Year 2008

VCE

Specialist Mathematics

Trial Examination 2

KILBAHA MULTIMEDIA PUBLISHING	**TEL: (03) 9817 5374**
PO BOX 2227	**FAX: (03) 9817 4334**
KEW VIC 3101	**kilbaha@gmail.com**
AUSTRALIA	**http://kilbaha.googlepages.com**

Victorian Certificate of Education 2008

STUDENT NUMBER

										Letter
Figures										
Words										

SPECIALIST MATHEMATICS

Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved graphics calculator or CAS (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer booklet of 30 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Answer sheet for multiple choice questions.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, and sign your name in the space provided to verify this.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section I

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
A correct answer scores 1 mark, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than one answer is completed for any question.
Take the acceleration due to gravity to have magnitude g m/s^2, where $g = 9.8$

Question 1

Consider the graph of the function with the rule $f(x) = \dfrac{1}{c - 4x - x^2}$ over its maximal domain, where *c* is a non-zero real constant.
Which one of the following statements is **false?**

A. The *x*-axis is a horizontal asymptote.

B. The graph crosses the *y*-axis at $y = \dfrac{1}{c}$.

C. The graph has a minimum turning point at $x = -2$.

D. If $c < -4$, then the graph has two vertical asymptotes.

E. If $c = -4$, then the graph has only one vertical asymptote.

Question 2

Consider the hyperbola with the equation $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$ where *a* and *b* are non-zero real numbers, and *h* and *k* are real constants. Which one of the following statements is **false?**

A. The graph crosses the *x*-axis at $x = h \pm \dfrac{a}{b}\sqrt{b^2 + k^2}$.

B. The graph crosses the *y*-axis at $y = k \pm \dfrac{b}{a}\sqrt{a^2 + h^2}$.

C. The domain is $(-\infty, h - a] \cup [a + h, \infty)$ and the range is R.

D. The equations of the asymptotes are $y = \pm\dfrac{b}{a}(x - h) + k$.

E. If $a = b$, the asymptotes are perpendicular.

Question 3

Let $\underset{\sim}{i}$ be a unit vector in the east direction and $\underset{\sim}{j}$ be a unit vector in the north direction. A force of magnitude $4\sqrt{3}$ acting in the direction south 30^0 west, could be

A. $-4\sqrt{3}\left(\underset{\sim}{i}+\sqrt{3}\,\underset{\sim}{j}\right)$

B. $-4\sqrt{3}\left(\sqrt{3}\,\underset{\sim}{i}+\underset{\sim}{j}\right)$

C. $-2\sqrt{3}\,\underset{\sim}{i}-6\,\underset{\sim}{j}$

D. $-6\,\underset{\sim}{i}-2\sqrt{3}\,\underset{\sim}{j}$

E. $2\sqrt{3}\left(-\underset{\sim}{i}+2\underset{\sim}{j}\right)$

Question 4

The position vector of a particle at a time $t \ge 0$ is given by

$\underset{\sim}{r}(t)=\left(\cos(t)-\sin(t)\right)\underset{\sim}{i}+\left(\cos(t)+\sin(t)\right)\underset{\sim}{j}$. The path of the particle is

A. a parabola.

B. a circle.

C. a hyperbola.

D. an ellipse.

E. a straight line.

Question 5

Consider the graph with the equation $y=\dfrac{ax^3+b}{x^2}$ where *a* and *b* are non-zero real numbers. Which one of the following statements is **false?**

A. If $a>0$ and $b>0$ the graph has a maximum turning point.

B. If $a<0$ and $b>0$ the graph has a minimum turning point.

C. the graph has $x=0$ as a horizontal asymptote.

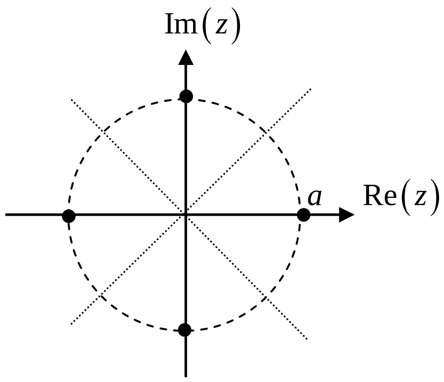
D. the graph has the line $y=ax$ as an oblique asymptote.

E. the graph has an *x*-intercept at $x=\sqrt[3]{-\dfrac{b}{a}}$.

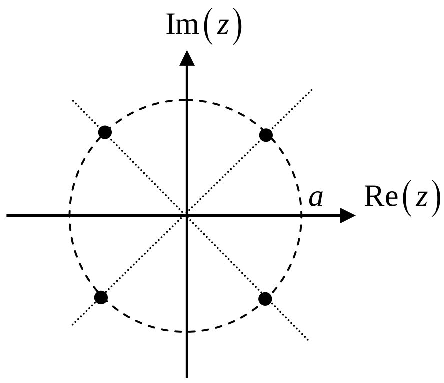
Question 6

Which one of the following diagrams represents the roots of the equation $z^4 + a^4 i = 0$ in the complex plane where $a \in R$?

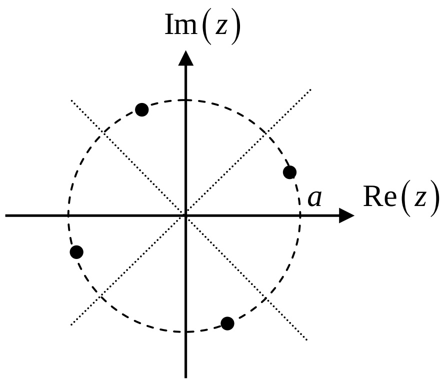
A.



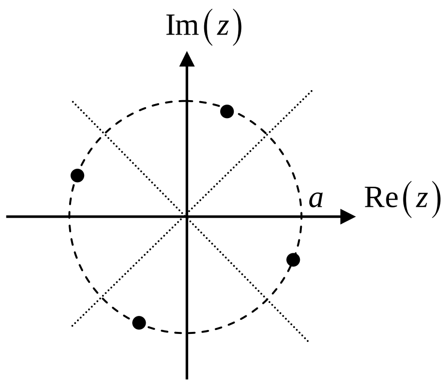
B.



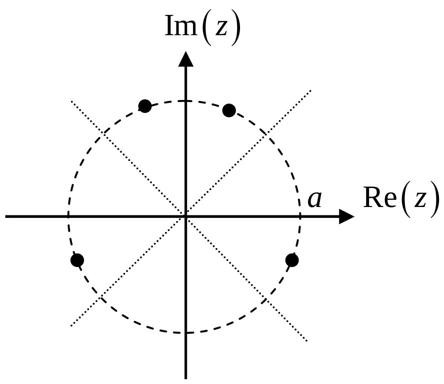
C.



D.



E.



Question 7

If a and b are positive real numbers and $\text{Arg}(a+bi)=\theta$, then $\text{Arg}(b-ai)$ is equal to

A. $\theta+\frac{\pi}{2}$

B. $\theta-\frac{\pi}{2}$

C. $\frac{\pi}{2}-\theta$

D. $\theta+\pi$

E. $-\theta$

Question 8

Let $P(z)=z^3+bz^2+cz+d$. If $P(ki)=0$ where b, c, d and k are all non-zero real constants, then which of the following is **false?**

A. $P(-ki)=0$

B. $c=k^2$

C. $d=bc$

D. $\left(z+\frac{d}{k^2}\right)$ is a factor of $P(z)$

E. z^2-k^2 is a factor of $P(z)$

Question 9

If the vector $x\underset{\sim}{i}+y\underset{\sim}{j}-2\underset{\sim}{k}$ makes an angle of $\pi-\cos^{-1}\left(\frac{1}{3}\right)$ with the positive *y*-axis, then

A. $x=0$ and $y=-1$.

B. $x=6$ and $y=-1$.

C. $x=2$ and $y=1$.

D. $x=-2$ and $y=1$.

E. $x=\pm 2$ and $y=-1$.

Question 10

If a and b are all positive real constants, and given each of the following functions, which one does **not** have the correct maximal domain or range?

		domain	range
A.	$f(x)=a\sin^{-1}(bx)$	$\left[-\frac{1}{b},\frac{1}{b}\right]$	$[0,\ a\pi]$
B.	$f(x)=a\cos^{-1}(bx)$	$\left[-\frac{1}{b},\frac{1}{b}\right]$	$\left[-\frac{a\pi}{2},\ \frac{a\pi}{2}\right]$
C.	$f(x)=a\tan^{-1}(bx)$	R	$\left(-\frac{a\pi}{2},\frac{a\pi}{2}\right)$
D.	$f(x)=\dfrac{a}{\sqrt{1-b^2x^2}}$	$\left(-\frac{1}{b},\frac{1}{b}\right)$	$(0,a]$
E.	$f(x)=\dfrac{ab}{b^2x^2+1}$	R	$(0,a]$

Question 11

Using a suitable substitution, which of the following is **not** equal to $\int_0^8 \frac{x}{\sqrt{2x+9}}\,dx$?

A. $\frac{1}{4}\int_9^{25} \frac{s-9}{\sqrt{s}}\,ds$

B. $\int_4^8 \frac{t}{\sqrt{2t+9}}\,dt$

C. $\frac{1}{4}\int_0^{16} \frac{u}{\sqrt{u+9}}\,du$

D. $\frac{1}{2}\int_9^{25} \frac{2v-9}{\sqrt{2v}}\,dv$

E. $\frac{1}{2}\int_3^5 \left(z^2-9\right)dz$

Question 12

If a is a positive real constant, then which of the following graphs does **not** have a period of $2a$?

A. $y=\frac{a}{\pi}\operatorname{cosec}\left(\frac{\pi}{a}(x-2)\right)$

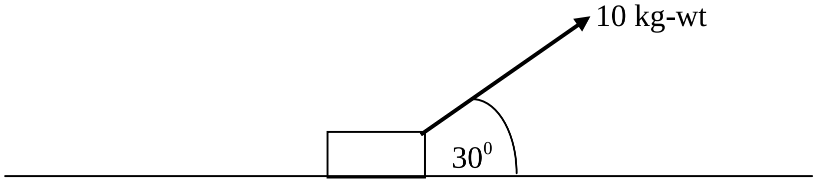
B. $y=\frac{\pi}{a}\sec\left(\frac{\pi}{a}(x+2)\right)$

C. $y=\frac{a}{\pi}\cot\left(\frac{\pi}{a}\left(\frac{x+2}{2}\right)\right)$

D. $y=\frac{\pi}{a}\tan\left(\frac{\pi}{a}(x+2)\right)$

E. $y=\frac{a}{\pi}\cos\left(\frac{\pi}{a}(x+2)\right)$

Question 13



A box of mass 20 kg is at rest on a horizontal plane. A force of magnitude 10 kg-wt acting at an angle of 30^0 to the horizontal is applied to the block. For equilibrium to be maintained, the coefficient of friction between the box and the plane must be

A. at least $\frac{\sqrt{3}}{3}$

B. less than $\frac{\sqrt{3}}{3}$

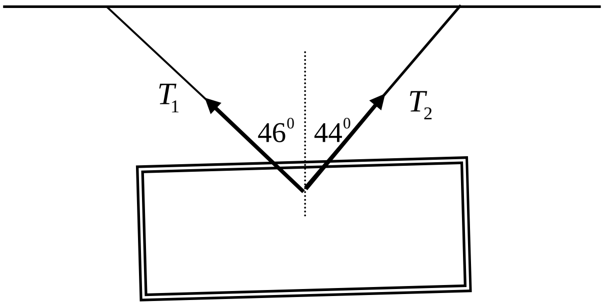
C. at least $\frac{\sqrt{3}}{4}$

D. at least $\frac{\sqrt{3}}{4g}$

E. less than $\frac{\sqrt{3}}{4g}$

Question 14

A painting of mass one kilogram is to be hung on a wall using two light strings. Unfortunately the painting is not quite horizontal. One string makes an angle of 46^0 with the vertical and has a tension of magnitude T_1 newtons. The other string makes an angle of 44^0 with the vertical and has a tension of magnitude T_2 newtons, as shown in the diagram below.



Which of the following is true?

A. $\frac{T_1}{T_2} = \tan\left(44^0\right)$

B. $\frac{T_1}{T_2} = \tan\left(46^0\right)$

C. $T_1 + T_2 = g$

D. $T_1^2 + T_2^2 = g^2$

E. $T_1 = T_2$

Question 15

Given the two vectors $\underset{\sim}{a} = x\underset{\sim}{i} + y\underset{\sim}{j} - 2\underset{\sim}{k}$ and $\underset{\sim}{b} = -3\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}$ then which of the following is **false?**

A. If $x = 9$ and $y = -6$, then vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are collinear.

B. If $x = 6$ and $y = -4$, the vector $\underset{\sim}{a}$ is parallel to the vector $\underset{\sim}{b}$.

C. If $x = y = \sqrt{5}$, then $|\underset{\sim}{a}| = |\underset{\sim}{b}|$.

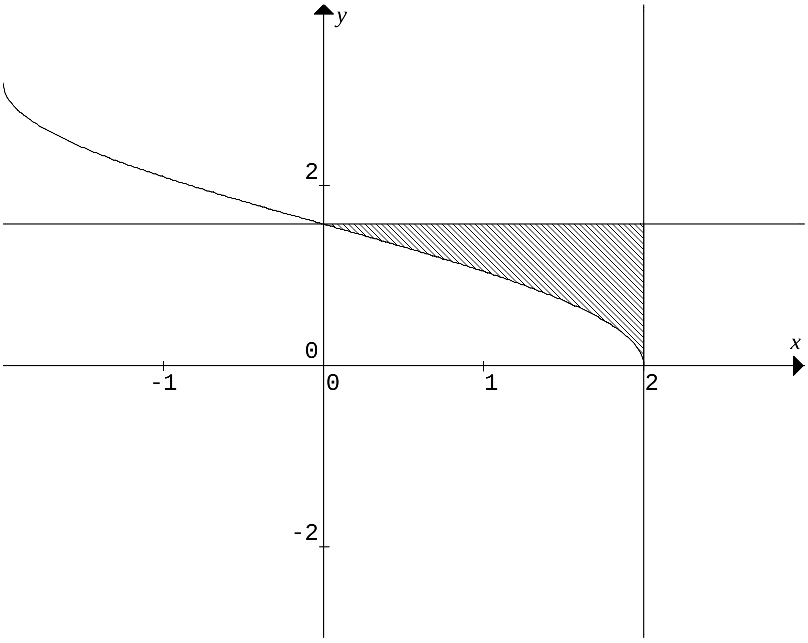
D. If $x = -4$ and $y = -5$, the vector $\underset{\sim}{a}$ is perpendicular to the vector $\underset{\sim}{b}$.

E. If $x = 0$ and $y = 2$, then the scalar resolute of $\underset{\sim}{a}$ in the direction of $\underset{\sim}{b}$ is $\frac{2}{\sqrt{14}}$

Question 16

The graph of the function $f(x) = \cos^{-1}\left(\frac{x}{2}\right)$ is shown below. The shaded region is the area bounded by the graph of f and the lines with the equations $y = \frac{\pi}{2}$ and $x = 2$.

The shaded region is rotated about the *x*-axis to form a solid of revolution.



The volume of the solid in cubic units is given by

A. $\pi\int_0^2 \left(\frac{\pi^2}{4} - \cos^{-2}\left(\frac{x}{2}\right)\right)dx$

B. $\pi\int_0^2 \left(4 - \cos^{-2}\left(\frac{x}{2}\right)\right)dx$

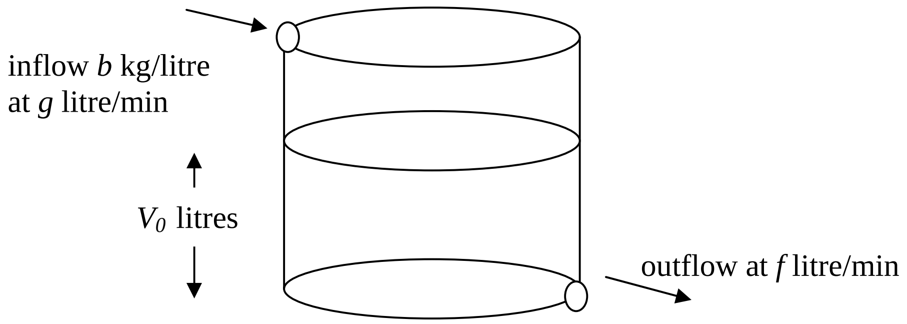
C. $\pi\int_0^2 \left(\frac{\pi^2}{4} - \left(\cos^{-1}\left(\frac{x}{2}\right)\right)^2\right)dx$

D. $\pi\int_0^2 \left(\frac{\pi}{2} - \cos^{-1}\left(\frac{x}{2}\right)\right)^2 dx$

E. $2\pi\int_0^{\frac{\pi}{2}} \left(1 - \cos(2x)\right)dx$

Question 17

Consider a tank, which initially holds V_0 litres of a solution, in which a kilograms of salt has been dissolved. Another solution containing b kilograms of salt per litre is poured into the tank at a rate of g litres per minute. The well-stirred mixture leaves the tank at a rate of f litres per minutes.



The differential equation for Q, the amount of salt in kilograms in the tank at a time t minutes, is given by

A. $\frac{dQ}{dt} = bg - \frac{f\,Q}{V_0 + (g-f)t} \qquad Q(0) = a$

B. $\frac{dQ}{dt} = bg - \frac{f\,Q}{V_0 + (f-g)t} \qquad Q(0) = a$

C. $\frac{dQ}{dt} = bg + \frac{f\,Q}{V_0 + (g-f)t} \qquad Q(0) = a$

D. $\frac{dQ}{dt} = bg - \frac{f\,Q}{V_0} \qquad Q(0) = a$

E. $\frac{dQ}{dt} = bg - \frac{f\,Q}{(g-f)V_0} \qquad Q(0) = a$

Question 18

A hot air balloon is accelerating vertically upwards with an acceleration of 1 m/s^2. A stone is dropped from the balloon when it is h metres above the ground. The stone strikes the ground 8 seconds later. Assuming the air resistance is negligible, the value of h is closest to

A. 282.

B. 290.

C. 314.

D. 322.

E. 346.

Question 19

The velocity of a particle at a time *t*, $t \geq 0$, is given by $\dot{\underset{\sim}{r}}(t) = 4\sin(2t)\underset{\sim}{i} + 6e^{-\frac{t}{3}}\underset{\sim}{j}$.
The initial position of the particle is given by $\underset{\sim}{r}(0) = \underset{\sim}{0}$. The position vector $\underset{\sim}{r}(t)$ is given by

A. $\underset{\sim}{r}(t) = -2\cos(2t)\underset{\sim}{i} - 18e^{-\frac{t}{3}}\underset{\sim}{j}$

B. $\underset{\sim}{r}(t) = 8\cos(2t)\underset{\sim}{i} - 2e^{-\frac{t}{3}}\underset{\sim}{j}$

C. $\underset{\sim}{r}(t) = 4\sin^2(t)\underset{\sim}{i} + 18\left(1 - e^{-\frac{t}{3}}\right)\underset{\sim}{j}$

D. $\underset{\sim}{r}(t) = -4\sin^2(t)\underset{\sim}{i} + 18\left(1 - e^{-\frac{t}{3}}\right)\underset{\sim}{j}$

E. $\underset{\sim}{r}(t) = 2(\cos(2t) - 1)\underset{\sim}{i} + 18\left(1 - e^{-\frac{t}{3}}\right)\underset{\sim}{j}$

Question 20

If $\frac{dx}{dt} = e^{t^2}$ and $x = 1$ when $t = 0$, then the value of *x* when $t = 2$ can be found by evaluating

A. $\int_1^2 e^{u^2}\, du$

B. $\int_0^2 \left(e^{u^2} - 1\right) du$

C. $\int_1^2 e^{u^2}\, du - 1$

D. $\int_0^2 \left(e^{u^2} + 1\right) du$

E. $\int_0^2 e^{u^2}\, du + 1$

Question 21

The velocity of a particle at a time *t* seconds is given by $v(t)=\cos^{-1}(2t-1)-1$.
Over the first second of the motion, the distance travelled by the particle is equal to

A. $\int_0^1 \left(\cos^{-1}(2t-1)-1\right)dt$

B. $\left|\int_0^1 \left(\cos^{-1}(2t-1)-1\right)dt\right|$

C. $\int_0^{0.77} \left(\cos^{-1}(2t-1)-1\right)dt+\int_{0.77}^1 \left(\cos^{-1}(2t-1)-1\right)dt$

D. $\int_0^{0.77} \left(\cos^{-1}(2t-1)-1\right)dt-\int_{0.77}^1 \left(\cos^{-1}(2t-1)-1\right)dt$

E. $\int_0^{0.8} \left(\cos^{-1}(2t-1)-1\right)dt+\int_{0.8}^1 \left(1-\cos^{-1}(2t-1)\right)dt$

Question 22

A particle of mass 3 kg travels in a straight line with velocity *v* m/s and its displacement is *x* metres. If $v=3\log_e(3x)$ for $x>0$, then the maximum force in newtons acting on the particle is closest to

A. 90.

B. 30.

C. 24.

D. 10.

E. 3.

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$

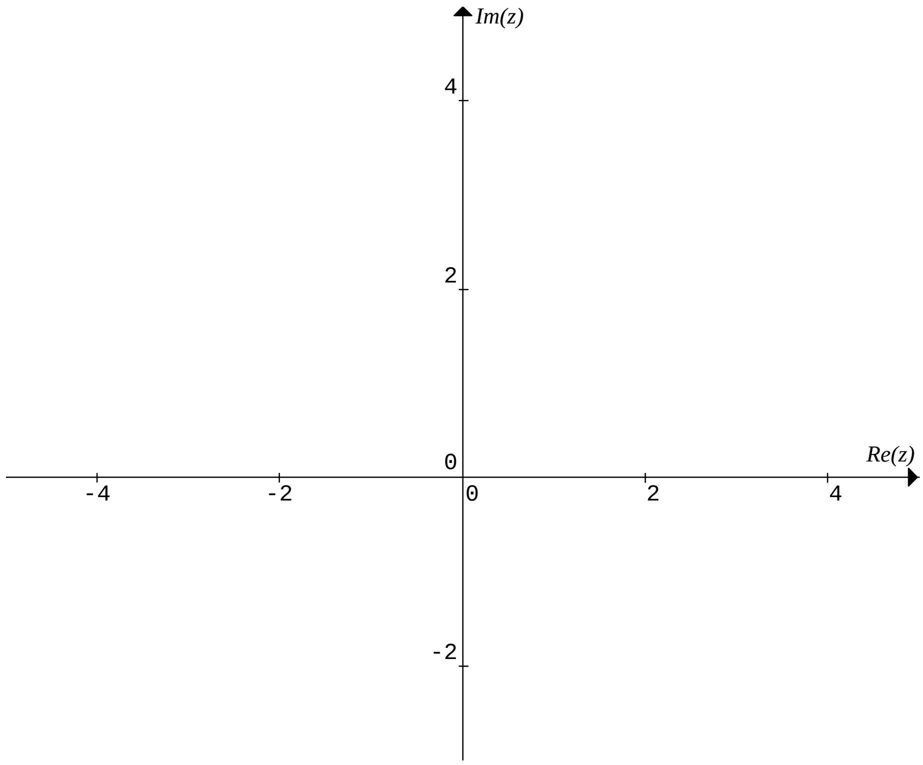
Question 1

Consider the cubic equation $z^3 - 4z^2 + bz + c = 0$ where b and c are real numbers.
The complex number $u = 1 + 3i$ is a root of this equation.

a. Find the values of b and c.

3 marks

b. Let P be the complex number u and Q represent the complex number iu.
Plot the points P and Q on the Argand diagram below.



1 mark

Let $\underset{\sim}{i}$ be a unit vector in the direction of the real axis and $\underset{\sim}{j}$ be a unit vector of the imaginary axis.

c. If the origin is the point O, using vectors, show that $\overrightarrow{OP}$ is perpendicular to $\overrightarrow{OQ}$.

1 mark

d. Find $\left|\overrightarrow{PQ}\right|$.

1 mark

e. The line perpendicular to PQ can be described as the subset of the complex plane $S = \{z : |z - p| = |z - q|\}$. Write down the values of p and q and find the Cartesian equation of S. Sketch the set S, on the Argand diagram above in **b.**

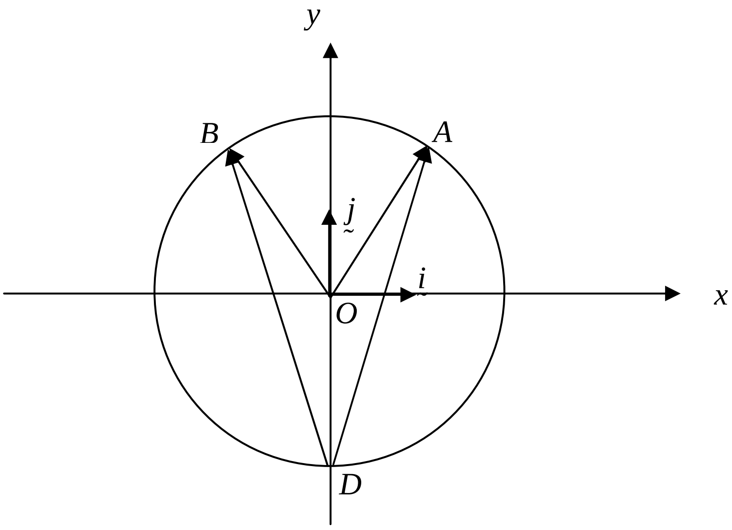
2 marks

f. The circle which passes through the points P, Q and the origin O, can be described as the subset of the complex plane $R = \{z : |z - c| = r\}$.
Write down the values of c and r and find the Cartesian equation of R.
Sketch the set R, on the Argand diagram above in **b.**

3 marks
Total 11 marks

Question 2

The diagram below, shows a circle of radius *r*, with center at the origin *O* with *x* and *y* axes. The three points *A*, *B* and *D* all lie on the circle and have coordinates $A(a,b)$ $B(-a,b)$ and $D(0,-r)$ where *a* , *b* and *r* are all positive real constants.



a. Let θ be the angle between the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Using vectors express $\cos(\theta)$ in terms of *a* and *b*.

3 marks

b. Let α be the angle between the vectors $\overrightarrow{DA}$ and $\overrightarrow{DB}$. Using vectors show that $\cos(\alpha) = \dfrac{b}{\sqrt{a^2 + b^2}}$.

4 marks

c. Hence, show that $2\alpha = \theta$.

3 marks
Total 10 marks

Question 3

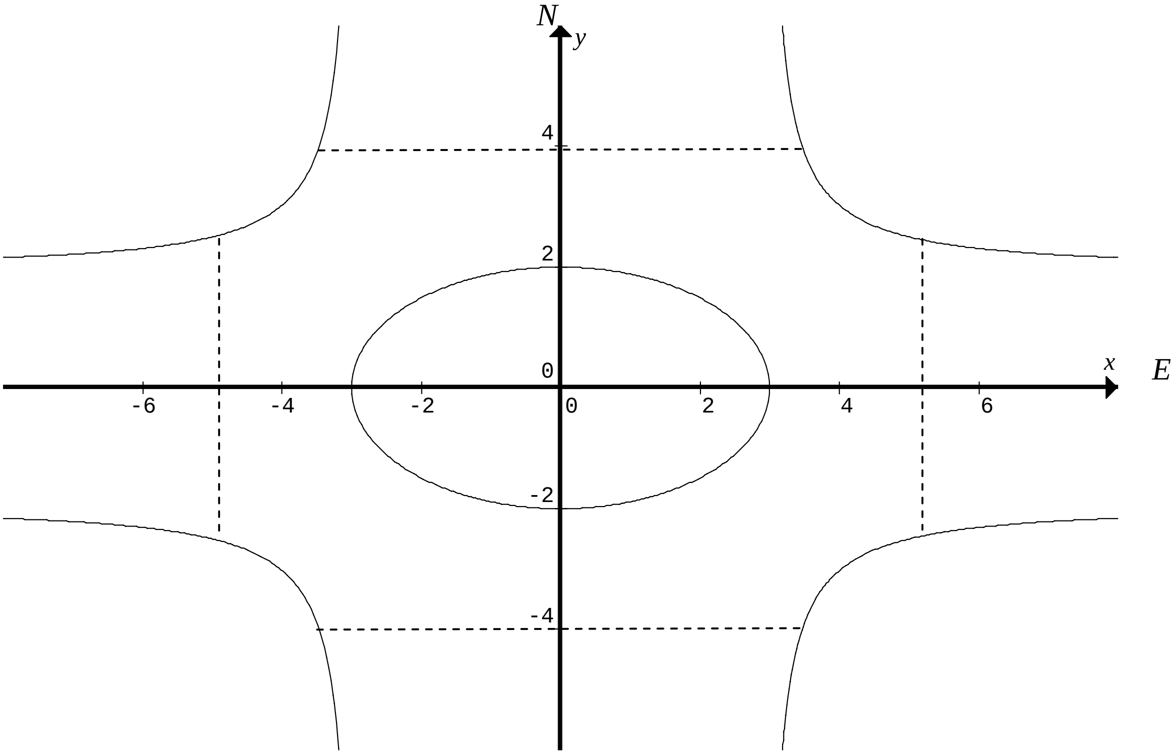
a. Find, using calculus, the exact value of $\int_{2\sqrt{3}}^{5} \frac{x}{\sqrt{x^2-9}}\,dx$.

2 marks

b. Using the substitution $x = 3\cos(t)$ and calculus, show that $\int_{0}^{3} \sqrt{9-x^2}\,dx = \frac{9\pi}{4}$.

3 marks

The diagram below, shows a traffic intersection, consisting of a major roadway running north-south and a minor roadway running east-west. There is an elliptical roundabout at the centre of the intersection, with x and y axes as shown.



c. Write down the equation of the elliptical roundabout.

1 mark

The edge of the roadway, can be represented by the curve $\frac{9}{x^2}+\frac{4}{y^2}=1$.

d. Show that the roadway edge can be represented by the parametric equations $x=3\sec(\theta)$ and $y=2\operatorname{cosec}(\theta)$ for $\theta>0$.

1 mark

e. Find the exact gradient of the roadway edge at the north-west point where $y = 4$.

3 marks

f. The slope of the roadway edge can be expressed in the form $b\cot^{n}(\theta)$.
Find the values of b and n.

2 marks

The council have decided to put new asphalt down in the intersection. The elliptical roundabout is grassed and will not need to be asphalted. The intersection is the area bounded by the roadway edge, the dotted lines $x = \pm 5$ and $y = \pm 4$ as shown on the diagram on page 22.

g. Use definite integrals to write an expression for the total area in square metres that needs to be asphalted.

2 marks

h. Hence, find the exact total area in square metres that needs to be asphalted.

1 mark
Total 15 marks

Question 4

Ashley has just purchased a car. The mass of Ashley and the car is 1800 kg.

a. Ashley is driving his car away from the intersection and the roundabout and heading east to watch a soccer game. While he is driving on a level section of roadway the engine of his car produces a propulsive force of $\frac{P}{v}$ newtons, where P is the power output in watts of the engine and v is its speed in m/s. The wind and frictional resistance forces are $0.5\sqrt{v}$ newtons.

i. Find the acceleration of the car in terms of v, when the power output is 200,000 watts.

1 mark

ii. **Hence,** write down a definite integral, which gives the distance that Ashley covers in attaining a speed of 10 m/s from rest.

1 mark

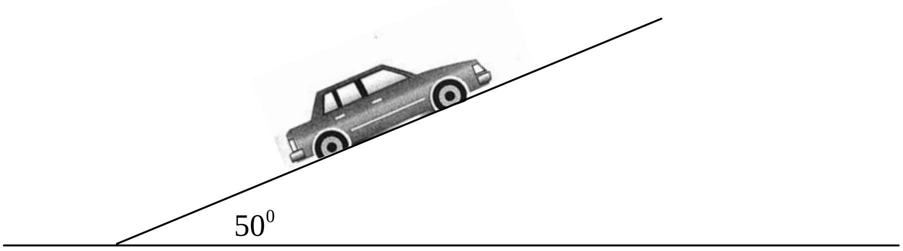
iii. Find this distance, giving your answer to the nearest metre.

1 mark

Unfortunately, when Ashley was driving on a roadway up an incline, a tyre burst and a tow-truck was called. The coefficient of friction between the car and the asphalt on the roadway is 0.2 and the roadway is inclined at an angle of 50^0 to the horizontal.

b. The tow-truck operator lowers a cable which is parallel to the roadway and is connected to the car.

i. On the diagram below mark in all the forces acting on the car.



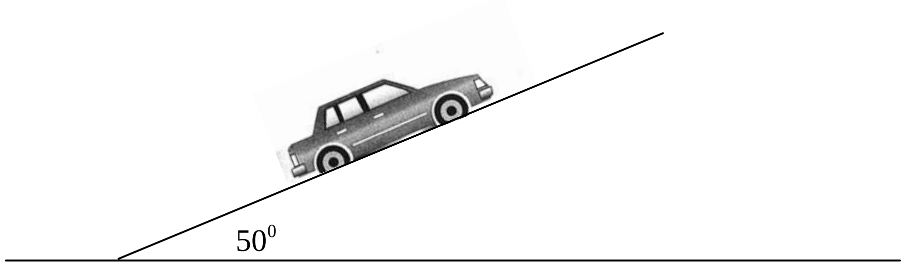
1 mark

ii. Find the greatest and least values of the tension in the cable consistent with the car being in equilibrium. Give your answers in newtons, correct to two decimal places.

4 marks

c. The tow-truck operator now adjusts the cable so that it is inclined at an angle of α to the roadway and increases the tension in the cable to 16,000 newtons. The car now accelerates up the incline at 0.1 m/s^2.

i. On the diagram below mark in all the forces acting on the car.



1 mark

ii. Find the value of α giving your answer to the nearest tenth of a degree.

4 marks
Total 13 marks

Question 5

A soccer ball is kicked off the ground, by a goal-keeper. Its position vector is given by $\underset{\sim}{r}(t) = 8t\,\underset{\sim}{i} + 26t\,\underset{\sim}{j} + \frac{5\sqrt{2}}{2}\sin\left(\frac{\pi t}{2}\right)\underset{\sim}{k}$, where $\underset{\sim}{i}$ and $\underset{\sim}{j}$ are unit vectors horizontally and vertically forward respectively, and $\underset{\sim}{k}$ is a unit vector in the vertical direction. Displacements are measured in metres and t is the time in seconds after the ball is kicked.

a. Find the initial speed at which the soccer ball is kicked. Give your answer correct to two decimal places.

2 marks

b. At what angle from the ground, correct to the nearest tenth of a degree, was the soccer ball kicked?

1 mark

c. Determine when the soccer ball reaches its maximum height above the ground, and find its position vector at this time.

1 mark

After the soccer ball has reached its maximum height and is on its downwards trajectory, a player jumps and heads the soccer ball when it is 2.5 metres above the ground.

d. What time has elapsed from the instant that the goal-keeper kicks the soccer ball until the player heads the ball?

1 mark

e. How far, measured along the ground, was the player from the goal-keeper? Give your answer correct to the nearest metre.

2 marks

f. If the soccer ball has a mass of 430 gm, find the magnitude of the momentum in kg m/s, correct to two decimal places, of the ball at the instant when it strikes the player's head.

2 marks
Total 9 marks

END OF EXAMINATION

EXTRA WORKING SPACE

END OF QUESTION AND ANSWER BOOKLET

SPECIALIST MATHEMATICS

Written examination 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of triangle:	$\frac{1}{2}bc\sin(A)$
sine rule:	$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$
cosine rule:	$c^2=a^2+b^2-2ab\cos(C)$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigonometric) functions

$\cos^2(x)+\sin^2(x)=1$

$1+\tan^2(x)=\sec^2(x)$ $\cot^2(x)+1=\operatorname{cosec}^2(x)$

$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$ $\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$

$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$ $\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$

$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$

$\sin(2x)=2\sin(x)\cos(x)$ $\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1,1]$	$[-1,1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0,\pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex Numbers)

$z = x + yi = r\left(\cos(\theta) + i\sin(\theta)\right) = r\,\text{cis}(\theta)$

$|z| = \sqrt{x^2 + y^2} = r$ $\qquad -\pi < \text{Arg}(z) \le \pi$

$z_1 z_2 = r_1 r_2 \,\text{cis}(\theta_1 + \theta_2)$ $\qquad \dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\,\text{cis}(\theta_1 - \theta_2)$

$z^n = r^n \text{cis}(n\theta)$ (de Moivre's theorem)

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$ $\qquad \underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \dfrac{d\underset{\sim}{r}}{dt} = \dfrac{dx}{dt}\underset{\sim}{i} + \dfrac{dy}{dt}\underset{\sim}{j} + \dfrac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

sliding friction: $F \le \mu N$

constant (uniform) acceleration:

$v = u + at$ $\qquad s = ut + \frac{1}{2}at^2$ $\qquad v^2 = u^2 + 2as$ $\qquad s = \frac{1}{2}(u+v)t$

acceleration: $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\tfrac{1}{2}v^2\right)$

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$	$\int x^n dx=\frac{1}{n+1}x^{n+1}+c\ ,\ n\neq -1$
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$	$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$
$\frac{d}{dx}\left(\log_e\left(x\right)\right)=\frac{1}{x}$	$\int \frac{1}{x}dx=\log_e\left(x\right)+c,\ \text{for}\ \ x>0$
$\frac{d}{dx}\left(\sin\left(ax\right)\right)=a\cos\left(ax\right)$	$\int \sin\left(ax\right)dx=-\frac{1}{a}\cos\left(ax\right)+c$
$\frac{d}{dx}\left(\cos\left(ax\right)\right)=-a\sin\left(ax\right)$	$\int \cos\left(ax\right)dx=\frac{1}{a}\sin\left(ax\right)+c$
$\frac{d}{dx}\left(\tan\left(ax\right)\right)=a\sec^2\left(ax\right)$	$\int \sec^2\left(ax\right)dx=\frac{1}{a}\tan\left(ax\right)+c$
$\frac{d}{dx}\left(\sin^{-1}\left(x\right)\right)=\frac{1}{\sqrt{1-x^2}}$	$\int \frac{dx}{\sqrt{a^2-x^2}}=\sin^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\cos^{-1}\left(x\right)\right)=\frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}}dx=\cos^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\tan^{-1}\left(x\right)\right)=\frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2}dx=\tan^{-1}\left(\frac{x}{a}\right)+c$

product rule: $\frac{d}{dx}\left(uv\right)=u\frac{dv}{dx}+v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

Euler's method: If $\frac{dy}{dx}=f(x),\ x_0=a$ and $y_0=b,$ then $x_{n+1}=x_n+h$ and $y_{n+1}=y_n+hf\left(x\right)$

END OF FORMULA SHEET

ANSWER SHEET

STUDENT NUMBER

									Letter
Figures									
Words									

SIGNATURE ____

SECTION 1

1	**A**	**B**	**C**	**D**	**E**
2	**A**	**B**	**C**	**D**	**E**
3	**A**	**B**	**C**	**D**	**E**
4	**A**	**B**	**C**	**D**	**E**
5	**A**	**B**	**C**	**D**	**E**
6	**A**	**B**	**C**	**D**	**E**
7	**A**	**B**	**C**	**D**	**E**
8	**A**	**B**	**C**	**D**	**E**
9	**A**	**B**	**C**	**D**	**E**
10	**A**	**B**	**C**	**D**	**E**
11	**A**	**B**	**C**	**D**	**E**
12	**A**	**B**	**C**	**D**	**E**
13	**A**	**B**	**C**	**D**	**E**
14	**A**	**B**	**C**	**D**	**E**
15	**A**	**B**	**C**	**D**	**E**
16	**A**	**B**	**C**	**D**	**E**
17	**A**	**B**	**C**	**D**	**E**
18	**A**	**B**	**C**	**D**	**E**
19	**A**	**B**	**C**	**D**	**E**
20	**A**	**B**	**C**	**D**	**E**
21	**A**	**B**	**C**	**D**	**E**
22	**A**	**B**	**C**	**D**	**E**