

**Year 2008**

**VCE**

**Specialist Mathematics**

**Trial Examination 1**



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**Victorian Certificate of Education  
2008**

**STUDENT NUMBER**

Figures										Letter
Words										

**SPECIALIST MATHEMATICS**

**Trial Written Examination 1**

Reading time: 15 minutes  
Total writing time: 1 hour

**QUESTION AND ANSWER BOOK**

**Structure of book**

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

**Materials supplied**

- Question and answer book of 12 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

**Instructions**

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

**Instructions**

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Take the acceleration due to gravity to have magnitude  $g \text{ m/s}^2$ , where  $g = 9.8$

**Question 1**

Consider the relation  $x^3 - 4x^2y^2 + 2y^2 = 7$ . Find an expression for  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ .

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2 marks

**Question 2**

Find the volume generated when the region enclosed by the curve with the equation

$y = \frac{3}{\sqrt{9+4x^2}}$ , the  $x$ -axis, the  $y$ -axis and the line  $x = \frac{3}{2}$  is rotated about the  $x$ -axis to

form a solid of revolution.

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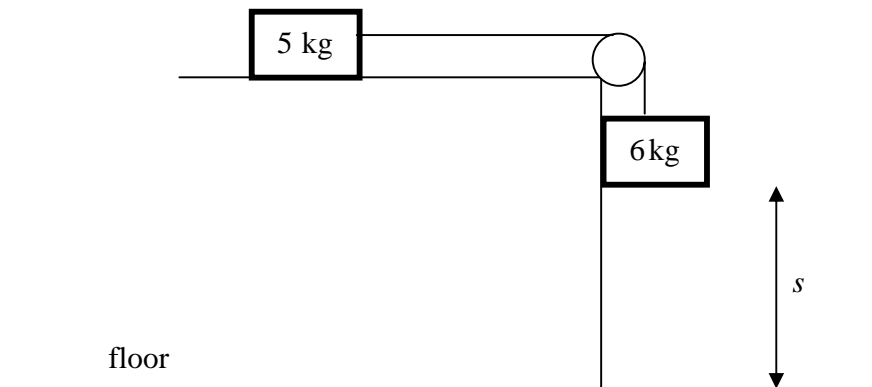
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2 marks

**Question 3**

A block of mass 5 kg rests on a horizontal table. The coefficient of friction between the block and the table surface is 0.2. The block is connected by a light string which passes over a smooth pulley at the edge of the table to another block of mass 6 kg which is hanging vertically at the edge of the table. The system is released from rest, when the 6 kg block is  $s$  metres above the floor. After one half of a second, the 6 kg mass hits the floor. Find the value of  $s$ .



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4 marks

**Question 4**

- a.** Solve the quadratic equation  $z^2 + 2zi - 4 = 0$ , expressing your answers in exact cartesian form.

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2 marks

- b.** If  $z = -\sqrt{3} - i$ , express  $z$  in polar form and, hence, find  $z^6$  giving your answer in exact cartesian form.

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3 marks

**Question 5**

a. Show that  $\frac{d}{dx}\left(\sin^{-1}\left(\frac{3}{\sqrt{x}}\right)\right) = \frac{-3}{2x\sqrt{x-9}}$  for  $x > 9$ .

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2 marks

b. Hence, find the exact value of  $\int_{12}^{18} \frac{1}{x\sqrt{x-9}} dx$ .

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3 marks

**Question 6**

- a.** Use Euler's method to find  $y_2$  if  $\frac{dy}{dx} = \log_e(2x-3)$ , given that  $y_0 = y(2) = 1$  and  $h = 0.5$ . Express your answer in the form  $\log_e(p)$ , where  $p$  is a real positive constant.

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2 marks

- b.** Differentiate  $(2x-3)\log_e(2x-3)$  and, hence, solve the differential given in part **a.** to find the value of  $y$  which is estimated by  $y_2$ . Express your answer in the form  $\log_e(q)$ , where  $q$  is a real positive constant.

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4 marks



**Question 7**

The position vector of a moving particle is given by  $\underline{r}(t) = e^{-t}\underline{i} + 2e^{-2t}\underline{j}$  for  $t \geq 0$ .

- a. Find the Cartesian equation of the path.

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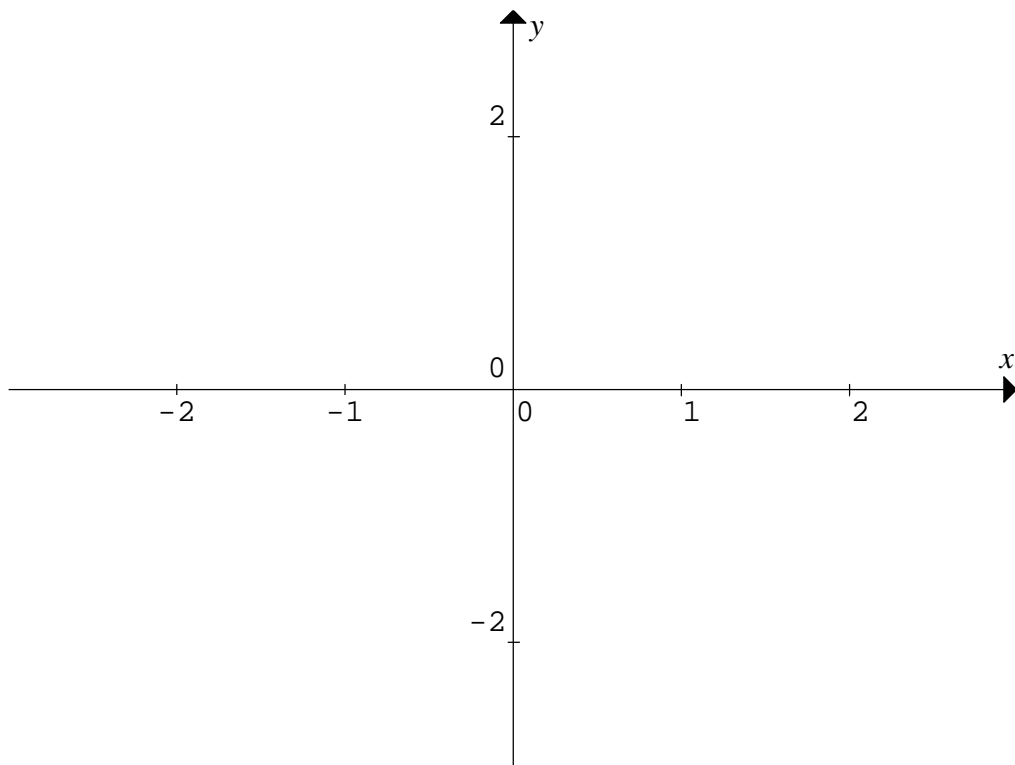
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1 mark

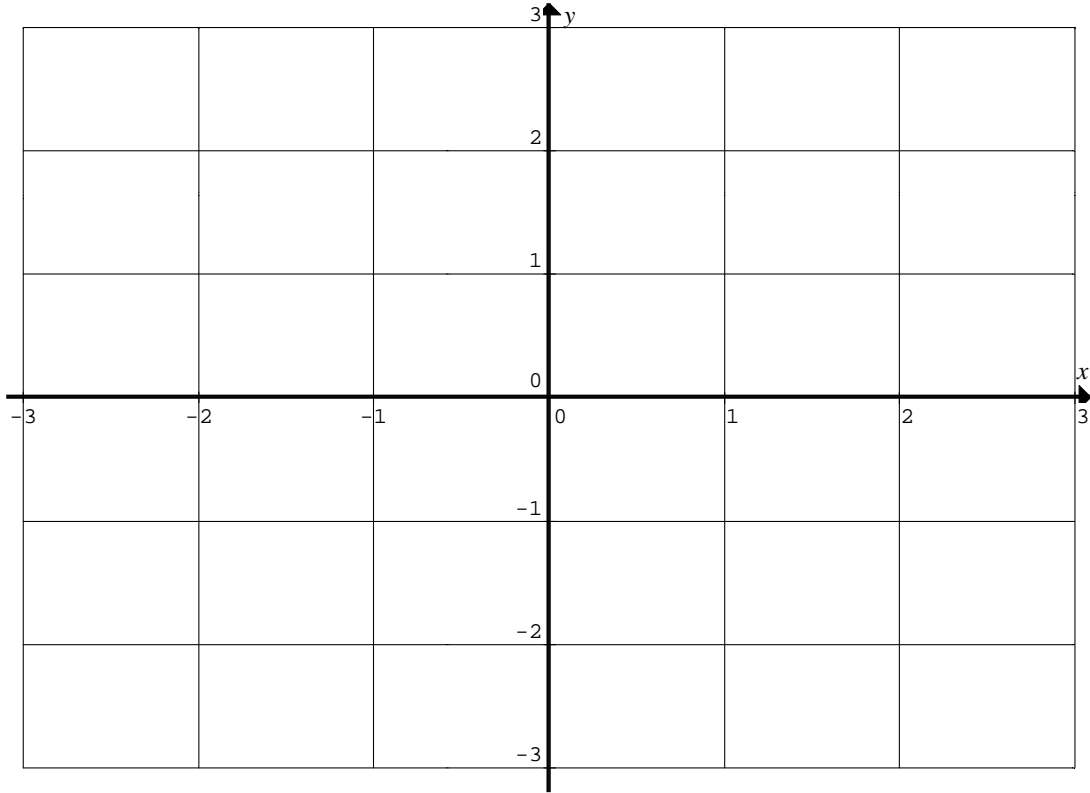
- b. Sketch the path of the particle on the axes provided.



1 mark

**Question 8**

- a. Sketch the slope field of the differential equation  $2\frac{dy}{dx} + y = 0$  for  $y = -2, -1, 0, 1, 2$  at each of the values  $x = -2, -1, 0, 1, 2$  on the axes below.



2 marks

- b. If  $y = -1$  when  $x = 0$ , solve the differential equation given in part a. to find  $y$  in terms of  $x$ .

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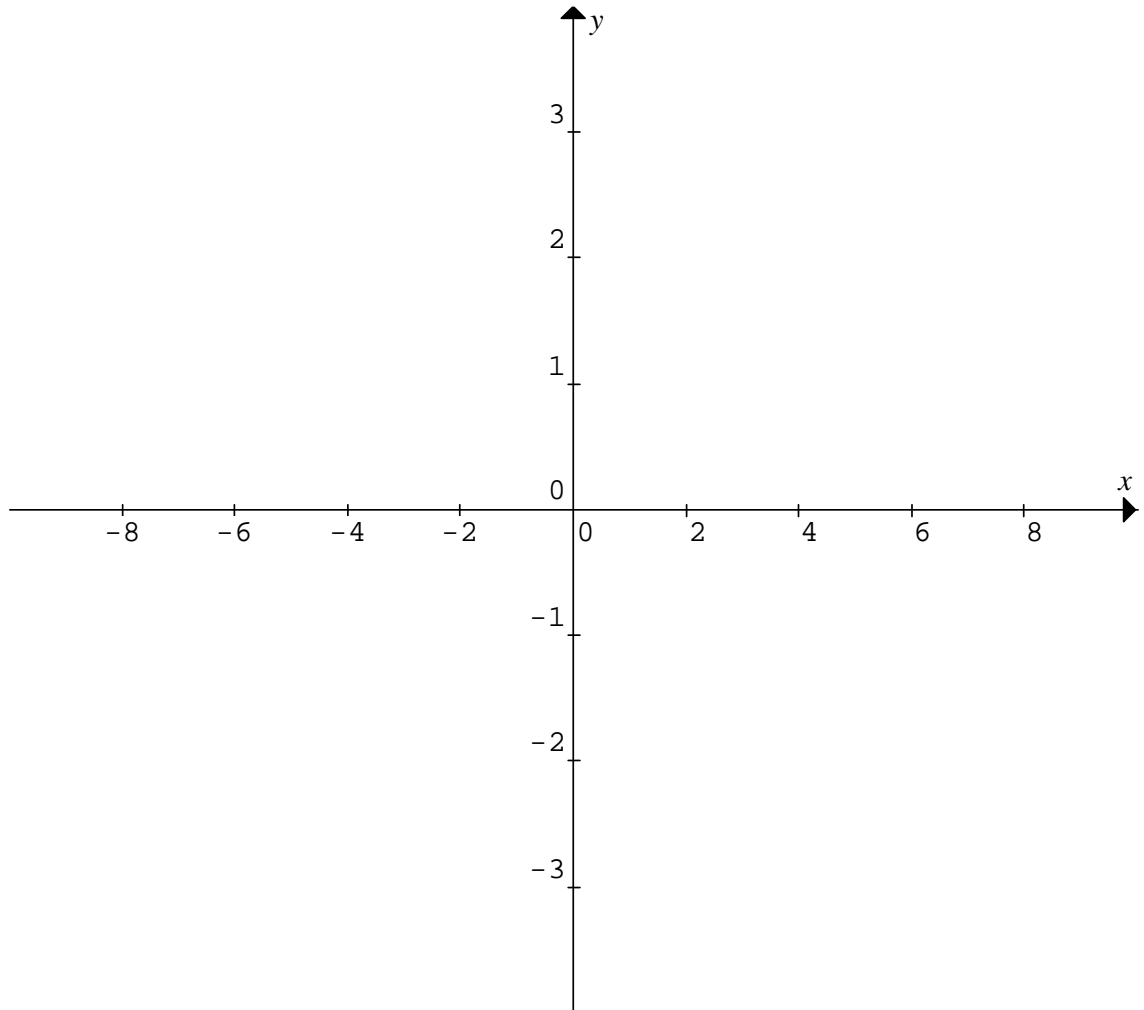
2 marks

- c. Sketch the graph of the solution curve found in part b. on the slope field in part a.

1 mark

**Question 9**

- a. Sketch the graph with the equation  $y = \frac{32}{x^2 - 16}$ , on the axes below, clearly indicating the location of all asymptotes, any turning points and axial intercepts.



2 marks

- b. Find the area bounded by  $y = \frac{32}{x^2 - 16}$ , the  $x$ -axis, the  $y$ -axis and the line  $x = -2$ .  
Express your answer in the form  $\log_e(a)$ , where  $a$  is a real positive constant.

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4 marks

**Question 10**

Given that  $\cos(x) - \sin(x) = \frac{1}{3}$  and  $0 < x < \frac{\pi}{4}$ , find the exact value of  $\cot(2x)$ .

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3 marks

**END OF EXAMINATION**

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# **SPECIALIST MATHEMATICS**

## **Written examination 1**

### **FORMULA SHEET**

#### **Directions to students**

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

## Specialist Mathematics Formulas

### Mensuration

area of a trapezium:  $\frac{1}{2}(a+b)h$

curved surface area of a cylinder:  $2\pi rh$

volume of a cylinder:  $\pi r^2 h$

volume of a cone:  $\frac{1}{3}\pi r^2 h$

volume of a pyramid:  $\frac{1}{3}Ah$

volume of a sphere:  $\frac{4}{3}\pi r^3$

area of triangle:  $\frac{1}{2}bc \sin(A)$

sine rule:  $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$

cosine rule:  $c^2 = a^2 + b^2 - 2ab \cos(C)$

### Coordinate geometry

ellipse:  $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$     hyperbola:  $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

### Circular ( trigonometric ) functions

$$\cos^2(x) + \sin^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$$

$$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$$

$$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$

$$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$$

$$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	$R$
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

## Algebra ( Complex Numbers )

$$z = x + yi = r(\cos(\theta) + i \sin(\theta)) = r \operatorname{cis}(\theta)$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$-\pi < \operatorname{Arg}(z) \leq \pi$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

$$z^n = r^n \operatorname{cis}(n\theta) \text{ (de Moivre's theorem)}$$

## Vectors in two and three dimensions

$$\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$$

$$|\underline{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\underline{r}_1 \cdot \underline{r}_2 = r_1 r_2 \cos \theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\underline{r}} = \frac{d\underline{r}}{dt} = \frac{dx}{dt} \underline{i} + \frac{dy}{dt} \underline{j} + \frac{dz}{dt} \underline{k}$$

## Mechanics

momentum:  $\underline{p} = m\underline{v}$

equation of motion:  $\underline{R} = m\underline{a}$

sliding friction:  $F \leq \mu N$

constant ( uniform ) acceleration:

$$v = u + at \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as \quad s = \frac{1}{2}(u + v)t$$

acceleration:  $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx} \left( \frac{1}{2}v^2 \right)$

## Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, \quad n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e(x) + c, \quad \text{for } x > 0$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c, \quad a > 0$$

$$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, \quad a > 0$$

$$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule:  $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$

quotient rule:  $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

chain rule:  $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

Euler's method If  $\frac{dy}{dx} = f(x)$ ,  $x_0 = a$  and  $y_0 = b$ , then  $x_{n+1} = x_n + h$  and  $y_{n+1} = y_n + hf(x)$

**END OF FORMULA SHEET**