Student Name: _____

SPECIALIST MATHEMATICS

Units 3 & 4 – Written examination 1

TSSM
Creating VCE Success

2007 Trial Examination

Reading Time: 15 minutes
Writing Time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	Number of questions to be answered	Number of marks
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, and rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 11 pages.
- Working space is provided throughout the book.

Instructions
- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic devices into the examination room.

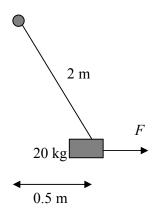**This page is blank**

Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

A 20 kg child is sitting on a swing of length 2 m which is pulled to one side 0.5 m by a force of F newtons. Let T newtons be the magnitude of the force in the rope of the swing. The situation is shown in the diagram below.

a. Draw a diagram showing all the forces acting on the child on the swing.

1 mark

b. Calculate the value of the force F, giving your answer in the form $\dfrac{ag\sqrt{b}}{c}$ where a, b and c are positive integers.

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3 marks

Question 2

Solve the differential equation $\dfrac{dy}{dx} = \dfrac{e^{2x}}{e^{4x}+3}$ given that $y = 0$ when $x = 0$.

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<div align="right">5 marks</div>

Question 3

a. Express $-2 - 2\sqrt{3}\,i$ in polar form.

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1 mark

b. Solve the quadratic equation $(z - 2 + i)^2 = -2 - 2\sqrt{3}\,i$ expressing your answer in exact Cartesian form.

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3 marks

Question 4

Consider the relation $x^2 + 4xy + 2y = -11$.

a. Find an expression for $\dfrac{dy}{dx}$ in terms of x and y.

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2 marks

b. **Hence** find the exact value of $\dfrac{dy}{dx}$ when $x = 1$.
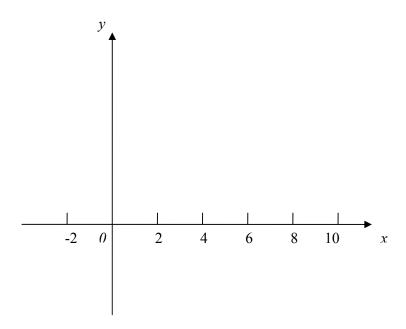
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2 marks

Question 5

The position vector of a moving particle is given by $r(t) = (t+3)\,i + \dfrac{4}{t^2}\,j$ for $1 \le t \le 3$.

a. Find the Cartesian equation of the path followed by the particle.

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2 marks

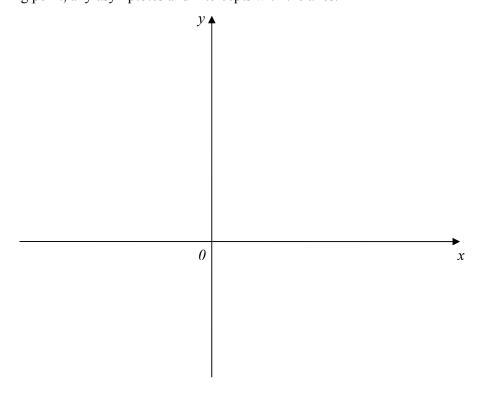**b.** Sketch the path of the particle on the axes provided.

2 marks

TURN OVER

Question 6

a. Sketch the graph with equation $y = \dfrac{8}{(4-x)(x+2)}$, clearly indicating the location of the turning point, any asymptotes and intercepts with the axes.

3 marks

b. Find the exact area bounded by $y = \dfrac{8}{(4-x)(x+2)}$, the x axis and the lines $x = 0$ and $x = 2$ in the form $\dfrac{a}{b}\log_e c$, where a, b and c are positive integers.

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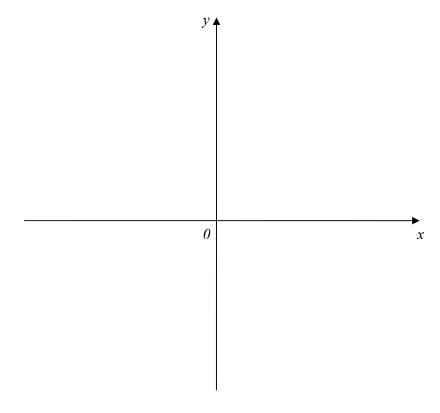
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4 marks

Question 7

a. Find the solution of the differential equation $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{4-x^2}}$ $|x| < 2$ with $y(0) = \dfrac{\pi}{4}$.

2 marks

b. Sketch the graph of the solution curve of this equation on the axes below, specifying scales on both axes.

2 marks

TURN OVER

Question 8

Use the change of variable $u = \sin 3x$ to evaluate the integral $\int \dfrac{dx}{\cos(3x)}$.

Express your answer in the form $a \log_e f(x) + c$ where a and c are real constants and $f(x)$ is a function of x. Specify explicitly a and $f(x)$.

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4 marks

Question 9

The region in the first quadrant enclosed by the coordinate axes, the graph of $y = \cos \frac{x}{4}$ and the line $x = a$ is rotated about the x axis to form a solid of revolution

a. Express the volume of the solid of revolution as a definite integral.

1 mark

b. Calculate the volume of the solid of revolution in terms of a.

2 marks

c. Find the exact value of a if the volume is $\frac{\pi}{2}\left(\frac{\pi}{3}+1\right)$.

1 mark

END OF QUESTION AND ANSWER BOOK