

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

	STUDEN	Γ NUMBE	R				Letter
Figures							
Words							

SPECIALIST MATHEMATICS

Written examination 1

Monday 5 November 2007

Reading time: 3.00 pm to 3.15 pm (15 minutes) Writing time: 3.15 pm to 4.15 pm (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	Number of questions to be answered	Number of marks
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are not permitted to bring into the examination room: notes of any kind, a calculator of any type, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 12 pages with a detachable sheet of miscellaneous formulas in the centrefold
- Working space is provided throughout the book.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer all questions in the spaces provided.

A decimal approximation will not be accepted if an exact answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g m/s², where g = 9.8.

Ouestion	1
Question	1

Express $\frac{2\sqrt{3} + 2i}{1 - \sqrt{3}i}$ in polar form.			

Λ	estion	~
	ACTIAN	•

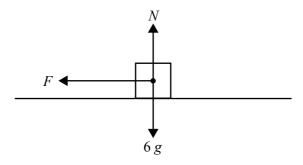
Show that $\sqrt{5} - i$ is a solution of the equation $z^3 - (\sqrt{5} - i)z^2 + 4z - 4\sqrt{5} + 4i = 0$	
	1
Find all other solutions of the equation $z^3 - (\sqrt{5} - i)z^2 + 4z - 4\sqrt{5} + 4i = 0$.	

Question 3
Find the equation of the tangent to the curve $x^3 - 2x^2y + 2y^2 = 2$ at the point $P(2, 3)$.

O	uestion	4
v	ucsuon	7

Find the volume generated when the region enclosed by the curve line $x = -\frac{1}{2}$ is rotated about the <i>x</i> -axis to form a solid of revolution	$y = \frac{1}{\sqrt{1 - x^2}}$, the x-axis, the y-axis and the on.

A block of mass 6 kg is given an initial push. As a result of this push, the block's initial velocity is 4 m/s and it travels across a horizontal floor in a straight line. It comes to rest 3 metres from where it was pushed due to the frictional force, *F*, between the block and the floor.



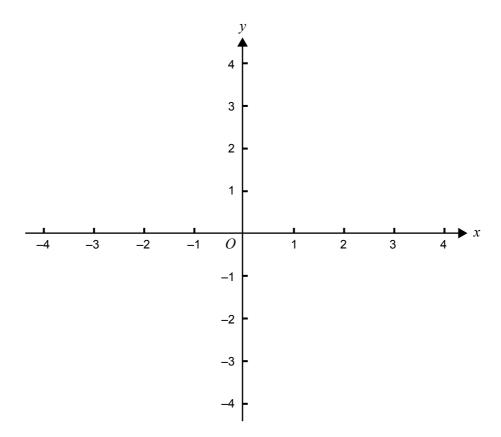
	lculate the acceleration of the block across the floor.
	2 mark
Са	ilculate the value of μ , the coefficient of friction between the block and the floor. Give your answer is
the	e form $\frac{b}{cg}$ where b and c are positive integers.
_	
_	

A particle moves so that its velocity at time t is given by

$$v(t) = -4\sin(2t)i + 6\cos(2t)j \text{ for } 0 \le t \le \frac{\pi}{2}.$$

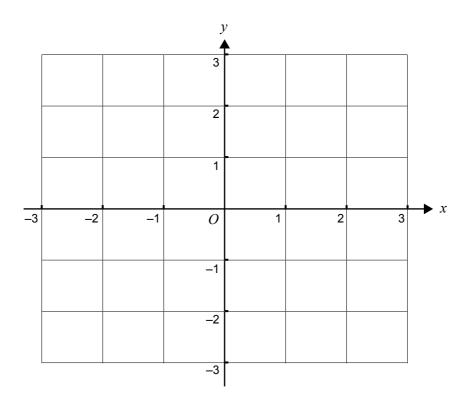
	2
Find the cartesian equation of the path followed by the particle.	_

c. Sketch the path followed by the particle on the axes below.



Use Euler's method to find y_2 if $\frac{dy}{dx} = \frac{1}{x}$, given that $y_0 = y(1) = 1$ and $h = 0.1$. Express your answer as a fraction.	
Solve the differential equation given in part at a find the value of a which is estimate	2 d by v
	d by y_2 .
Solve the differential equation given in part a. to find the value of y which is estimate. Express your answer in the form $\log_e(a) + b$, where a and b are positive real constants.	d by y_2 .

a. Sketch the slope field of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{2}$ for y = -2, -1, 0, 1, 2 at each of the values x = -2, -1, 0, 1, 2 on the axes below.



2 marks

b. If y = -1 when x = 0, solve the differential equation given in **part a.** to find y in terms of x.

3 marks

c. Sketch the graph of the solution curve found in part b. on the slope field in part a.

1 mark

Question	9
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A particle moves in the cartesian plane with position vector $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$ where x and y are functions of t. If its velocity vector is $\mathbf{v} = -y\mathbf{i} + x\mathbf{j}$, find the acceleration vector of the particle in terms of the position
vector \mathbf{r} .
3 marks
Question 10
Given that $\tan(2x) = \frac{4\sqrt{2}}{7}$ where $x \in \left[0, \frac{\pi}{4}\right]$, find the exact value of $\sin(x)$.

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

SPECMATH

Specialist Mathematics Formulas

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Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc\sin A$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$$\cos^2(x) + \sin^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$\cot^2(x) + 1 = \csc^2(x)$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$

$$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$$

$$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$
 $\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$

function	sin ⁻¹	cos ⁻¹	tan^{-1}
domain	[-1, 1]	[-1, 1]	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0,\pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (complex numbers)

$$z = x + yi = r(\cos \theta + i \sin \theta) = r \operatorname{cis} \theta$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$

$$-\pi < \operatorname{Arg} z \le \pi$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

$$\frac{z_1 z_2 - r_1 r_2 \cos(\sigma_1 + \sigma_2)}{z_2} = \frac{z_2}{r_2} \cos(\sigma_1 + \sigma_2)$$

 $z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e |x| + c$$

$$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = a\sec^2(ax)$$

$$\int \sec^2(ax) dx = \frac{1}{a}\tan(ax) + c$$

$$\int \frac{d}{dx} \left(\sin^{-1}(x) \right) = \frac{1}{\sqrt{1 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\int \frac{d}{dx} \left(\cos^{-1}(x) \right) = \frac{-1}{\sqrt{1 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\int \frac{d}{dx} \left(\tan^{-1}(x) \right) = \frac{1}{1+x^2}$$

$$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule:
$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

quotient rule:
$$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

chain rule:
$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Euler's method: If
$$\frac{dy}{dx} = f(x)$$
, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration:
$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx} \left(\frac{1}{2}v^2\right)$$

constant (uniform) acceleration:
$$v = u + at$$
 $s = ut + \frac{1}{2}at^2$ $v^2 = u^2 + 2as$ $s = \frac{1}{2}(u + v)t$

SPECMATH

Vectors in two and three dimensions

$$\underline{\mathbf{r}} = x\underline{\mathbf{i}} + y\underline{\mathbf{j}} + z\underline{\mathbf{k}}$$

$$|\overset{\mathbf{r}}{_{\sim}}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\dot{\mathbf{r}} = \frac{d\mathbf{r}}{dt} = \frac{dx}{dt} \dot{\mathbf{i}} + \frac{dy}{dt} \dot{\mathbf{j}} + \frac{dz}{dt} \dot{\mathbf{k}}$$

Mechanics

momentum: p = mv

equation of motion: $\underset{\sim}{R} = m \underset{\sim}{a}$

friction: $F \leq \mu N$