Specialist Mathematics Exam 2: SOLUTIONS

Multiple-choice Answers

1. D	**2.** B	**3.** C	**4.** E	**5.** A
6. B	**7.** D	**8.** D	**9.** A	**10.** C
11. B	**12.** E	**13.** D	**14.** C	**15.** A
16. D	**17.** B	**18.** E	**19.** B	**20.** A
21. B	**22.** C			

Section 1: Multiple-choice Solutions

Question 1 Answer D

The range of $\cos^{-1}(3x)$ is $[0, \pi]$ and so the range of $y = a\cos^{-1}(3x) + \frac{\pi}{2}$ will be:
$\left[a \times 0 + \frac{\pi}{2}, a \times \pi + \frac{\pi}{2}\right] = \left[\frac{\pi}{2}, (2a+1)\frac{\pi}{2}\right]$

Question 2 Answer B

The hyperbola is $\frac{(x-1)^2}{4} - \frac{y^2}{9} = 1$ which has centre at $(1, 0)$. The vertices are 2 units either side of the centre, given by $\sqrt{4}$ in the equation. Hence they are 4 units apart.

Equations of asymptotes: $y = \pm\frac{3}{2}(x-1)$ which can be written as: $3x - 2y - 3 = 0$ and $-3x - 2y + 3 = 0$.

Question 3 Answer C

$$\begin{aligned} z &= \left[8\text{cis}\left(-\frac{5\pi}{6}\right)\right]^3 \\ &= 512\text{cis}\left(-\frac{5\pi}{2}\right) \\ &= 512\text{cis}\left(-\frac{\pi}{2}\right) \end{aligned}$$

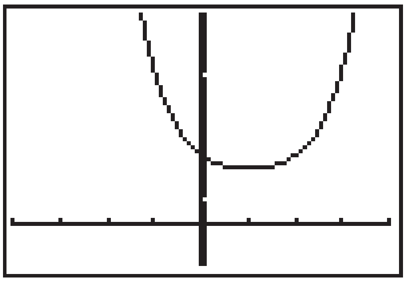
and so $|z|$ and $\text{Arg}(z)$ are 512 and $-\frac{\pi}{2}$

Question 4 Answer E

$z^4 - 4z^3 + 8z^2 - 8z + 16 = 0, z \in C$

If there are any complex solutions then they will be in pairs as all of the coefficients are real. This eliminates alternatives A and C.

Graphing this polynomial onto the graphics calculator shows no x-intercepts and so there are 4 complex solutions.



Question 5 Answer A

$\underset{\sim}{a} = \sqrt{2}\,\underset{\sim}{i} - \underset{\sim}{j} + 7\underset{\sim}{k}$ has a magnitude of $\sqrt{2 + 1 + 49} = \sqrt{52}$ or $2\sqrt{13}$.

$\therefore$ unit vector $= \dfrac{1}{2\sqrt{13}}\left(\sqrt{2}\,\underset{\sim}{i} - \underset{\sim}{j} + 7\underset{\sim}{k}\right)$

Question 6 Answer B

From the graph, the gradient is clearly positive and so alternatives D and E are incorrect.
Differentiate the equation:

$x^2 - 4xy + 4y^2 + x - 12y - 10 = 0$

$2x - \left(4y + 4x\dfrac{dy}{dx}\right) + 8y\dfrac{dy}{dx} + 1 - 12\dfrac{dy}{dx} = 0$

Substituting $x = 6$ and $y = 1$ gives:

$12 - \left(4 + 24\dfrac{dy}{dx}\right) + 8\dfrac{dy}{dx} + 1 - 12\dfrac{dy}{dx} = 0$

$9 = 28\dfrac{dy}{dx}$

$\dfrac{dy}{dx} = \dfrac{9}{28}$

Question 7 Answer D

Let $z = x + iy$ in $|z| = |z + 4|$.

$|x + iy| = |(x + 4) + iy|$

$\sqrt{x^2 + y^2} = \sqrt{(x + 4)^2 + y^2}$

$x^2 + y^2 = (x + 4)^2 + y^2$

$x^2 + y^2 = x^2 + 8x + 16 + y^2$

$8x = -16$ and so $x = -2$

Now $x = \text{Re}(z)$ and so the answer is D.

Question 8 Answer D

$$\int_0^{\frac{\pi}{6}} \cos(3x)e^{\sin(3x)}\,dx$$

Let $u = \sin(3x)$ so $\frac{du}{dx} = 3\cos(3x)$

If $x = 0$ then $u = 0$ and if $x = \frac{\pi}{6}$ then $u = 1$.

$\frac{1}{3}\int_0^1 e^u\,du$ is the result.

Question 9 Answer A

$$\overrightarrow{BA} \bullet \overrightarrow{BC} = \left|\overrightarrow{BA}\right|\left|\overrightarrow{BC}\right|\cos\angle ABC$$

$$\begin{aligned}\overrightarrow{BA} &= \overrightarrow{BO} + \overrightarrow{OA}\\ &= -\left(\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}\right) + 2\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}\\ &= \underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k} \text{ and so } \left|\overrightarrow{BA}\right| = 3\end{aligned}$$

$$\begin{aligned}\overrightarrow{BC} &= \overrightarrow{BO} + \overrightarrow{OC}\\ &= -\left(\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}\right) + 3\underset{\sim}{i} + \underset{\sim}{j}\\ &= 2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k} \text{ and so } \left|\overrightarrow{BC}\right| = 3\end{aligned}$$

$\overrightarrow{BA} \bullet \overrightarrow{BC} = 2 + 4 + 2$ and therefore $\cos\angle ABC = \frac{8}{9}$

Question 10 Answer C

$$\begin{aligned}f(x+h) &\approx f(x) + h\log_e(x) \text{ where}\\ f(1+0.1) &\approx f(1) + 0.1 \times \ln(1)\\ &= 3\end{aligned}$$

$$\begin{aligned}f(1.1+0.1) &\approx f(1.1) + 0.1 \times \ln(1.1)\\ &= 3 + 0.009531\\ &= 3.009531\end{aligned}$$

$$\begin{aligned}f(1.2+0.1) &\approx f(1.2) + 0.1 \times \ln(1.2)\\ &= 3.009531 + 0.01823\\ &= 3.02776\end{aligned}$$

Answer: 3.0278

Question 11 Answer B

A volume of revolution around the y-axis is found by evaluating $\int \pi x^2 \, dy$. The lower limit is $y = 1$ and the upper is e.

$y = e^{2x}$

$2x = \log_e(y)$

$x = \frac{1}{2}\log_e(y)$

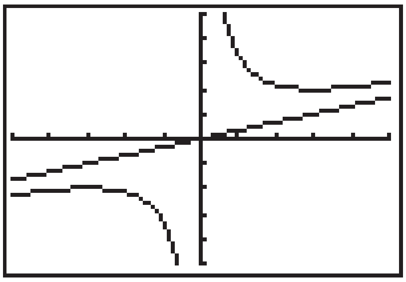
The volume formed by rotating the curve around the y-axis $= \frac{\pi}{4}\int_1^e (\log_e(y))^2 \, dy$

The volume formed by rotating the line $x = \frac{1}{2}$ around the y-axis $= \pi\int_1^e \left(\frac{1}{2}\right)^2 dy$

Required volume $\frac{\pi}{4}\int_1^e (1 - \log^2(y))dy$

Question 12 Answer E

The graph of the function $y = \frac{x^2 + 9}{3x}$ has two asymptotes, one vertical $(x = 0)$ and the other oblique $\left(y = \frac{x}{3}\right)$. It also has two turning points as can be seen from the graph of this function. The oblique asymptote with equation $y = \frac{x}{3}$ is also shown.



Question 13 Answer D

A point of inflection occurs where the second derivative is zero and the first derivative does **not** change sign. So, at the point of inflection $\frac{d^2y}{dx^2}$ changes sign and $\frac{dy}{dx}$ does not change sign.

Question 14 Answer C

A stationary point of inflection occurs at $x = -1$ since $f'(x) = 0$ and the derivative of $f'(x)$ is zero as well at that point. $f'(x)$ changes from negative to positive through zero at $x = 2$ and so there is a local minimum at this point for the graph of $y = f(x)$. The rate of change of the gradient of $f'(x)$ is zero at $x = 1$ (but the gradient of $f(x)$ is about -4) and so there is a point of inflection at this point.

Question 15 Answer A

It should be noted that $\frac{dy}{dx}$ is

- positive if $y > 0$
- negative if $y < 0$
- infinite (shown by vertical steepness) as y approaches zero.

Only alternative A displays these properties.

Question 16 Answer D

The direction of motion at $t = 2$ is given by $\dot{\underset{\sim}{r}}(2)$.

$$\dot{\underset{\sim}{r}}(t) = (3 - 4t)\underset{\sim}{i} + 6\underset{\sim}{j} + \frac{1}{2\sqrt{t}}\underset{\sim}{k}$$

Substituting $t = 2$: $-5\underset{\sim}{i} + 6\underset{\sim}{j} + \frac{1}{2\sqrt{2}}\underset{\sim}{k}$ which is equivalent to $-5\underset{\sim}{i} + 6\underset{\sim}{j} + \frac{\sqrt{2}}{4}\underset{\sim}{k}$.

Question 17 Answer B

Let the coordinates of P be $(-a, -b)$ where a and b are both positive. Hence $z = -a - bi$.
Q (by inspection) is $-b + ai$.

Testing each of the alternatives given:

A. $iz = i(-a - bi) = b - ai$ which is not Q. However, it is the negative of what is required and so the answer is **B**.

Alternatively, rotation clockwise through a right angle is equivalent to multiplication by $-i$.

Question 18 Answer E

$$v = \int (5\sin(2t) - 1)dt$$
$$= -\frac{5}{2}\cos(2t) - t + c$$

At $t = 0$, $v = 0$ and so $c = 2.5$

$$v = -\frac{5}{2}\cos(2t) - t + 2.5$$

At $t = 1$, $v = -2.5\cos(2) - 1 + 2.5$

Answer: $v = 2.54$

Question 19 Answer B

Acceleration down slope $= g\sin(\theta)$ where

$\theta = \arctan\left(\frac{1}{5}\right)$.

$\therefore a = \frac{g}{\sqrt{26}}$ is Jimmy's uniform acceleration.

$u = 0, s = 250, a = \frac{g}{\sqrt{26}}$ and $v^2 = u^2 + 2as$

$\therefore v^2 = 0 + 2 \times \frac{g}{\sqrt{26}} \times 250$ and so $v = 30.9995$

$$\begin{aligned}\text{Momentum} &= mv \\ &= 75 \times 30.9995 \\ &= 2325 \text{ kgms}^{-1}\end{aligned}$$

Question 20 Answer A

Let $\underset{\sim}{i}$ and $\underset{\sim}{j}$ be unit vectors acting east and north respectively.

The sum of the two forces are:

$8\underset{\sim}{j} + \left(-6\cos 60°\underset{\sim}{j} - 6\sin 60°\underset{\sim}{i}\right)$

$= -6\sin 60°\underset{\sim}{i} + (8 - 6\cos 60°)\underset{\sim}{j}$

$= -3\sqrt{3}\,\underset{\sim}{i} + 5\underset{\sim}{j}$

$$\begin{aligned}\text{Magnitude} &= \sqrt{(27 + 25)} \\ &= \sqrt{52} \\ &= 2\sqrt{13}\end{aligned}$$

Question 21 Answer B

$$-mg = \frac{mv^2}{1000} = ma$$

$$a = -\frac{1000g + v^2}{1000}$$

$$v\frac{dv}{dx} = -\frac{1000g + v^2}{1000}$$

$$\frac{dv}{dx} = -\frac{1000g + v^2}{1000v}$$

$$\frac{dx}{dv} = -\frac{1000v}{1000g + v^2}$$

$$x = \int -\frac{1000v}{1000g + v^2}dv$$

$$x = -500\ln\left(1000g + v^2\right) + c$$

When $x = 0,\ v = 400$

$$x = 500\ln\frac{9800 + 400^2}{1000g + v^2}$$

When $v = 0, x = 1426$ m

Question 22 Answer C

$s = -60, a = -9.8, u = 20, t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$-60 = 20t - 4.9t^2$$

$$4.9t^2 - 20t - 60 = 0$$

Solving this quadratic using TI-83 QUAD PRGM: $t = -2.01, 6.09$

Ignoring the negative solution, $t = 6.09$ seconds.

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Section 2

Question 1

a. $-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i = 1\text{cis}\left(\frac{3\pi}{4}\right)$

Apply de Moivre's theorem:

$z^2 = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$

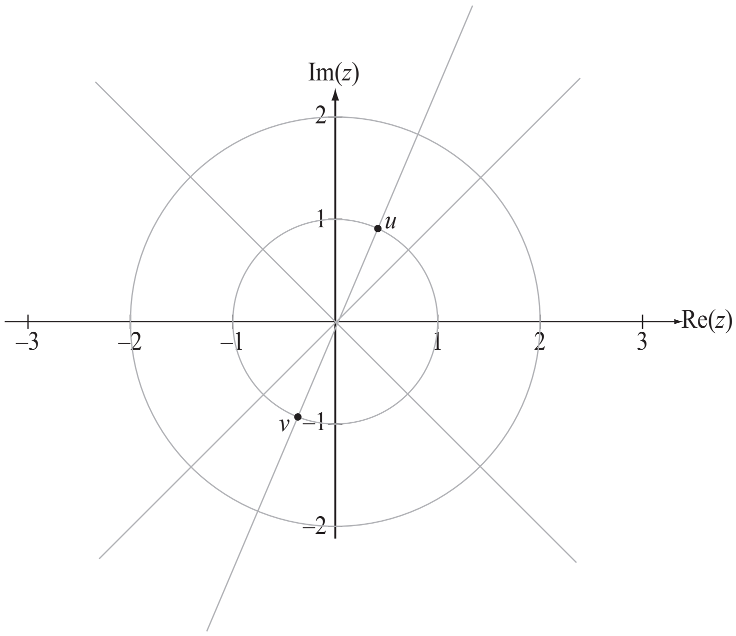
$r^2\,\text{cis}\,(2\theta) = 1\text{cis}\left(\frac{3\pi}{4}\right)$ **M1**

$r^2 = 1 \therefore r = 1, u = 1\text{cis}\left(\frac{3\pi}{4}\right)$

$2\theta = \frac{3\pi}{4} + 2k\pi, k = 0, 1$

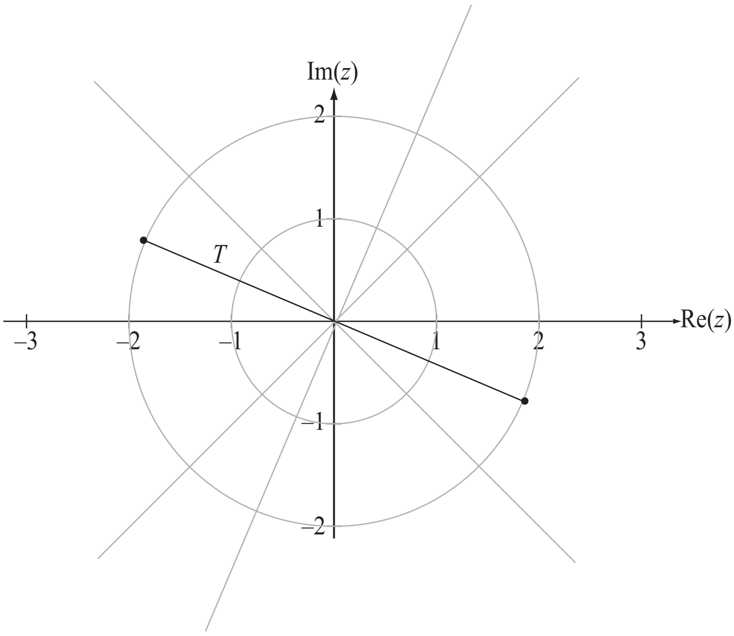
$u = 1\text{cis}\left(\frac{3\pi}{8}\right)$ and $v = 1\text{cis}\left(-\frac{5\pi}{8}\right)$ **A2**

b.



A2 for u and v in correct positions
A1 for u and v swapped

c.



Region T is the intersection of the inside of the circle centre $(0, 0)$ and radius 2 with the perpendicular bisector of the line segment joining u and v.

A1 perpendicular bisector

A1 inside of the circle

A1 intersection of the two clearly labelled.

d. For what values of n is u^n purely imaginary?

$$u^n = \left[1\text{cis}\left(\frac{3\pi}{8}\right)\right]^n$$
$$= 1\text{cis}\left(\frac{3n\pi}{8}\right)$$

If this is purely imaginary then $\cos\left(\frac{3n\pi}{8}\right) = 0$ **M1**

$\frac{3n\pi}{8} = \frac{(2k-1)\pi}{2}$ where $k \in Z$ or alternatively

$\frac{3n\pi}{8} = \frac{(2k+1)\pi}{2}$ **M1** for equating angle to $\frac{\text{odd } \pi}{2}$

Hence $n = \frac{4}{3}(2k-1)$ where $k \in Z$ **or** $n = \frac{4}{3}(2k+1)$ for any multiple of $\frac{\pi}{2}$ **A1**

Question 2

a. $1+\log_e x = 0$

$\log_e x = -1$ so $x = e^{-1}$

Coordinates of $A = \left(e^{-1}, 0\right)$ **A1** for $\left(e^{-1}, 0\right)$

b. **i.** $f'(x) = \dfrac{x\left(\frac{1}{x}\right) - 1\left(1+\log_e(x)\right)}{x^2}$

$= \dfrac{1-1-\log_e(x)}{x^2}$ **A1**

$= \dfrac{-\log_e(x)}{x^2}$

ii. $f'(x) = 0$

$\therefore \dfrac{-\log_e(x)}{x^2} = 0$

$\log_e(x) = 0$

When $x = 1$, $y = 1$

Coordinates of $B = (1, 1)$ **A1** for $(1, 1)$

c. $f''(x) = \dfrac{x^2\left(-\frac{1}{x}\right) + 2x\log_e(x)}{x^4}$

$= \dfrac{-x + 2x\log_e(x)}{x^4}$ **A1**

M1 for equating second derivative to zero

A1 for both coordinates correct

$= \dfrac{-1 + 2\log_e(x)}{x^3}$

$= 0$ if $x = e^{0.5}$

Coordinates of $C = \left(e^{0.5}, 1.5e^{-0.5}\right)$

d. $\sqrt{e} \approx 1.65$

$f''(1.6) = -0.0146$

$f''(1.7) = 0.0125$

$f''(x)$ changes sign so the concavity changes, hence the point of inflection. **A1**

It should be noted that the first derivative does **not** change sign at $x = 1.65$.

e. Equate the first derivative to the gradient between the points $(0, 0)$ and $\left(\frac{1 + \log_e(x)}{x}\right)x$.

$$-\frac{\log_e(x)}{x^2} = \frac{\frac{1 + \log_e(x)}{x} - 0}{x - 0}$$

$$-\frac{\log_e(x)}{x^2} = \frac{1 + \log_e(x)}{x^2}$$

$$-\log_e(x) = 1 + \log_e(x)$$

$$-2\log_e(x) = 1 \text{ and so } \log_e(x) = -\frac{1}{2}$$

$$\therefore x = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$$

f. $$\int_0^{\frac{1}{\sqrt{e}}} \left(\frac{e}{2}x\right)dx - \int_{\frac{1}{e}}^{\frac{1}{\sqrt{e}}} \left(\frac{1 + \log_e(x)}{x}\right)dx$$

(This first integral represents the area under the tangent which passes through the origin.)

M1 the difference of two integrals

A1 correct limits

$$= \left[\frac{ex^2}{4}\right]_0^{\frac{1}{\sqrt{e}}} - \frac{1}{2}\left[(1 + \log_e(x))^2\right]_{\frac{1}{e}}^{\frac{1}{\sqrt{e}}}$$

A1 (correct anti-derivative of the $\frac{1 + \log_e(x)}{x}$ function)

$$= 0.25 - \frac{1}{2}\left[(1 - 0.5)^2 - (1 - 1)^2\right]$$

$$= \frac{1}{8}$$

Question 3

a. $\overrightarrow{OA} = -\underset{\sim}{i} + 3\underset{\sim}{j} + 2\underset{\sim}{k}$, $\overrightarrow{OB} = 3\underset{\sim}{i} + 6\underset{\sim}{j} + \underset{\sim}{k}$ and $\overrightarrow{OC} = -4\underset{\sim}{i} + 4\underset{\sim}{j} + 3\underset{\sim}{k}$

$$\overrightarrow{BC} \cdot \overrightarrow{BA} = \left(\overrightarrow{BO} + \overrightarrow{OC}\right) \cdot \left(\overrightarrow{BO} + \overrightarrow{OA}\right)$$

M1 dot product

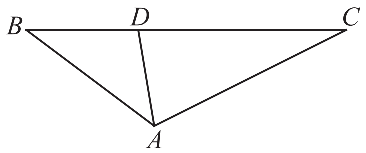
$$= \left(-3\underset{\sim}{i} - 6\underset{\sim}{j} - \underset{\sim}{k} - 4\underset{\sim}{i} + 4\underset{\sim}{j} + 3\underset{\sim}{k}\right) \cdot \left(-3\underset{\sim}{i} - 6\underset{\sim}{j} - \underset{\sim}{k} - \underset{\sim}{i} + 3\underset{\sim}{j} + 2\underset{\sim}{k}\right)$$

$$= \left(-7\underset{\sim}{i} - 2\underset{\sim}{j} + 2\underset{\sim}{k}\right) \cdot \left(-4\underset{\sim}{i} - 3\underset{\sim}{j} + \underset{\sim}{k}\right)$$

A1 for these vectors

$$= 28 + 6 + 2$$

$$= 36$$



b. $\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD}$ **A1**

$$= -\underset{\sim}{q} + k\underset{\sim}{p}$$

Let D be a point on the side BC and let $\overrightarrow{BC} = \underset{\sim}{p}$, $\overrightarrow{BA} = \underset{\sim}{q}$ *and* $\overrightarrow{BD} = k\underset{\sim}{p}$, $k \in R$.

c. If $\overrightarrow{AD} \perp \overrightarrow{BC}$ then $\overrightarrow{AD} \bullet \overrightarrow{BC} = 0$.

Find k such that $\left(-\underset{\sim}{q} + k\underset{\sim}{p}\right).\underset{\sim}{p} = 0$

So $-\underset{\sim}{q}.\underset{\sim}{p} + k\underset{\sim}{p}.\underset{\sim}{p} = 0$

$$k = \frac{\underset{\sim}{q}.\underset{\sim}{p}}{\underset{\sim}{p}.\underset{\sim}{p}}$$

$\underset{\sim}{q}.\underset{\sim}{p} = 36$ (from part (a))

$\underset{\sim}{p}.\underset{\sim}{p} = \left(-7\underset{\sim}{i} - 2\underset{\sim}{j} + 2\underset{\sim}{k}\right)\bullet\left(-7\underset{\sim}{i} - 2\underset{\sim}{j} + 2\underset{\sim}{k}\right)$ **M1**

$= 57$

So $k = \frac{36}{57} = \frac{12}{19}$ **A1**

d. Find the magnitude of $\overrightarrow{AD}$ with $k = \frac{12}{19}$ for the shortest distance because we need $\overrightarrow{AD} \perp \overrightarrow{BC}$.

$\overrightarrow{AD} = -\underset{\sim}{q} + k\underset{\sim}{p}$ with $k = \frac{12}{19}$ **M1**

$\overrightarrow{AD} = 4\underset{\sim}{i} + 3\underset{\sim}{j} - \underset{\sim}{k} + \frac{12}{19}\left(-7\underset{\sim}{i} - 2\underset{\sim}{j} + 2\underset{\sim}{k}\right)$ **M1**

$= -\frac{8}{19}\underset{\sim}{i} + \frac{33}{19}\underset{\sim}{j} + \frac{5}{19}\underset{\sim}{k}$ **A1**

$$\left|\overrightarrow{AD}\right| = \sqrt{\left(\frac{-8}{19}\right)^2 + \left(\frac{33}{19}\right)^2 + \left(\frac{5}{19}\right)^2}$$

$$= \sqrt{\frac{62}{19}}$$

$\left|\overrightarrow{AD}\right| = 1.81$ to 2 decimal places **A1**

Question 4

a. **i.** Rate of mass going in $= 0.1$ kg/litre $\times$ 5 litre/min **M1**

$= 0.5$ kg/min **A1**

Rate of mass going out $= \frac{x}{800}$ kg/litre $\times$ 5 litre/min

$= \frac{5x}{800}$ kg/min **A1**

$$\frac{dx}{dt} = 0.5 - \frac{x}{160} = \frac{80 - x}{160}$$

ii. $a = 80$ and $b = 160$ **A1**

iii. $\frac{dt}{dx} = \frac{160}{80 - x}$ **M1** for inversion

$t = \int \frac{160}{80 - x} dx = -160 \log_e |80 - x| + c$ **M1** log recognition

When $t = 0$, $x = 10$ so $c = 160 \log_e 70$

$t = 160 \log_e \left(\frac{70}{80 - x} \right)$ **A1**

$$\frac{80 - x}{70} = e^{-\frac{t}{160}}$$

$80 - 70e^{-\frac{t}{160}} = x$ **A1**

iv. When $t = 12$, $x = 15.058$ kg **A1**

b. Rate of mass going in $= 0.1$ kg/litre $\times$ 5 litre/min

$= 0.5$ kg/min

Number of litres in the tank after t minutes $800 + 4t$ **M1** for volume $800 + At$ where A is a constant

Rate of mass going out $= \frac{x}{800 + 4t}$ kg/litre $\times$ 1 litre/min

$= \frac{x}{800 + 4t}$ kg/min

$\frac{dx}{dt} = 0.5 - \frac{x}{800 + 4t}$ **A1**

Question 5

a. $\underset{\sim}{a} = 0\underset{\sim}{i} - 9.8\underset{\sim}{j}$

$\underset{\sim}{v} = \int \left(0\underset{\sim}{i} - 9.8\underset{\sim}{j}\right)dt$ **M1**

$= c\underset{\sim}{i} - (9.8t + d)\underset{\sim}{j}$ where c and d are constants.

When $t = 0$, $\underset{\sim}{v} = 5\underset{\sim}{i}$ and so $c = 5$ and $d = 0$.

$\underset{\sim}{r} = \int \left(5\underset{\sim}{i} - 9.8t\underset{\sim}{j}\right)dt$

$= (5t + e)\underset{\sim}{i} - \left(4.9t^2 + f\right)\underset{\sim}{j}$ where e and f are constants.

When $t = 0$, $\underset{\sim}{a} = 0\underset{\sim}{i} + 40\underset{\sim}{j}$ and so $e = 0$ and $f = 40$.

$\underset{\sim}{r} = 5t\underset{\sim}{i} + \left(40 - 4.9t^2\right)\underset{\sim}{j}$ as required. **A1**

b. The $\underset{\sim}{j}$ component of $\underset{\sim}{r} = 5t\underset{\sim}{i} + \left(40 - 4.9t^2\right)\underset{\sim}{j}$ is zero when the toy touches the ground.

$40 - 4.9t^2 = 0$ so $t = \sqrt{\dfrac{40}{4.9}}$

It reaches the ground after 2.86 seconds. **A1**

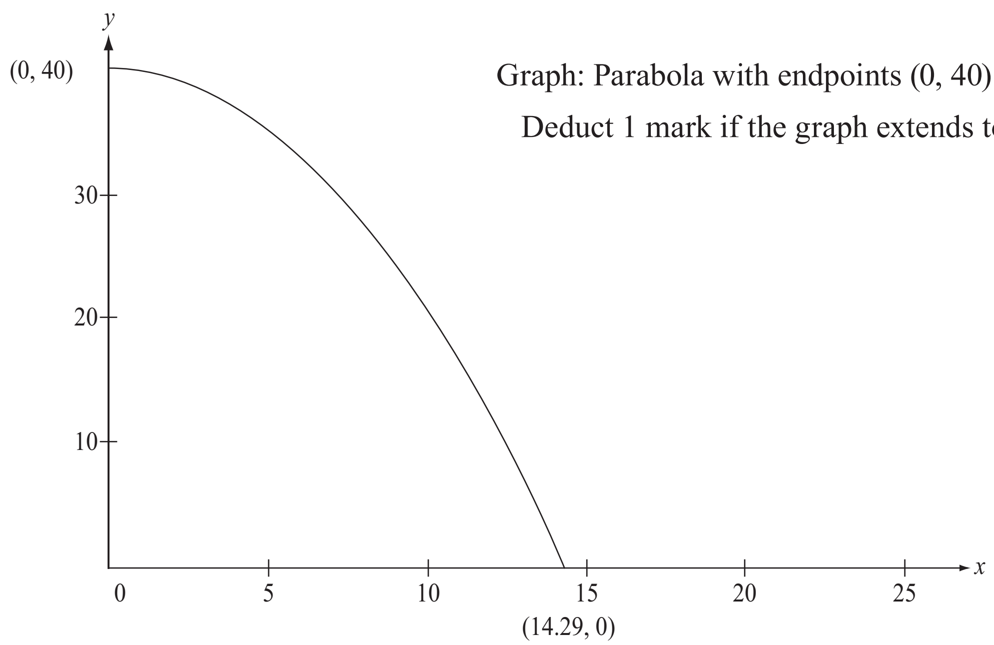
The horizontal distance reached is the $\underset{\sim}{i}$ component: $5t$

It travels 14.29 metres before reaching the ground. **A1**

c. $x = 5t$ so $t = \dfrac{x}{5}$ and $y = 40 - 4.9t^2$ **A1**

Substituting for t into y gives: $y = 40 - 4.9\left(\dfrac{x}{5}\right)^2$

$\therefore y = 40 - \dfrac{49x^2}{250}$ **A1**



Graph: Parabola with endpoints (0, 40) and (14.29, 0) **A2**

Deduct 1 mark if the graph extends to $x < 0$ or $y < 0$

d. $\underset{\sim}{v} = 5\underset{\sim}{i} - 9.8t\underset{\sim}{j}$ and $t = \sqrt{\dfrac{40}{4.9}}$ when the plane hits the ground. **M1**

The magnitude of the velocity at the point of impact with the ground is

$\sqrt{\left(25 + 9.8^2 \times \dfrac{40}{49}\right)}$ which is $\sqrt{809}$. **A1**

Magnitude of momentum $= |mv|$

$= 1.5 \times \sqrt{809}$

$= 42.7$ kg m/s to 3 significant figures. **A1**

e. $\tan(\theta) = \dfrac{40}{x}$

$$\frac{d}{dt}(\tan(\theta)) = \frac{d}{dt}\left(\frac{40}{x}\right)$$

So $\dfrac{d}{d\theta}(\tan(\theta))\dfrac{d\theta}{dt} = \dfrac{d}{dx}\left(\dfrac{40}{x}\right)\dfrac{dx}{dt}$ **M1**

$\therefore \dfrac{d\theta}{dt} = -\dfrac{40}{x^2\sec^2(\theta)}\dfrac{dx}{dt}$ **A1**

f. $\dfrac{dx}{dt} = 5$ as it is moving with a constant speed of 5 m/s.

$$\frac{d\theta}{dt} = -\frac{200\cos^2(\theta)}{x^2}$$

Now $\cos(\theta) = \dfrac{x}{\sqrt{x^2 + 1600}}$ **M1**

When $x = 10$, $\cos(\theta) = \dfrac{1}{\sqrt{17}}$, $\sec^2(\theta) = 17$ and so $\dfrac{d\theta}{dt} = -\dfrac{2}{17}$ **A1**