Neap:

Trial Examination 2007

VCE Specialist Mathematics Units 3 & 4

Written Examination 1

Suggested Solutions

Neap Trial Exams are licensed to be photocopied or placed on the school intranet and used only within the confines of the school purchasing them, for the purpose of examining that school's students only. They may not be otherwise reproduced or distributed. The copyright of Neap Trial Exams remains with Neap. No Neap Trial Exam or any part thereof is to be issued or passed on by any person to any party inclusive of other schools, non-practising teachers, coaching colleges, tutors, parents, students, publishing agencies or websites without the express written consent of Neap.

Copyright © 2007 Neap ABN 49 910 906 643 96–106 Pelham St Carlton VIC 3053 Tel: (03) 8341 8341 Fax: (03) 8341 8300 TEVSMU34EX1_SS_07.FM

Question 1

$xy - y^2 - 4 = 0$

When $x = -5$, we obtain a quadratic equation in *y*, i.e. $-5y - y^2 - 4 = 0$.

Solve $-5y - y^2 - 4 = 0$ or equivalent for *y*. M1

Hence $y = -4$ or -1, but as $y > -4$ we obtain $y = -1$. A1

Using implicit differentiation, we obtain $y + x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$. M1

Substituting $x = -5$ and $y = -1$ to find $\frac{dy}{dx}$, we obtain $-1 - 5\frac{dy}{dx} + 2\frac{dy}{dx} = 0$.

Hence $\frac{dy}{dx} = -\frac{1}{3}$. A1

Question 2

$\frac{dy}{dx} = \cos(2x)e^{-\sin(2x)}$

$y = \int \cos(2x)e^{-\sin(2x)}dx$

Let $u = \sin(2x)$, so $\frac{du}{dx} = 2\cos(2x)$. M1

$y = \frac{1}{2}\int e^{-u}du$ A1

Hence $y = -\frac{1}{2}e^{-u} + c$ or $y = -\frac{1}{2}e^{-\sin(2x)} + c$. A1

We now apply the condition $y = 1$ when $x = 0$ to find the value of *c* using either of the above.

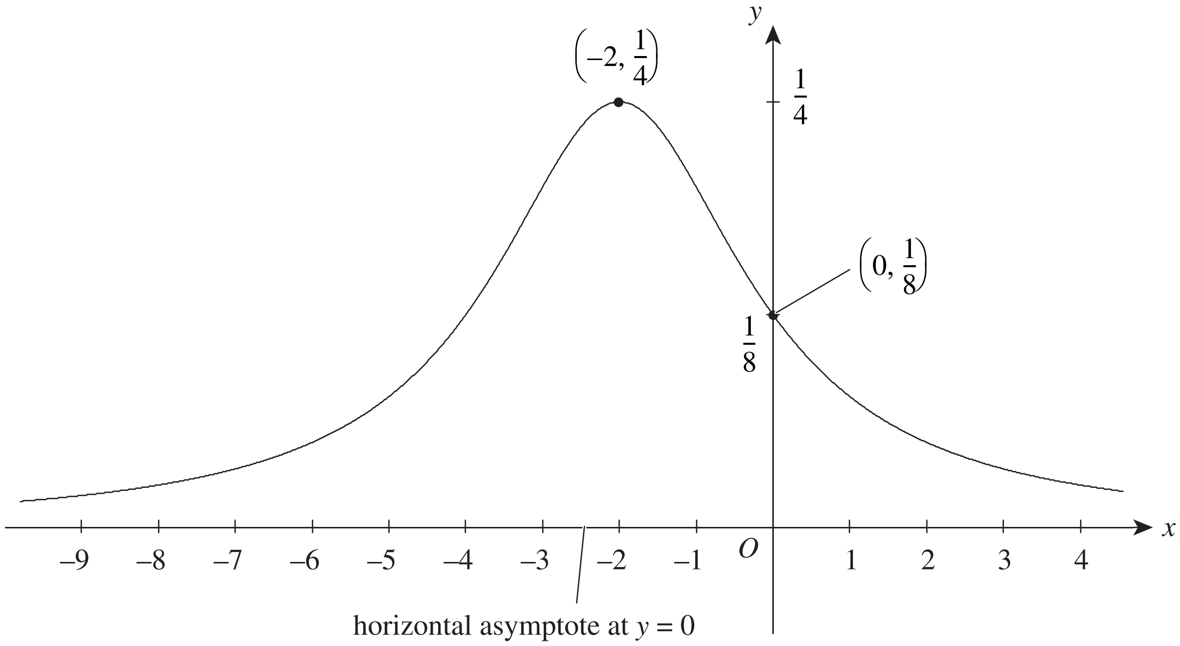
$1 = -\frac{1}{2} + c$, so $c = \frac{3}{2}$.

Hence $y = \frac{3}{2} - \frac{1}{2}e^{-\sin(2x)}$. A1

Copyright © 2007 Neap

Question 3

a.



Correct shape ("camel's hump") and horizontal asymptote at $y = 0$ is shown. A1

Maximum stationary point at $\left(-2, \frac{1}{4}\right)$ is shown. A1

y-intercept at $\left(0, \frac{1}{8}\right)$ is shown. A1

b. The area between $x = -4$ and $x = 0$ is given by $\int_{-4}^{0} \frac{dx}{x^2 + 4x + 8}$. A1

$$= \int_{-4}^{0} \frac{dx}{(x+2)^2 + 4}$$

$$= \frac{1}{2}\int_{-4}^{0} \frac{2}{(x+2)^2 + 2^2}dx \qquad \text{A1}$$

$$= \frac{1}{2}\left[\tan^{-1}\left(\frac{x+2}{2}\right)\right]_{-4}^{0}$$

$$= \frac{1}{2}[\tan^{-1}(1) - \tan^{-1}(-1)]$$

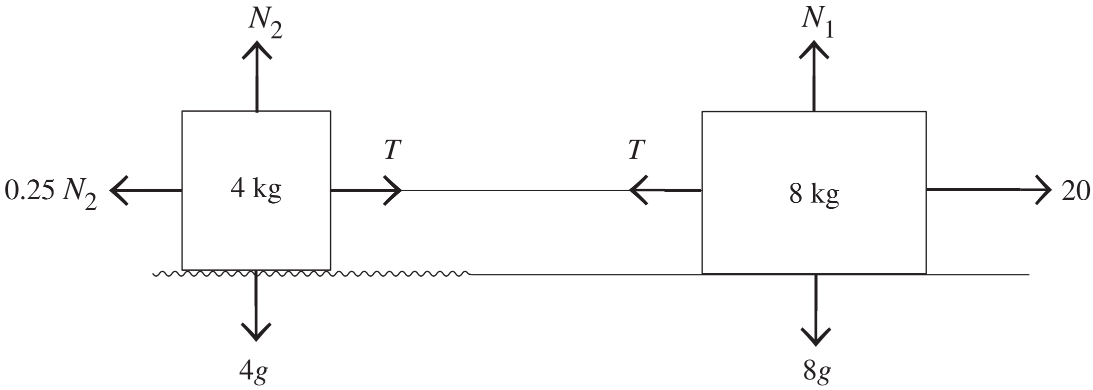
$$= \frac{1}{2}\left[\frac{\pi}{4} + \frac{\pi}{4}\right]$$

$$= \frac{\pi}{4} \text{ square units} \qquad \text{A1}$$

Copyright © 2007 Neap

Question 4

a.



All forces are correctly shown. A1

b. Resolving horizontally for the 8 kg mass, $20 - T = 8a$ (1) A1
Resolving vertically for the 4 kg mass, $N_2 = 4g$ (2)
Resolving horizontally for the 4 kg mass, $T - 0.25N_2 = 4a$ (3)
Substituting (2) into (3) gives $T - g = 4a$ (4) A1
Adding (1) and (4) gives $20 - g = 12a$

Thus $a = \dfrac{20 - g}{12}$. A1

Question 5

a.

$$\tan(15°) = \tan(45° - 30°)$$

$$= \frac{\tan(45°) - \tan(30°)}{1 + \tan(45°)\tan(30°)} \quad \left(\text{using } \tan(x - y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}\right) \qquad \text{M1}$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + (1)\left(\frac{1}{\sqrt{3}}\right)}$$

$$= \frac{\frac{\sqrt{3} - 1}{\sqrt{3}}}{\frac{\sqrt{3} + 1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1} \qquad \text{A1}$$

$$= \frac{4 - 2\sqrt{3}}{2}$$

$$= 2 - \sqrt{3} \qquad \text{A1}$$

Hence if $\tan(15°) = a + b\sqrt{3}$, $a = 2$ and $b = -1$.

Copyright © 2007 Neap

b. $\text{LHS} = \dfrac{\cos(2A)}{1 + \sin(2A)}$

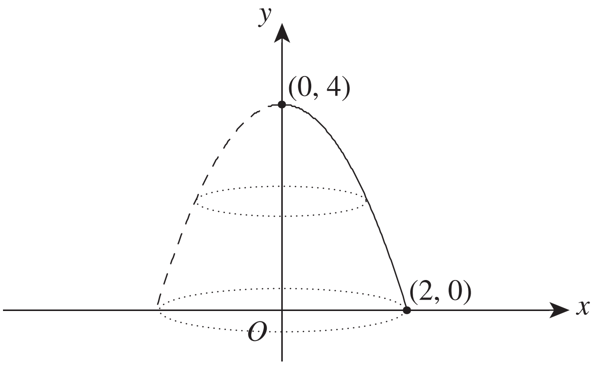
$= \dfrac{\cos^2(A) - \sin^2(A)}{\cos^2(A) + \sin^2(A) + 2\sin(A)\cos(A)}$ M1

$= \dfrac{(\cos(A) - \sin(A))(\cos(A) + \sin(A))}{(\cos(A) + \sin(A))^2}$ A1

$= \dfrac{\cos(A) - \sin(A)}{\cos(A) + \sin(A)}$ A1

$= \text{RHS}$

Question 6



When $x = 0,\ y = 4$ and when $x = 2,\ y = 0$.

Since $y = 4 - x^2,\ x^2 = 4 - y$ over the interval $x \in [0, 2]$. M1

The volume is given by $\pi\displaystyle\int_0^4 x^2\,dy = \pi\int_0^4 (4 - y)\,dy$ A1

$= \pi\left[4y - \dfrac{y^2}{2}\right]_0^4$ A1

$= \pi[(16 - 8) - (0 - 0)]$

$= 8\pi$ cubic units A1

Question 7

a. The parametric equations are:

$x = \cos^2(t)$ (1)

$y = 4\sin^2(t)$ (2)

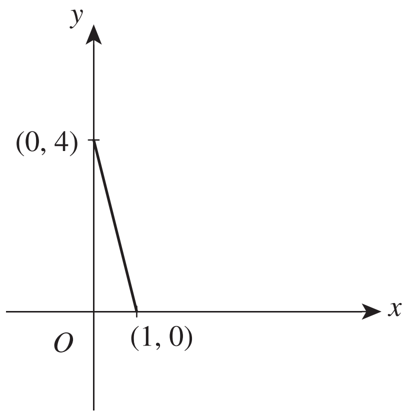
$\dfrac{y}{4} = \sin^2(t)$ (3)

(1) + (3) gives $x + \dfrac{y}{4} = 1$. M1

Hence $y = 4 - 4x$, or equivalent. A1

Copyright © 2007 Neap

b. The path is a straight line with equation $y = 4 - 4x$ for $0 \le x \le 1$.



Straight line through (0, 4) and (1, 0). A1
Line ends at the intercepts. A1

Question 8

a. $P(i) = i^2 + bi + 1 + i$

$= (b + 1)i$

Given that $P(i) = 0$, we obtain $b = -1$. A1

b. Let the roots of $z^2 - z + (1 + i) = 0$ be α and β.

$(z - \alpha)(z - \beta) = z^2 - (\alpha + \beta)z + \alpha\beta$ M1

By equating coefficients, $\alpha + \beta = 1$ and $\alpha\beta = 1 + i$. A1

From **a.**, $\alpha = i$, and from $\alpha + \beta = 1$ we obtain $\beta = 1 - i$. A1

Question 9

$\frac{x^2}{x^2 - 4} = \frac{4}{x^2 - 4} + 1$ (by division) A1

Using partial fractions on $\frac{4}{x^2 - 4}$ gives M1

$\frac{4}{x^2 - 4} \equiv \frac{A}{x - 2} + \frac{B}{x + 2}$

$4 \equiv A(x + 2) + B(x - 2)$

When $x = 2, A = 1$ and when $x = -2, B = -1$.

Hence $\int \frac{x^2}{x^2 - 4}dx = \int\left(\frac{1}{x - 2} - \frac{1}{x + 2} + 1\right)dx$. A1

$\int\left(\frac{1}{x - 2} - \frac{1}{x + 2} + 1\right)dx = \log_e\left|\frac{x - 2}{x + 2}\right| + x$ A1

Copyright © 2007 Neap