^2}+c$

C. $-\frac{1}{2}\log_e\left(1+4x^2\right)+c$

D. $\frac{1}{2}\tan^{-1}(2x)+c$

E. $2\tan^{-1}\left(\frac{x}{2}\right)+c$

Question 13 The volume of the solid formed by revolving the curve $y = \log_e(-x)$ where $-2 \le x \le -1$ about the x-axis is given by

A. $2\pi \int_{-2}^{-1} \log_e(-x)dx$

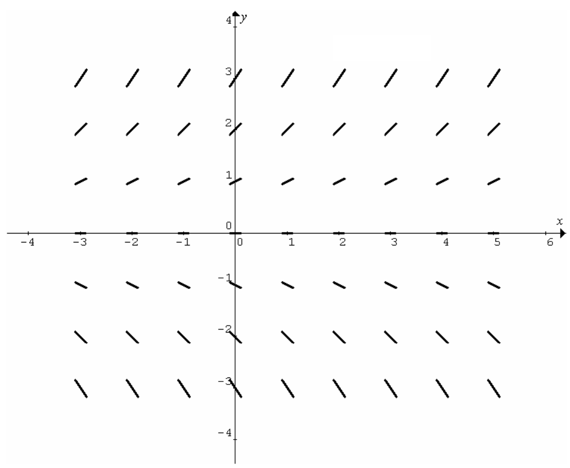
B. $2\pi \int_{-1}^{-2} \log_e(-x)dx$

C. $\pi \int_{1}^{2} [\log_e(x)]^2 dx$

D. $\pi \int_{1}^{2} e^{2x} dx$

E. $\pi \int_{-2}^{-1} e^{-2x} dx$

Question 14 Consider the direction field shown below.



A possible differential equation for the direction field is

A. $\frac{dy}{dx} = -\frac{y}{2}$

B. $\frac{dy}{dx} = \frac{y}{2}$

C. $\frac{dy}{dx} = -\frac{x}{2}$

D. $\frac{dy}{dx} = \frac{x}{2}$

E. $\frac{dy}{dx} = \pm\frac{e^x}{2}$

Question 15 Given $\frac{dy}{dx} = \sin(x^2)$ and $y = 2$ when $x = 0$. When $x = \frac{\pi}{2}$, the value of y is closest to

A. 2.828 B. 2.172 C. 1.172 D. 0.828 E. 0.172

Question 16 The velocity of a particle at position x is given by $v(x) = 2\sin^{-1}(x)$, its acceleration at x is

A. $\frac{2}{\sqrt{1-x^2}}$

B. $\frac{4\sin^{-1}(x)}{\sqrt{1-x^2}}$

C. $\frac{1}{\sqrt{1-x^2}}\frac{dx}{dt}$

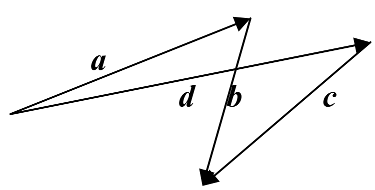
D. $\frac{4}{\sqrt{1-x^2}}\frac{dx}{dt}$

E. $\frac{4\sin^{-1}(x)}{\sqrt{1-x^2}}\frac{dx}{dt}$

Question 17 Vector $\boldsymbol{a} = \sqrt{3}\,\boldsymbol{i} - 3\,\boldsymbol{j}$ makes angles $\alpha°$, $\beta°$ and $\gamma°$ with the orthogonal unit vectors $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ respectively. The values of α, β and γ are respectively

A. 60, 150, 90 B. 60, 30, 90 C. 30, 150, 0 D. 30, 60, 0 E. 30, 60, 90

Question 18 Four coplanar vectors ***a***, ***b***, ***c*** and ***d*** are shown below.



Which one of the following statements is ***false***?

A. $\boldsymbol{a} + \boldsymbol{b} - \boldsymbol{c} = \boldsymbol{d}$

B. ***a***, ***b*** and ***c*** are linearly dependent

C. ***a***, ***b***, ***c*** and ***d*** are linearly independent

D. ***b***, ***c*** and ***d*** are linearly dependent

E. $\boldsymbol{c} - \boldsymbol{a} = \boldsymbol{b} - \boldsymbol{d}$

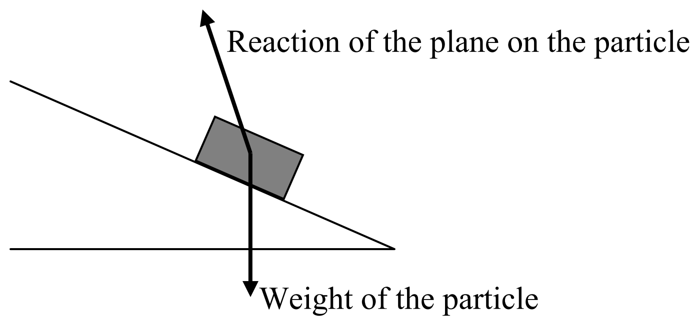
Question 19 A particle moves in a plane with orthogonal unit vectors $\boldsymbol{i}$ and $\boldsymbol{j}$. Its velocity at time t is given by $\boldsymbol{v}(t) = (0.2t)\boldsymbol{i} - \boldsymbol{j},\ t \geq 0$, and initial position is the origin O. The cartesian equation of the locus of the particle is

A. $y = 5x$ B. $y = -1$ C. $y = -\sqrt{10x}$ D. $y = \sqrt{10x}$ E. $y^2 = 10x$

Question 20 A force causes a 100-gram particle to change its velocity (in ms^{-1}) from $5\boldsymbol{i} - \boldsymbol{j}$ to $2\boldsymbol{j} + \sqrt{2}\,\boldsymbol{k}$. The magnitude of the change in momentum (in kg ms^{-1}) of the particle is

A. 0.06 B. 0.6 C. 6 D. 60 E. 600

Question 21 A particle is on an inclined plane. **All** forces on the particle are shown in the diagram below.



Which one of the following statements is definitely true at the moment as shown?

A. The particle moves with increasing speed.

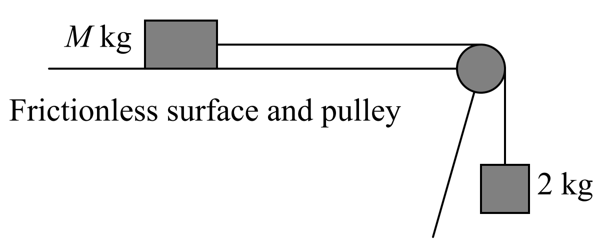
B. The particle moves with decreasing speed.

C. The particle moves at constant speed.

D. The particle is at rest.

E. The given information is insufficient to determine the motion of the particle.

Question 22 The acceleration of the system of two connected particles (see diagram below) is 2.0 ms^{-2}. The hanging particle has a mass of 2 kg. The one on the horizontal surface has a mass of M kg.



The value of M is

A. 4.8 B. 5.8 C. 6.8 D. 7.8 E. 8.8

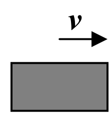
SECTION 2 Extended-answer questions

Instructions for Section 2

Answer **all** questions.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1 A 2000-kg flying object has a constant forward thrust of 2500 newtons. It experiences a retarding force of k^2v^2 newtons at speed v ms^{-1}, where $k \in R^+$.

a. Draw labeled arrows to show the forces on the flying object (shown as a rectangle below).



1 mark

b i. Write down the acceleration of the flying object in terms of k and v. 1 mark

ii. Hence show that $v = \dfrac{50\left(e^{\frac{kt}{20}} - 1\right)}{k\left(e^{\frac{kt}{20}} + 1\right)}$, given $v = 0$ at $t = 0$.

5 marks

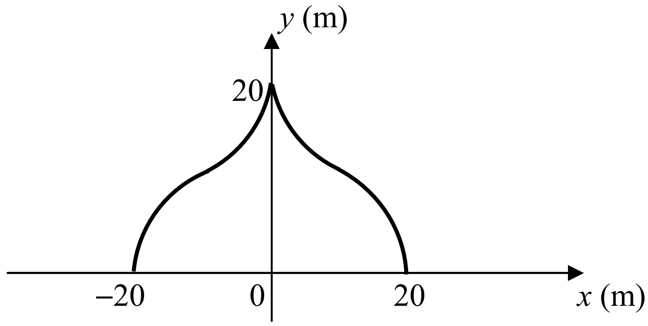
iii. Show that $k = \dfrac{e-1}{e+1}$, given $v = 50$ at $t = \dfrac{20}{k}$. 1 mark

c. Find t to the nearest second when $v = 50$. 1 mark

d. Find the speed v to the nearest ms^{-1} of the flying object closest to its maximum speed. 2 marks

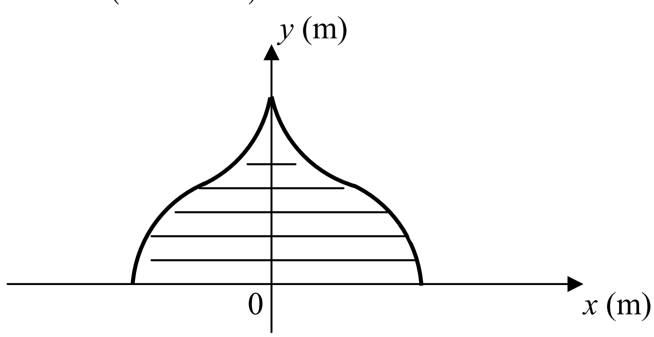
Total 11 marks

Question 2 The dome roof of a building has the following profile. The curve in the first quadrant is in the form $y = A\cos^{-1}(bx - c)$.



a. Find the values of A, b and c. 3 marks

The dome roof of another building has a profile that is the dilation of the one shown above. The curve in the first quadrant is now given by $y = 5\cos^{-1}(0.125x - 1)$.



b. Find the area of the shaded region to the nearest m^2. 2 marks

c. Find the exact volume of the shaded dome roof. 5 marks

Total 10 marks

Question 3 The straight-line motion of a 5-kg particle is given by the differential equation $\frac{d}{dx}\left(\frac{v^2}{2}\right) = -2x$ and $v = -1$ at $x = 0$ when $t = 0$, where x (m) is the displacement of the particle from the origin O at time t (s).

a i. Show that $v = \pm\sqrt{1-2x^2}$. 2 marks

ii. Hence show that $x = -\frac{1}{\sqrt{2}}\sin\left(t\sqrt{2}\right)$. 4 marks

b. Express the acceleration of the particle in terms of t. 1 mark

c. Find the exact total distance travelled by the particle between $t = \frac{\pi}{\sqrt{2}}$ and $t = \frac{3\pi}{\sqrt{2}}$. 2 marks

d. Find the change in momentum of the particle between $t = 0$ and $t = \frac{\pi}{\sqrt{2}}$. 2 marks

Total 11 marks

Question 4 The position of a spacecraft at time *t* is given by the complex number $z = \dfrac{1.5}{2+\cos\left(\frac{\pi}{6}t\right)} cis\left(\frac{\pi}{6}t\right)$ for $t \geq 0$.

a. Find the positions of the spacecraft at $t = 3$ and $t = 9$. 2 marks

b. Find the maximum and minimum values of $|z|$. 3 marks

c i. Show that $|z| = \dfrac{1.5}{2 + \dfrac{\text{Re}(z)}{|z|}}$. 2 marks

ii. Hence write a cartesian equation for the path of the spacecraft. 2 marks

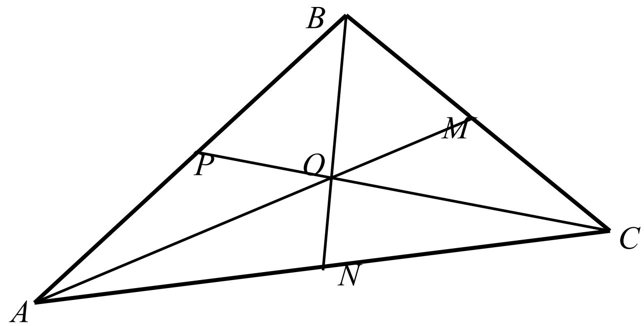
iii. Express the cartesian equation in the form $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$. 2 marks

d. Sketch the path of the spacecraft in a complex plane. Indicate the direction of motion with an arrowhead on the path.

3 marks

Total 14 marks

Question 5 In the following diagram $\overline{AM}$ and $\overline{BN}$ are medians, and $\overline{CP}$ passes through the intersection O of $\overline{AM}$ and $\overline{BN}$.



Let $\overrightarrow{OA} = \boldsymbol{a}$, $\overrightarrow{OB} = \boldsymbol{b}$ and $\overrightarrow{OC} = \boldsymbol{c}$.

a. Show that $\overrightarrow{OM} = \frac{1}{2}(\boldsymbol{b} + \boldsymbol{c})$. 2 marks

It can also be shown that $\overrightarrow{ON} = \frac{1}{2}(\boldsymbol{c} + \boldsymbol{a})$.

Let $\overrightarrow{OM} = -m\,\boldsymbol{a}$, $\overrightarrow{ON} = -n\,\boldsymbol{b}$ and $\overrightarrow{OP} = -p\,\boldsymbol{c}$, where m, n and $p \in R^+$.

b i. Show that $(1-2m)\boldsymbol{a} - (1-2n)\boldsymbol{b} = \boldsymbol{0}$. 2 marks

ii. Hence show that $\boldsymbol{b} = -(\boldsymbol{c} + \boldsymbol{a})$. 3 marks

c i. Express $\overrightarrow{AP}$ and $\overrightarrow{AB}$ in terms of $\boldsymbol{a}$, $\boldsymbol{c}$ and p. 2 marks

ii. Hence show that $\overline{CP}$ is also a median. 3 marks

Total 12 marks

End of Exam 2