Insight Publications

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Trial Exam Paper

2008

SPECIALIST MATHEMATICS

Written examination 1

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes
Writing time: 1 hour

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring sheets of paper, notes of any kind or white out liquid/tape into the examination.
- Calculators are not permitted in this examination.

Materials provided

- The question and answer book of 13 pages with a separate sheet of miscellaneous formulas.
- Working space is provided throughout this book.

Instructions

- Write your **name** in the box provided.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

Instructions

Answer **all** questions in the spaces provided.
A decimal approximation **will not** be accepted if an exact answer is required.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, diagrams in this book **are not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$

Question 1

Let $u = 10 - 5i$ and $v = 2 - i$.

Find $\frac{iu}{\overline{v}}$ in Cartesian form.

2 marks

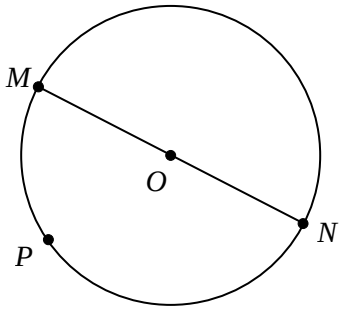
Question 2

The position of particles *A* and *B* at any time *t* seconds, $t \geq 0$, is given by $\underset{\sim}{r}_A(t) = (t^2 - 2t)\underset{\sim}{i} + (6t - 2)\underset{\sim}{j}$ and $\underset{\sim}{r}_B(t) = (5t - 12)\underset{\sim}{i} + (t^2 + 6)\underset{\sim}{j}$, respectively.

Determine the time when the particles collide.

3 marks

Question 3



MN is the diameter of circle centre *O*.

P is a point on the circumference of this circle.

Let $\vec{OP} = \underset{\sim}{p}$ and $\vec{OM} = \underset{\sim}{m}$.

Use vectors to prove that $\angle MPN$ is a right angle.

3 marks

Question 4

a. Show that $\cot(x) - \operatorname{cosec}(2x) = \cot(2x)$

2 marks

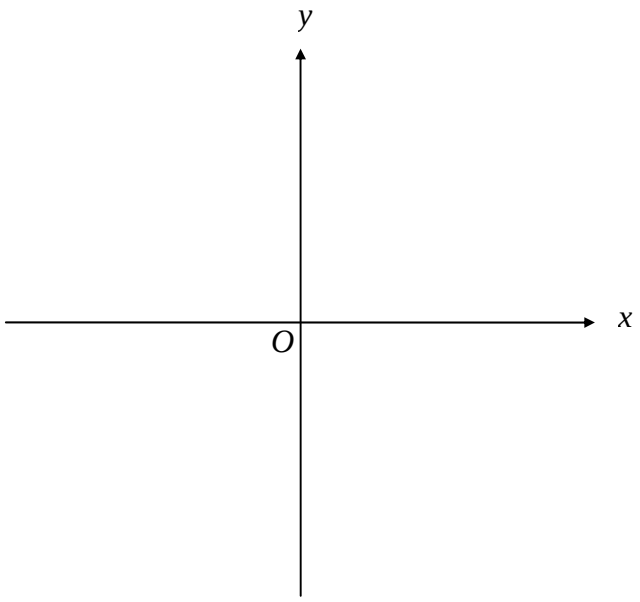
b. Hence, solve the equation $\cot(x) - \operatorname{cosec}(2x) = \sqrt{3}$, $x \in [-\pi, \pi]$.

2 marks
Total: 2 + 2 = 4 marks

TURN OVER

Question 5

a. Sketch the graph of $f(x) = 3\arcsin(x+1) - \frac{\pi}{2}$ on the axes below, showing the intercepts and endpoints in exact form.



3 marks

b. Find $f^{-1}(x)$ stating its domain.

2 marks

Total 3 + 2 = 5 marks

Question 6

Find $\int \frac{x+1}{x^2+2}\,dx$.

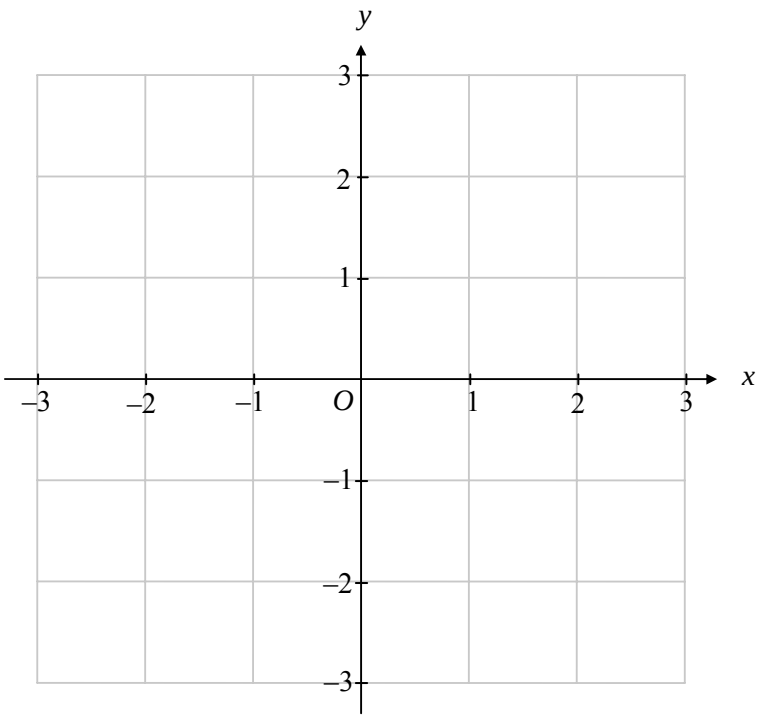
3 marks

TURN OVER

Question 7

Given the differential equation $\frac{dy}{dx} = \frac{y+3}{2}$

a. Use $y = -3, -2, -1, 0, 1, 2, 3$ to sketch a slope field of the differential equation at each of the values $x = -3, -2, -1, 0, 1, 2, 3$.



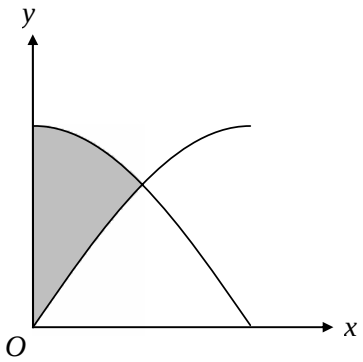
2 marks

b. If $y = -2$ when $x = 1$, solve the differential equation to find y in terms of x.

3 marks
Total 2 + 3 = 5 marks

TURN OVER

Question 8



The area between the graphs of $y = \sin(3x)$ and $y = \cos(3x)$ shaded in the diagram above is rotated around the x-axis to form a solid of revolution.

Find the exact volume of this solid.

4 marks

Question 9

Let $y = x\sqrt{1-x^2} - \cos^{-1}(x)$.

a. Show that $\frac{dy}{dx} = 2\sqrt{1-x^2}$.

3 marks

b. Hence, determine the exact value of $\int_{\frac{1}{2}}^{1} \sqrt{1-x^2}\, dx$.

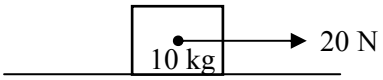
2 marks
Total 3 + 2 = 5 marks

TURN OVER

Question 10

A crate of toys of mass 10 kg is sitting on the floor of a room.

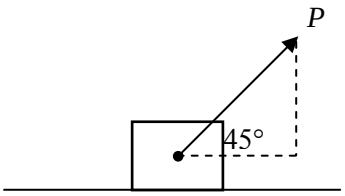
a. A child starts to pull the crate with a horizontal force of 20 newtons so that it is on the point of moving. Show that the coefficient of friction between the floor and the crate of toys is $\frac{2}{g}$.



2 marks

b. Determine the maximum force, P newtons, that can be applied to the crate at an angle of 45° to the horizontal level without moving it.

Express your answer in the form $P = \dfrac{ag}{g + b}$, where $a, b \in R$.



4 marks
Total 2 + 4 = 6 marks

END OF QUESTION AND ANSWER BOOK