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Specialist Mathematics

2008

Trial Examination 2

SECTION 1 Multiple-choice questions

Instructions for Section 1

Answer **all** questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1 For any defined value of x, the value of $\left(\sin^{-1} x\right)^2 - \left(\sin x\right)^{-2}$ is

A. 0

B. less than 1

C. in $(-\infty, 10]$

D. in $[0,1]$

E. in $[-4820, 4820]$

Question 2 $\cos(x - y + z)$ can be expressed as

A. $\cos(x)\cos(y+z) + \sin(x)\sin(y+z)$

B. $\cos(x+z)\cos(y) - \sin(x+z)\sin(y)$

C. $\cos(x)\cos(y+z) - \sin(x)\sin(y+z)$

D. $\cos(x-y)\cos(z) + \sin(x-y)\sin(z)$

E. $\cos(x)\cos(y-z) + \sin(x)\sin(y-z)$

Question 3 The inverse function of $2\arctan\left[\frac{1}{2}(x+1)\right] - 1$ is

A. $\frac{1}{2}\tan[2(x+1)] - 1$, where $-\frac{\pi}{4} < x < \frac{\pi}{4}$

B. $\frac{1}{2}\tan[2(x+1)] - 1$, where $-\frac{\pi}{4} - 1 < x < \frac{\pi}{4} - 1$

C. $\frac{1}{2}\tan[2(x+1)] - 1$

D. $2\tan\left[\frac{1}{2}(x+1)\right] - 1$, where $-\pi - 1 < x < \pi - 1$

E. $2\tan\left[\frac{1}{2}(x+1)\right] - 1$

Question 4 The graph of $y=\left(x-a+\frac{1}{\sqrt{x-a}}\right)\left(x-a-\frac{1}{\sqrt{x-a}}\right)+b$ has straight-line asymptote(s)

A. $x=a$

B. $x=a$ and $y=b$

C. $x=-a$ and $y=0$

D. $y=b$

E. $x=a$ and $y=-b$

Question 5 The graph of $\frac{(x+1)^2}{4}+(y-2)^2=1$ undergoes the following transformations in the order shown.

Translate to the right by 1 unit, translate downward by 1 unit, dilate from the *x*-axis by a factor of 2. The transformed equation is

A. $\frac{x^2}{4}+\frac{(y-3)^2}{6}=1$

B. $x^2+(y-1)^2=4$

C. $x^2+(y-2)^2=4$

D. $(x+2)^2+(y-2)^2=4$

E. $2x^2+(y-1)^2=1$

Question 6 If $z=i\left(\cos\left(\frac{\pi}{2}+\theta\right)-i\sin\left(\frac{\pi}{2}-\theta\right)\right)$ and $0<\theta<\frac{\pi}{2}$, $Arg(z)$ is

A. $-\theta$ B. θ C. $\frac{\pi}{2}-\theta$ D. $\frac{\pi}{2}+\theta$ E. $\theta-\frac{\pi}{2}$

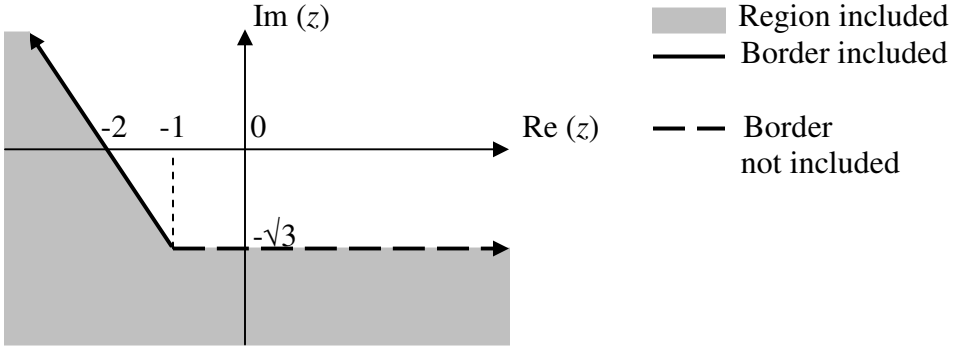
Question 7 $3-2i$ is a root of $2z^2-(5-i)z+3+11i=0$, the other root is

A. $\frac{3}{2}-i$ B. $-\frac{3}{2}-i$ C. $-\frac{1}{2}+\frac{3}{2}i$ D. $-\frac{1}{2}-\frac{3}{2}i$ E. $2-3i$

Question 8 $3cis\left(\frac{\pi}{3}\right)-icis\left(-\frac{\pi}{6}\right)=$

A. $2cis\left(\frac{\pi}{3}\right)$ B. $2cis\left(\frac{\pi}{6}\right)$ C. $2icis\left(\frac{\pi}{3}\right)$ D. $-2icis\left(\frac{\pi}{6}\right)$ E. $4cis\left(\frac{\pi}{6}\right)$

Question 9



The shaded region in the complex plane C represents

A. $C \setminus \left\{ z : 0 \le Arg\left(z+1+i\sqrt{3}\right) < \dfrac{2\pi}{3} \right\}$

B. $C \setminus \left\{ z : 0 < Arg\left(z+1+i\sqrt{3}\right) \le \dfrac{2\pi}{3} \right\}$

C. $\left\{ z : -\dfrac{4\pi}{3} \le \arg\left(z+1-i\sqrt{3}\right) < \dfrac{2\pi}{3} \right\}$

D. $C \setminus \left\{ z : 0 < Arg\left(z-1-i\sqrt{3}\right) \le \dfrac{2\pi}{3} \right\}$

E. $C \setminus \left\{ z : 0 < Arg\left(z-1-i\sqrt{3}\right) \le \dfrac{\pi}{3} \right\}$

Question 10 For constants $a, b \in R$, the graph of $y = 1000(x+a)^2(x-b)^{\frac{1}{3}}$ has

A. 3 stationary points and 2 inflection points

B. 2 stationary points and 2 inflection points

C. 3 stationary points and 1 inflection point

D. 2 stationary points and 1 inflection point

E. 3 stationary points and no inflection points

Question 11 For constant $a \in R$, $\dfrac{d}{dx}\left(\dfrac{-1}{\sqrt{a-x^2}}\right) =$

A. $\cos^{-1}\left(\dfrac{x}{a}\right)$ B. $\arcsin\left(\dfrac{x}{a}\right)$ C. $\arccos\left(\dfrac{x}{\sqrt{a}}\right)$ D. $\sin^{-1}\left(\dfrac{x}{\sqrt{a}}\right)$ E. $-\dfrac{x}{\left(a-x^2\right)\sqrt{a-x^2}}$

Question 12 For $2x^2y^2 = x^2 - y^2$, at $x = 1$, $\dfrac{dy}{dx} =$

A. $\dfrac{1}{\sqrt{3}}$ B. $-\dfrac{\sqrt{3}}{9}$ C. $\pm\dfrac{1}{3\sqrt{3}}$ D. $\dfrac{1}{3}$ E. $\pm\dfrac{1}{3}$

Question 13 An anti-derivative of $\dfrac{1}{1-x^2}$ with respect to x^2 for $x \in R \setminus [-1,1]$ is

A. $-\log_e\left(x^2-1\right)+2$

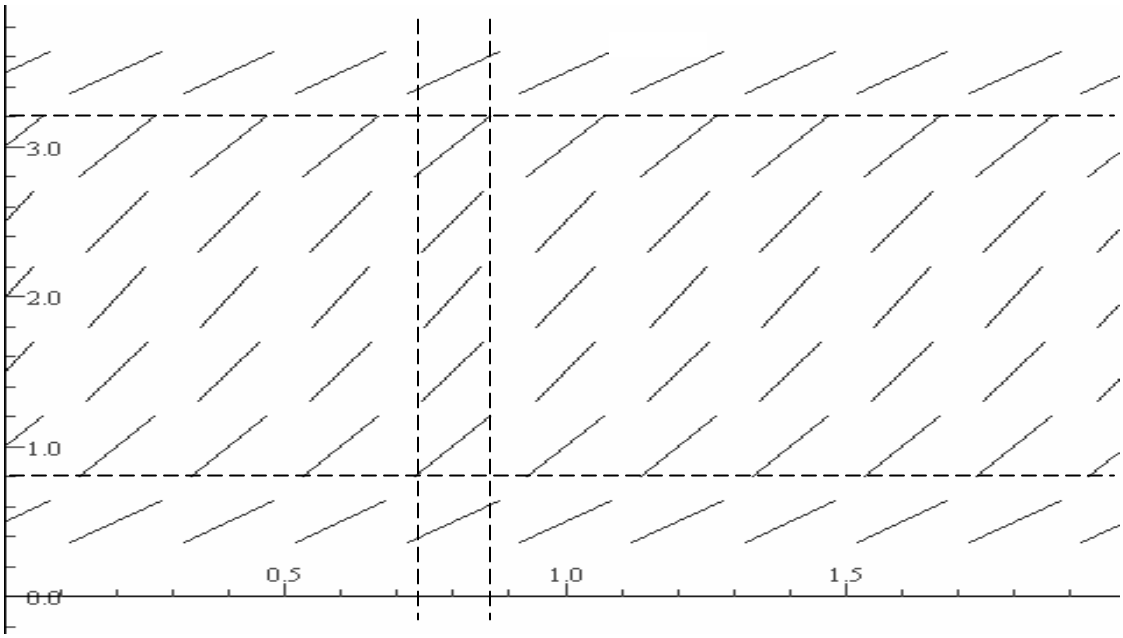
B. $\log_e\left|1-x^2\right|-3$

C. $\dfrac{1}{2}\log_e\left|\dfrac{1+x}{1-x}\right|$

D. $\dfrac{1}{2}\log_e\left|\dfrac{1-x}{1+x}\right|$

E. $\log_e\sqrt{\dfrac{x+1}{x-1}}-1$

Question 14



A slope field for $y' = y(a-y)$ is shown above, the approximate value of constant *a* is closest to

A. 2 B. 3 C. 4 D. 5 E. 6

Question 15 For $\frac{dy}{dx} = \tan\left(x^2\right)$, $y = -1$ when $x = -\frac{1}{2}$. When $x = -1$, the value of y is closest to

A. -1.4 B. -0.64 C. 1.4 D. 0.64 E. -0.5

Question 16 The angle between vector $\boldsymbol{p} = a\boldsymbol{i} + b\boldsymbol{j} + c\boldsymbol{k}$ and the zx-plane is

A. $\tan^{-1}\left(\frac{b}{\sqrt{c^2 + a^2}}\right)$

B. $\cos^{-1}\left(\frac{b}{\sqrt{a^2 + b^2 + c^2}}\right)$

C. $\tan^{-1}\left(\frac{c}{\sqrt{a^2 + b^2}}\right)$

D. $\tan^{-1}\left(\frac{a}{\sqrt{b^2 + c^2}}\right)$

E. $\sin^{-1}\left(\frac{c + a}{\sqrt{a^2 + b^2 + c^2}}\right)$

Question 17 For vectors $\boldsymbol{a} = 2\boldsymbol{i} - \boldsymbol{j}$, $\boldsymbol{b} = \boldsymbol{i} + 3\boldsymbol{j}$ and $\boldsymbol{c} = 4\boldsymbol{i} - 9\boldsymbol{j}$, which one of the following statements is **false**?

A. $\boldsymbol{a}$ and $\boldsymbol{b}$ are independent

B. $\boldsymbol{b}$ and $\boldsymbol{c}$ are independent

C. $\boldsymbol{c}$ and $\boldsymbol{a}$ are independent

D. $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are independent

E. $\frac{1}{2}\boldsymbol{c} + \boldsymbol{b}$ and $\boldsymbol{a}$ are dependent

Question 18 The position vector of a moving particle is given by $\boldsymbol{r}(t) = \tan(t)\boldsymbol{i} + \frac{1}{2}\sec^2(t)\boldsymbol{j} + \boldsymbol{k}$.

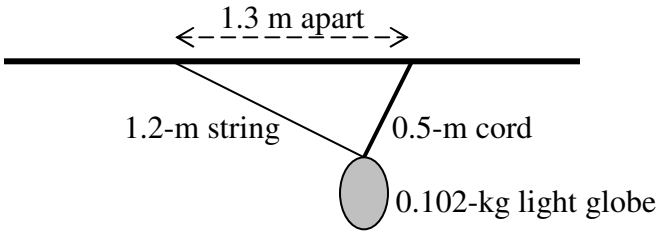
At $t = \frac{3\pi}{4}$, the direction of motion of the particle is given by

A. $\frac{\sqrt{2}}{2}\boldsymbol{i} - \frac{\sqrt{2}}{2}\boldsymbol{j}$ B. $-\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$ C. $\frac{1}{\sqrt{3}}(-\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k})$ D. $\frac{1}{\sqrt{2}}(\boldsymbol{i} + \boldsymbol{j})$ E. $\frac{1}{\sqrt{3}}(\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k})$

Question 19 The momentum of a 0.20-kg particle changes from $(3\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k})$ kg ms^{-1} to $(\boldsymbol{i} - \boldsymbol{k})$ kg ms^{-1}. The change of speed (in ms^{-1}) of the particle is closest to

A. 10 B. −10 C. 15 D. −15 E. 3

Question 20

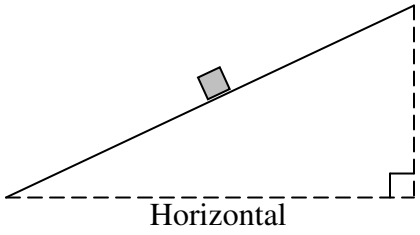


The diagram above shows a 0.102-kg light globe suspended from the ceiling by a 1.2-m string and a 0.5-m cord. The tension in the string is T_S newtons and the tension in the cord is T_C newtons.

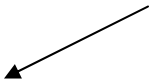
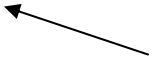
The value of the ratio $\frac{T_S}{T_C}$ is closest to

A. 2.4 B. 0.38 C. 1.08 D. 0.92 E. 0.42

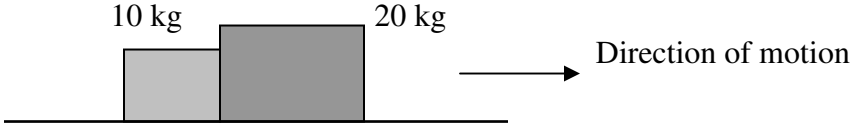
Question 21



The diagram above shows a particle sliding down a slope at constant speed. The direction of the reaction force of the slope on the particle is best indicated by arrow

A.  B.  C.  D.  E. 

Question 22



The diagram above shows two boxes in contact sliding along a rough horizontal floor. The sliding friction is 0.2 newtons per kg. The force (in newtons) exerted by the 20-kg box on the 10-kg box is closest to

A. 4 B. 2 C. 0 D. 1 E. 3

SECTION 2 Extended-answer questions

Instructions for Section 2

Answer **all** questions.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1 The acceleration a of a particle is a function of velocity v, where $a^2 = 1 - v^2$.
At time $t = 0$, the particle is at position $x = 1$.
The particle is at rest at positions $x = \pm 1$. **Note** that $0 \le t \le \pi$.

a. In terms of variables x and v, set up a differential equation to the rectilinear motion of the particle.

1 mark

b. i. Show that the solution to the differential equation is $v = A\sqrt{B + Cx^2}$. Find the values of A, B and C.

3 marks

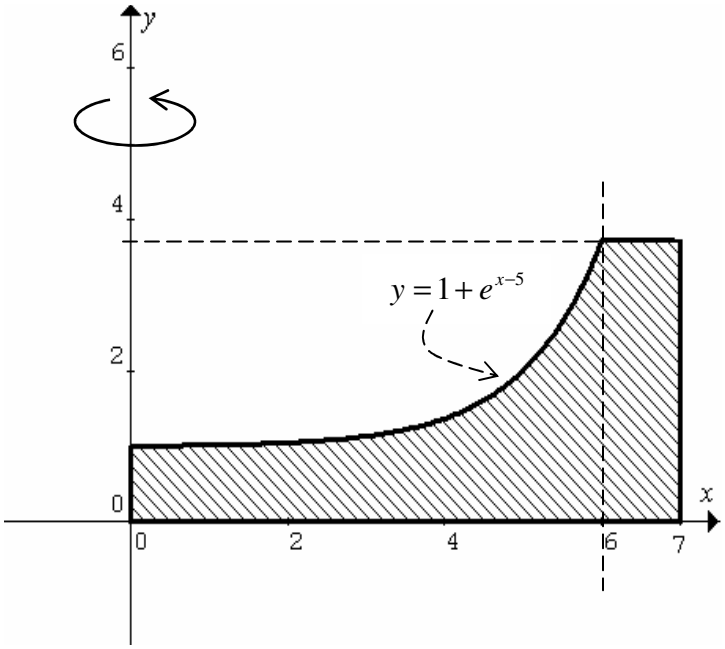
b. ii. Hence show that $x = D\cos(nt)$. Find the values of D and n. 2 marks

c. Find the value of $\int_0^{\pi} v\,dt$. 1 mark

d. Show that the direction of the acceleration of the particle is opposite to the direction of its displacement from the origin. 1 mark

Total 8 marks

Question 2 The following diagram shows the right-half of the upright cross-section of a large circular concrete structure for water storage. The structure is modelled by the solid of revolution of the shaded region in the y-axis. Length is measured in metres.



a. Find the exact area of the shaded region. 2 marks

b. i. Set up a definite integral for finding the **maximum** volume of water that the structure can hold.

b. ii. Find the volume of concrete (nearest m^3) required to build the structure.

3 + 3 = 6 marks

Water is filling the structure at a constant rate of 12 m^3 per minute.

c. Find the **average** rate (2 decimal places in mm per second) at which the water level is rising.

2 marks

d. i. When the structure is filled to a depth of h m, express the volume of water $V(h)$ m^3 as a definite integral. Integration is not required.

d. ii. Hence find an expression for $\frac{dh}{dt}$.

1 + 1 = 2 marks

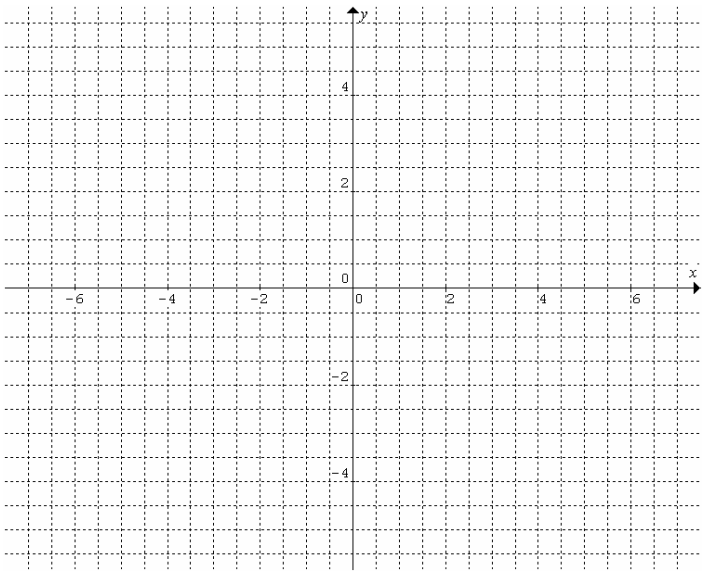
Total 12 marks

Question 3 The position of a particle is given by $\boldsymbol{r}(t) = \left(t + \frac{1}{t+1} + 1\right)\boldsymbol{i} + \left(t - \frac{1}{t+1} + 1\right)\boldsymbol{j},\ t \geq 0$.

a Find the Cartesian equation of the locus of the particle. Include domain and range in the answer.

4 marks

b. Sketch the graph of the locus. Indicate the direction of motion with an arrow head. Show asymptote(s) and coordinates of intercept(s).



3 marks

Point $P\left(\sqrt{2},3\right)$ is away from the locus of the particle.

c. i. Find a vector from point P to the particle at time *t*.

c. ii. Find the shortest distance between the particle and point P.

c. iii. Find the time when the particle is closest to point P.

2 + 3 + 1 = 6 marks

d. Find the speed (1 decimal place) of the particle at $t = \sqrt{2}$.

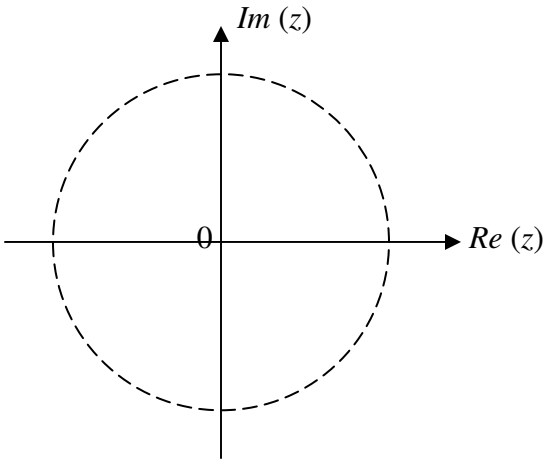
2 marks

Total 15 marks

Question 4 Let $z \in C$.

a. $\dfrac{z^7+1}{z+1}$ can be expressed as a polynomial of z, $P(z) = Az^6 + Bz^5 + Cz^4 + Dz^3 + Ez^2 + Fz + G$. Find the values of A, B, C, D, E, F and G. 2 marks

b. Plot accurately the roots of $P(z) = 0$ in the complex plane. 2 marks



c. Express the roots of $P(z) = 0$ in polar form. 2 marks

d. Find the products of the roots of $P(z) = 0$. 1 mark

e. Show that $\cos\left(\dfrac{\pi}{7}\right) + \cos\left(\dfrac{3\pi}{7}\right) + \cos\left(\dfrac{5\pi}{7}\right) = \dfrac{1}{2}$. 3 marks

Total 10 marks

Question 5 The coefficient of sliding friction between a 2.5-kg object and a plank is 0.7.
(**Round all answers to 1 decimal place**)

a. What is the force of friction if the object is at rest on the horizontal plank? 1 mark

b. What is the force of friction (2 decimal places) if the plank is tilted, making a 30° angle with the horizontal? 2 marks

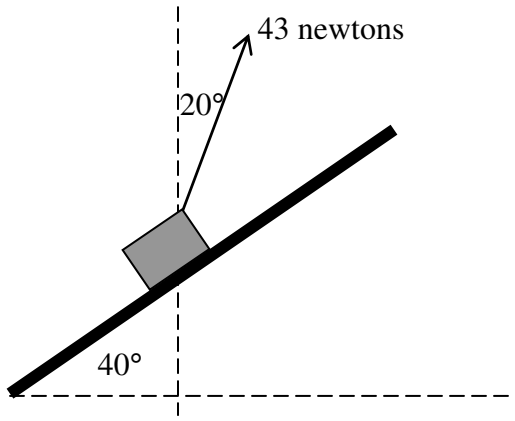
Now the angle is increased to 40° with the horizontal.

c i. Calculate the acceleration (2 decimal places) of the object down the plank.
2 marks

c. ii. The object starts from rest 3.0 m from the lower end of the plank. Find the magnitude of the momentum (2 decimal places) of the object when it reaches the lower end of the plank.

2 marks

The plank is left tilted at 40° with the horizontal. The object is placed on the plank, and a rope is attached to the object at an angle of 20° with the vertical. A student pulls the rope with a force of 43 newtons.



d. Calculate the magnitude of the resultant force (1 decimal place) on the object. 2 marks

e. Determine the direction (nearest degree) of motion of the object. 2 marks

f. Describe the path of the object and its motion in terms of direction, acceleration. 2 marks

Total 13 marks

End of Exam 2

