Year 2008

VCE

Specialist Mathematics

Trial Examination 1

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Victorian Certificate of Education
2008

STUDENT NUMBER

										Letter
Figures										
Words										

SPECIALIST MATHEMATICS

Trial Written Examination 1

Reading time: 15 minutes
Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 12 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.
Take the acceleration due to gravity to have magnitude g m/s^2, where $g = 9.8$

Question 1

Consider the relation $x^3 - 4x^2y^2 + 2y^2 = 7$. Find an expression for $\frac{dy}{dx}$ in terms of *x* and *y*.

2 marks

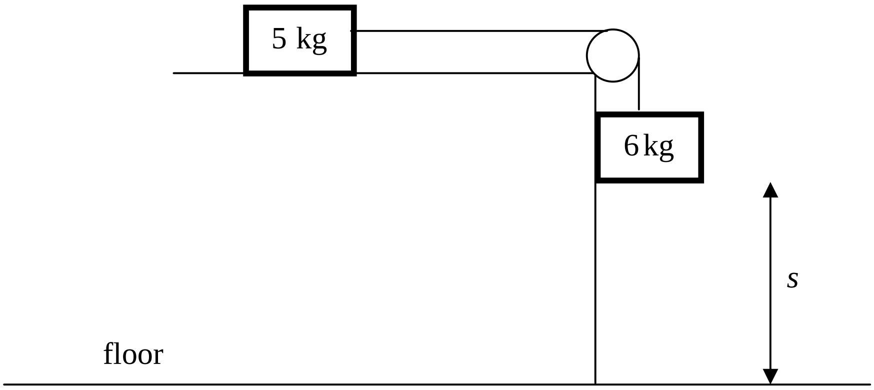
Question 2

Find the volume generated when the region enclosed by the curve with the equation $y = \frac{3}{\sqrt{9+4x^2}}$, the *x*-axis, the *y*-axis and the line $x = \frac{3}{2}$ is rotated about the *x*-axis to form a solid of revolution.

2 marks

Question 3

A block of mass 5 kg rests on a horizontal table. The coefficient of friction between the block and the table surface is 0.2. The block is connected by a light string which passes over a smooth pulley at the edge of the table to another block of mass 6 kg which is hanging vertically at the edge of the table. The system is released from rest, when the 6 kg block is s metres above the floor. After one half of a second, the 6 kg mass hits the floor. Find the value of s.



4 marks

Question 4

a. Solve the quadratic equation $z^2 + 2zi - 4 = 0$, expressing your answers in exact cartesian form.

2 marks

b. If $z = -\sqrt{3} - i$, express z in polar form and, hence, find z^6 giving your answer in exact cartesian form.

3 marks

Question 5

a. Show that $\frac{d}{dx}\left(\sin^{-1}\left(\frac{3}{\sqrt{x}}\right)\right) = \frac{-3}{2x\sqrt{x-9}}$ for $x > 9$.

2 marks

b. Hence, find the exact value of $\int_{12}^{18} \frac{1}{x\sqrt{x-9}}\,dx$.

3 marks

Question 6

a. Use Euler's method to find y_2 if $\dfrac{dy}{dx} = \log_e(2x-3)$, given that $y_0 = y(2) = 1$ and $h = 0.5$. Express your answer in the form $\log_e(p)$, where p is a real positive constant.

2 marks

b. Differentiate $(2x-3)\log_e(2x-3)$ and, hence, solve the differential given in part **a.** to find the value of y which is estimated by y_2. Express your answer in the form $\log_e(q)$, where q is a real positive constant.

4 marks

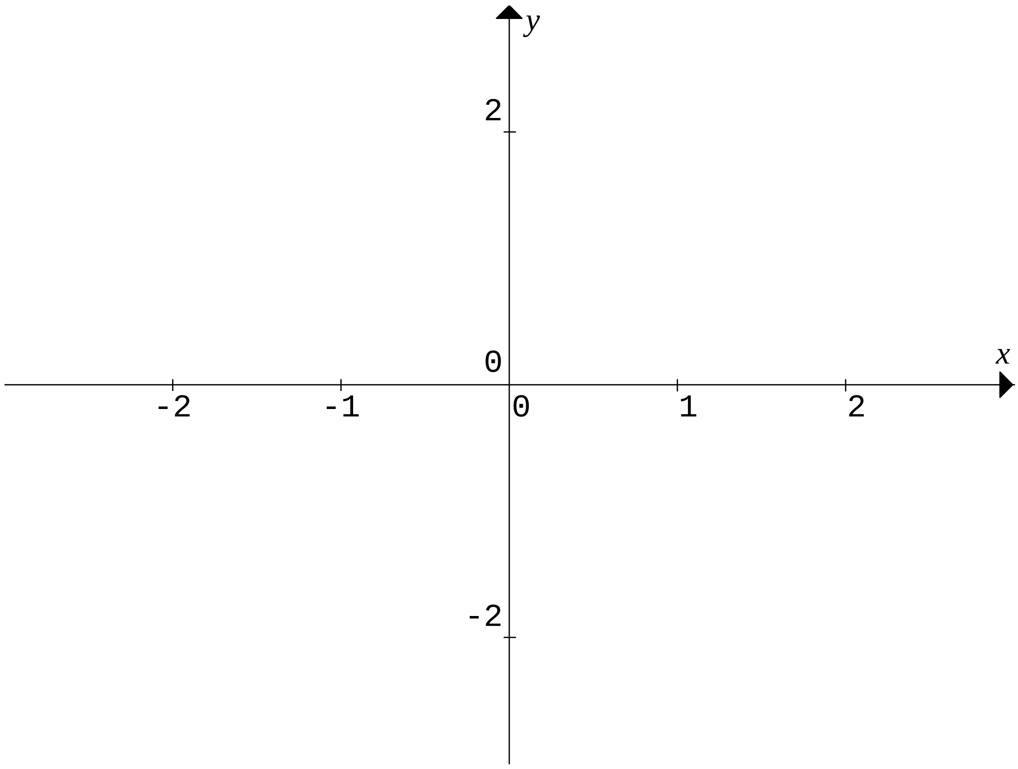
Question 7

The position vector of a moving particle is given by $\underset{\sim}{r}(t) = e^{-t}\underset{\sim}{i} + 2e^{-2t}\underset{\sim}{j}$ for $t \geq 0$.

a. Find the Cartesian equation of the path.

1 mark

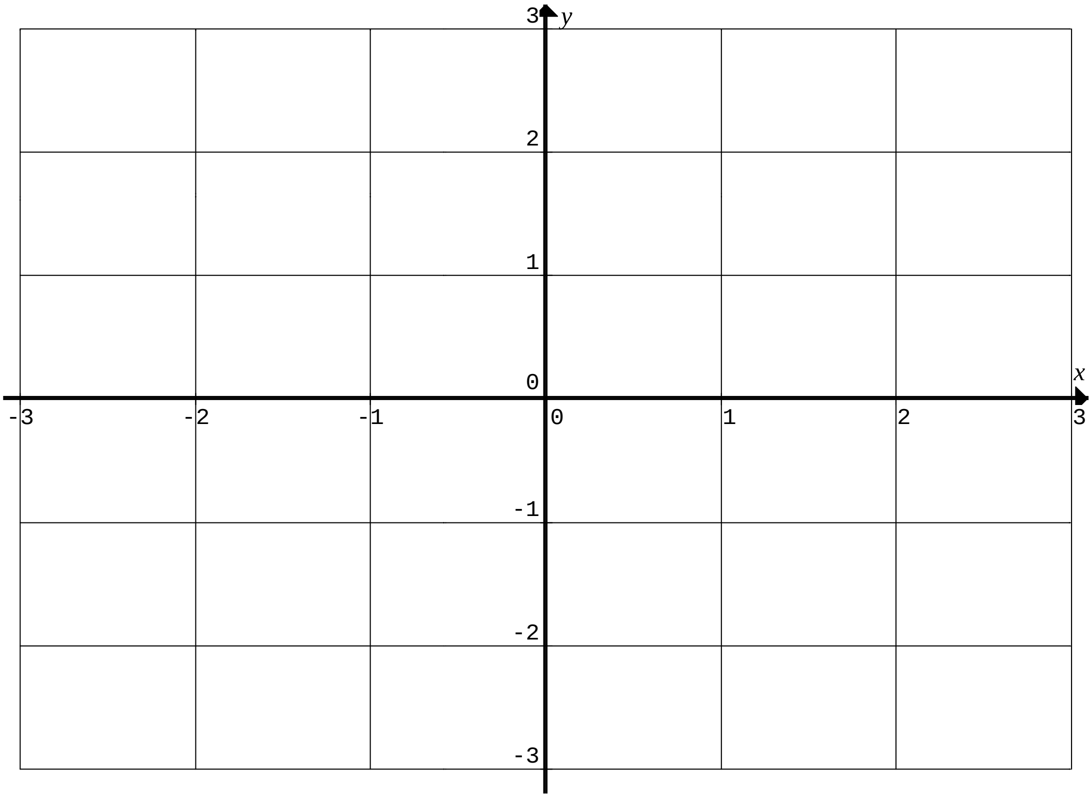
b. Sketch the path of the particle on the axes provided.



1 mark

Question 8

a. Sketch the slope field of the differential equation $2\frac{dy}{dx} + y = 0$ for $y = -2, -1, 0, 1, 2$ at each of the values $x = -2, -1, 0, 1, 2$ on the axes below.



2 marks

b. If $y = -1$ when $x = 0$, solve the differential equation given in part **a.** to find y in terms of x.

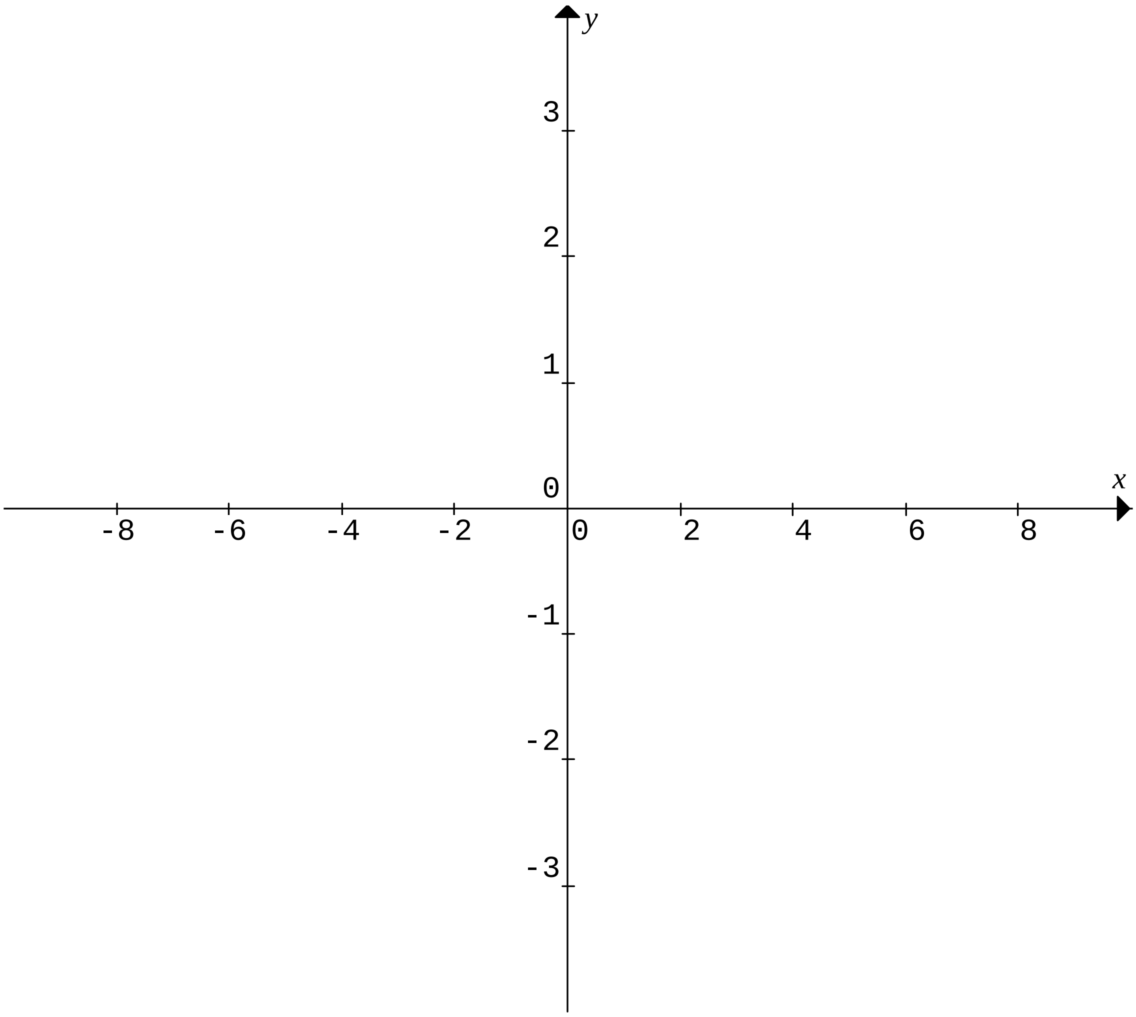
2 marks

c. Sketch the graph of the solution curve found in part **b.** on the slope field in part **a.**

1 mark

Question 9

a. Sketch the graph with the equation $y = \frac{32}{x^2 - 16}$, on the axes below, clearly indicating the location of all asymptotes, any turning points and axial intercepts.



2 marks

b. Find the area bounded by $y = \dfrac{32}{x^2 - 16}$, the x-axis, the y-axis and the line $x = -2$.

Express your answer in the form $\log_e(a)$, where a is a real positive constant.

4 marks

Question 10

Given that $\cos(x) - \sin(x) = \dfrac{1}{3}$ and $0 < x < \dfrac{\pi}{4}$, find the exact value of $\cot(2x)$.

3 marks

END OF EXAMINATION

SPECIALIST MATHEMATICS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of triangle:	$\frac{1}{2}bc\sin(A)$
sine rule:	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$ $\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ $\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$ $\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$ $\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1,1]$	$[-1,1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0,\pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex Numbers)

$z = x + yi = r\left(\cos(\theta) + i\sin(\theta)\right) = r\,\text{cis}(\theta)$

$|z| = \sqrt{x^2 + y^2} = r$ $\qquad$ $-\pi < \text{Arg}(z) \le \pi$

$z_1 z_2 = r_1 r_2 \,\text{cis}(\theta_1 + \theta_2)$ $\qquad$ $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\,\text{cis}(\theta_1 - \theta_2)$

$z^n = r^n \text{cis}(n\theta)$ (de Moivre's theorem)

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$ $\qquad$ $\underset{\sim}{r}_1 \,.\, \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \dfrac{d\underset{\sim}{r}}{dt} = \dfrac{dx}{dt}\underset{\sim}{i} + \dfrac{dy}{dt}\underset{\sim}{j} + \dfrac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

sliding friction: $F \le \mu N$

constant (uniform) acceleration:

$v = u + at$ $\qquad$ $s = ut + \frac{1}{2}at^2$ $\qquad$ $v^2 = u^2 + 2as$ $\qquad$ $s = \frac{1}{2}(u+v)t$

acceleration: $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\tfrac{1}{2}v^2\right)$

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c\ ,\ n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$$

$$\int \frac{1}{x}\,dx = \log_e(x) + c,\ \text{for}\ \ x > 0$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$$

$$\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$$

$$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$$

$$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$$

$$\int \frac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$$

$$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$$

$$\int \frac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Euler's method If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x)$

END OF FORMULA SHEET