2008 Specialist Maths GA 3: Written examination 2

GENERAL COMMENTS

The number of students who sat the 2008 examination was 4884 compared to 4760 in 2007. Examination 2 comprised 22 multiple-choice questions (worth 22 marks) and five extended answer questions (worth 58 marks). The time allowed – two hours – seemed adequate, although this year a significant number of students did not answer some of the questions.

Two students scored full marks, compared to five in 2007. In Section 2 the average score for the five questions, expressed as a percentage of the marks available for each question, was 53 per cent, 55 per cent, 27 per cent, 46 per cent and 43 per cent respectively. This compares with 60 per cent, 73 per cent, 49 per cent, 42 per cent, and 38 per cent for the 2007 examination. This year students found the analysis questions of Section 2 more challenging, in particular Question 3, which required basic parametric and vector manipulations in a non-routine context. More detailed statistical information can be found on the VCAA website.

In 2008 there were five 'show that' questions in Section 2 – Questions 1di., 2a., 2b., 3ai. and 4di. It should be emphasised once more that for this type of question, students need to show all steps to demonstrate that they are capable of a proper derivation of the given result. Students should appreciate that a 'show that' format is used specifically to enable access to later parts of the question.

There were two 'hence' questions – Questions 1diii. and 3aii. It should be restated that in such questions students must use a previously established result to answer the question at hand, in order to gain full credit.

The examination revealed areas of strength and weakness in student performance. Areas of strength included the ability to:

- use technology to solve equations numerically – Question 1bii. and to a lesser extent, Question 4e.
- resolve forces in a mechanics situation – Question 2eii.
- apply the chain rule for derivatives – Question 4di.
- verify a solution of a differential equation – Question 4dii.
- plot points and regions in the complex plane – Questions 5b., 5d. and 5e.

Areas of weakness included the inability to:

- read questions carefully – in Question 2c. many students left out the equation of motion and simply wrote down the acceleration. In Question 2eiii. many students solved for θ, instead of for $\tan\theta$ to three decimal places
- completely change the variable in an integral and then evaluate it – Questions 1dii. and 1diii.
- understand the meaning of 'cartesian equation', and how to eliminate a parameter to get one – Question 3aii.
- accurately reproduce a sketch graph on paper from a screen display – Questions 1e. and 3b.
- find the area of a relatively straightforward composite shape (major segment of a circle) – Question 5f.

SPECIFIC INFORMATION

Section 1

The table below indicates the percentage of students who chose each option. The correct answer is indicated by shading.

Question	% A	% B	% C	% D	% E	% No Answer	Comments
1	4	1	17	75	3	0	Only option D provides the correct asymptotes.
2	9	8	26	14	42	1	$\left(x+\frac{a}{2}\right)^2 + y^2 = \frac{a^2}{4} - 1$, $\frac{a^2}{4} - 1 > 0$, $a^2 > 4$ so $a < -2$ or $a > 2$.
3	4	77	4	11	4	0	
4	17	14	14	28	26	1	The graph is asymptotic to $y = \pm 2x$, $\Rightarrow \lvert\text{gradient}\rvert > 2$
5	4	58	7	6	26	0	
6	3	28	56	4	9	0	The imaginary part is the (real) coefficient of i, when z is in rectangular form.
7	10	10	5	71	4	1	The equation becomes $(x+2)^2 + y^2 = 4$, thus the correct answer is option D
8	20	3	1	75	1	0	
9	36	40	8	8	9	0	Only option B gives the correct gradients on the y axis and the correct period of the gradient line segments.
10	7	13	65	9	6	0	
11	3	9	11	63	14	0	
12	45	6	7	12	29	0	$F(0) - F(-3)$ where the graph of F is the cubic.
13	4	10	8	5	72	1	
14	2	83	10	3	2	0	$m^2 + 4m - 12 = 0$, hence option B is the correct answer.
15	15	12	24	36	12	1	$\left\lvert \underset{\sim}{i} + 4\times\frac{1}{2}\left(\underset{\sim}{i} + \sqrt{3}\underset{\sim}{j}\right)\right\rvert = \left\lvert 3\underset{\sim}{i} + 2\sqrt{3}\underset{\sim}{j}\right\rvert$
16	5	7	69	6	13	0	
17	43	20	14	15	7	1	$\underset{\sim}{r} = \underset{\sim}{p} + \frac{3}{2}\left(\underset{\sim}{q} - \underset{\sim}{p}\right)$
18	8	4	21	9	58	1	Need equivalent of $\underset{\sim}{F}.\hat{\underset{\sim}{d}}$
19	6	5	73	7	9	1	
20	4	7	5	13	70	1	$v\frac{dv}{dx} = \sin^{-1}(x) \times \frac{1}{\sqrt{1-x^2}}$
21	60	24	5	3	7	1	
22	16	45	16	15	8	1	$\frac{dt}{dv} = \frac{1}{f(v)}$, $t_1 - t_0 = \int_{v_0}^{v_1} \frac{1}{f(v)}\,dv$

The mean score for the multiple-choice section was 13.01 out of 22 and the standard deviation was 4.52. This compares to 12.32 and 4.59 respectively for 2007. There were 7 questions (Questions 2, 4, 9, 12, 15, 17 and 22) which were answered correctly by less than 50 per cent of students. This compares with eight questions for the 2007 paper. The evidence suggests that students found the multiple-choice section slightly easier compared to last year.

Section 2

Question 1a.

Marks	**0**	**1**	**2**	**Average**
%	12	27	61	**1.5**

$(1, 1.5)$. $f''(1) = -\frac{9}{8}$, which is negative, thus $(1, 1.5)$ is the maximum turning point.

This question was quite well done. The most frequent errors were the omission of the *y* coordinate, and the failure to verify the maximum turning point using the second derivative or other suitable test.

Question 1bi.

Marks	**0**	**1**	**Average**
%	63	37	**0.4**

$9x^4 - 26x^2 + 1 = 0$

This question was quite well done by those students who attempted this part. However, many students lacked understanding of the term 'polynomial' and simply equated $f''(x)$ to zero.

Question 1bii.

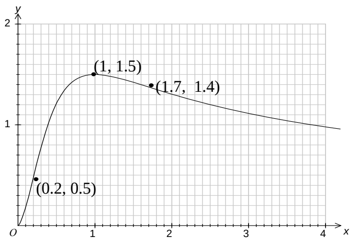
Marks	**0**	**1**	**2**	**Average**
%	42	12	46	**1.1**

$(0.2, 0.5)$, $(1.7, 1.4)$

Most students used the 'equation solver' feature of their calculator to find the points of inflection, although in some instances extra points were given which were outside the domain of *f*. A frequent error was to give only the *x* coordinates of the two points of inflection.

Question 1c.

Marks	**0**	**1**	**2**	**Average**
%	22	35	43	**1.3**



Most students could obtain the form of the required curve from their calculators, however drawing it accurately, passing through the correct points, proved to be a challenge for many. A number of students tried to draw their graphs with stationary points of inflection at (0.2, 0.5) and (1.7, 1.4).

Question 1di.

Marks	**0**	**1**	**Average**
%	26	74	**0.8**

$$V = \pi \int_0^{\frac{1}{\sqrt{3}}} \left(\frac{6x\sqrt{x}}{3x^2+1} \right)^2 dx \; = \; \pi \int_0^{\frac{1}{\sqrt{3}}} \left(\frac{36x^3}{\left(3x^2+1\right)^2} \right) dx \; = 2\pi \int_0^{\frac{1}{\sqrt{3}}} \left(\frac{18x^3}{\left(3x^2+1\right)^2} \right) dx$$

This question was well done. As it was a 'show that' question, the most common error was the omission of a suitable intermediate expression.

Question 1dii.

Marks	0	1	2	Average
%	49	18	33	**0.9**

$$V = 2\pi \int_1^2 \left(\frac{1}{u} - \frac{1}{u^2} \right) du$$

This question was not very well done. A significant number of students tried to force this to be a partial fractions problem. Only a small number of students attempted to change both the integrand and the terminals.

Question 1diii.

Marks	0	1	2	Average
%	56	18	26	**0.7**

$$V = 2\pi \log_e\left(2\right) - \pi$$

For students who managed the previous part correctly, this final part was done quite well. A number of students left out π and a few had $\log_e\left(u^2\right)$ for the antiderivative of their second term.

Question 2a.

Marks	0	1	Average
%	9	91	**1**

$$390 - 30 = 80a\text{, } a = \frac{360}{80} = 4.5 \text{ m/s}^2$$

This question was very well done with nearly all students able to apply Newton's second law to show the given result.

Question 2b.

Marks	0	1	Average
%	14	86	**0.9**

$$v^2 = 2 \times 4.5 \times 16,\ v^2 = 144, \text{ so } v = 12 \text{ m/s}$$

Question 2c.

Marks	0	1	2	Average
%	26	17	57	**1.4**

$$360 - 6v = 80a\text{ , } a = \frac{180 - 3v}{40}$$

While this question was generally well done, a large number of students omitted the equation of motion, despite the explicit instruction to 'write down', and went directly to the expression for a.

Question 2d.

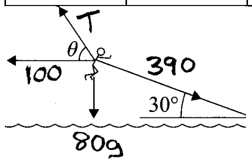
Marks	0	1	2	Average
%	59	12	30	**0.8**

$$t=\int_{12}^{18}\left(\frac{40}{(180-3v)}\right)dv\,,\ t=1.8\text{ s}$$

Most students set up the definite integral and then found the time numerically, while fewer students integrated first, found a constant of integration and then substituted to find the time.

Question 2ei.

Marks	0	1	Average
%	19	81	0.9



This question was very well done. A few students mixed up T and 390 and a small number introduced an extra force N acting on the skier.

Question 2eii.

Marks	0	1	2	Average
%	45	15	40	1

$T\sin(\theta)=80g+390\sin\left(30^\circ\right)$, $T\cos(\theta)+100=390\cos\left(30^\circ\right)$

Most errors in this question involved the omission of forces, the mixing up of signs on forces and, to a lesser extent, poor trigonometry.

Question 2eiii.

Marks	0	1	2	Average
%	51	21	28	0.8

$\tan\theta=4.118$

This question was generally well done by students who managed the previous part. A significant number of students misread the question and found θ instead of spelling out the value of $\tan\theta$ to three decimal places.

Question 2eiv.

Marks	0	1	Average
%	73	27	0.3

$T=1007$ N

Only a small number of students managed to follow Question 2 through to this final part.

Question 3ai.

Marks	0	1	Average
%	45	55	0.6

$$y=\frac{1}{2}\times 2\sin\left(\frac{t}{3}\right)\cos\left(\frac{t}{3}\right)\Rightarrow y^2=\sin^2\left(\frac{t}{3}\right)\cos^2\left(\frac{t}{3}\right)$$

Most students realised that the double angle formula for $\sin(2\theta)$ was to be used.

Question 3aii.

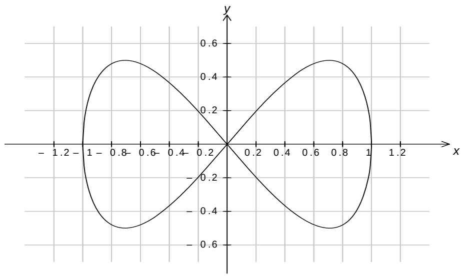
Marks	0	1	Average
%	68	32	**0.4**

$$y^2=x^2\left(1-x^2\right)$$

A significant number of students gave only the positive square root of the right side if they chose to make y the subject. Others gave non-cartesian forms of the relation.

Question 3b.

Marks	0	1	2	Average
%	65	16	19	**0.6**



The most common errors involved partial and inaccurate sketch graphs of the relation. Occasionally quartic polynomial looking graphs were seen.

Question 3c.

Marks	0	1	2	Average
%	76	14	10	**0.4**

6π

Few students considered the y component of the motion.

Question 3d.

Marks	0	1	2	Average
%	60	9	31	**0.8**

speed = $\dfrac{\sqrt{2}}{3}$ m/s

This question was reasonably well done by students who attempted it. Common errors centred on using the wrong time and not finding the magnitude of the velocity vector.

Question 3ei.

Marks	0	1	Average
%	81	19	**0.2**

$$\frac{1}{3}\int_0^{6\pi}\left(\sqrt{\cos^2\left(\frac{t}{3}\right)+\cos^2\left(\frac{2t}{3}\right)}\right)dt$$

This question was reasonably well done by the small number of students who attempted the last parts of Question 3. For those who tried it, most errors related to terminals.

Question 3eii.

Marks	0	1	Average
%	92	8	**0.1**

6.1 m

The small number of students who managed the correct integral set up of the previous part generally got the correct answer.

Question 4a.

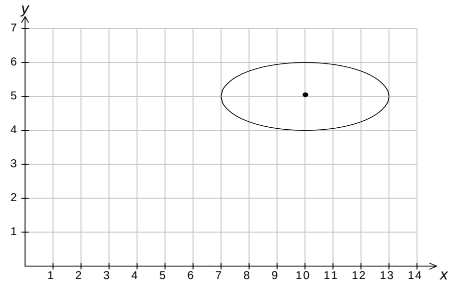
Marks	0	1	2	Average
%	30	6	63	**1.4**

$$\frac{(x-10)^2}{3^2}+(y-5)^2=1$$

A number of students found various non-cartesian forms involving inverse trigonometric functions.

Question 4b.

Marks	0	1	2	Average
%	35	4	61	**1.3**



This question was quite well done, with the most common error being to show only the top half of the ellipse.

Question 4ci.

Marks	0	1	Average
%	44	56	**0.6**

6 months

The most common error related to confusion of units.

Question 4cii.

Marks	0	1	Average
%	61	39	**0.4**

500 foxes

The most common error was confusion of units.

Question 4di.

Marks	0	1	Average
%	23	77	**0.8**

$$\frac{dy}{dt}\times\frac{dt}{dx}=\frac{-0.2y+0.02xy}{0.5x-0.1xy}\times\frac{50}{50}\quad=\frac{xy-10y}{25x-5xy}$$

Most students could successfully apply the chain rule to show the given result.

Question 4dii.

Marks	0	1	2	Average
%	56	5	39	**0.9**

$$25\times\frac{1}{y}\frac{dy}{dx}-5\frac{dy}{dx}-1+\frac{10}{x}=0\,,\ \frac{dy}{dx}=\frac{\left(1-\frac{10}{x}\right)}{\left(\frac{25}{y}-5\right)}\times\frac{xy}{xy}=\frac{xy-10y}{25x-5xy}$$

The implicit differentiation was handled fairly well, and most students continued on to successfully show the given result.

Question 4e.

Marks	0	1	2	3	Average
%	81	7	3	9	**0.4**

6590 and 14 430 rabbits

Only a small number of students deduced that the minimum and maximum values of x occurred where y = 5. Few students could use this value of y to solve numerically the relation in Question 4dii to find the minimum and maximum numbers of rabbits. Rounding to the nearest hundred rather than ten was more than just an occasional error.

Question 5a.

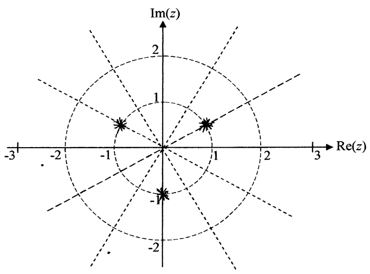
Marks	0	1	Average
%	39	61	**0.7**

$$\frac{\sqrt{3}}{2}+\frac{1}{2}i=\operatorname{cis}\left(\frac{\pi}{6}\right),\ \left(\operatorname{cis}\left(\frac{\pi}{6}\right)\right)^3=\operatorname{cis}\left(\frac{3\pi}{6}\right)=i$$

The simplest way to deal with this question was to use polar coordinates. Quite a number of students expanded out in cartesian form, but often got lost in the algebra.

Question 5b.

Marks	0	1	2	Average
%	28	25	47	**1.3**



The majority of students were able to plot at least one of the three roots – usually the one given. More than three roots were sometimes seen. A common error was the reflection of the roots across the x axis.

Question 5c.

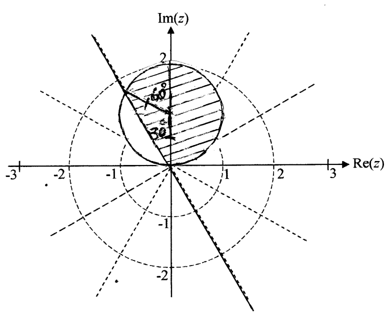
Marks	0	1	2	3	Average
%	45	9	13	33	**1.4**

$(0,0)$, $\left(-\frac{\sqrt{3}}{2},\frac{3}{2}\right)$ or equivalent complex numbers.

An algebraic attempt at the solution was often seen in this question, but this was followed through successfully in only a small number of cases. A fairly common error was interchanging *x* and *y* coordinates in the second point of intersection.

Question 5d.

Marks	0	1	2	Average
%	41	21	38	1



Circle and straight line

A variety of incorrect circles and straight lines was generally seen.

Question 5e.

Marks	0	1	2	Average
%	38	25	37	1.1

Shaded region on the diagram above in Question 5d.

The majority of students who attempted this question knew to shade a region somewhere inside their circle. A pleasing number of students managed to shade the region $0 \le \text{Arg}(z) \le \frac{2\pi}{3}$ correctly.

Question 5f.

Marks	0	1	2	Average
%	90	4	6	0.2

2.53

Only a small number of students managed to successfully answer this question. A variety of approaches was seen, ranging from integration (more often using incorrect boundaries and terminals) to more straightforward geometric methods. The most capable students simply found the area of a major segment of the circle. Creative approaches were seen where students relocated the region in order to apply geometric methods more easily.