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Students Name:………………………………………

SPECIALIST MATHEMATICS

TRIAL EXAMINATION 1

2009

Reading Time: 15 minutes
Writing time: 1 hour

Instructions to students

This exam consists of 10 questions.
All questions should be answered.
There is a total of 40 marks available.
The marks allocated to each of the ten questions are indicated throughout.
Students may not bring any notes or calculators into the exam.
Where more than one mark is allocated to a question, appropriate working must be shown.
Where an exact answer is required to a question, a decimal approximation will not be accepted.
Unless otherwise indicated, diagrams in this exam are not drawn to scale.
The acceleration due to gravity should be taken to have magnitude g m/s^2 where $g = 9.8$
Formula sheets can be found on pages 13-15 of this exam.

Question 1

Consider the function *f* with rule $f(x) = \cos^{-1}(3x - 2)$.

a. Write down the maximal domain of *f*.

1 mark

b. Find $f'(x)$.

1 mark

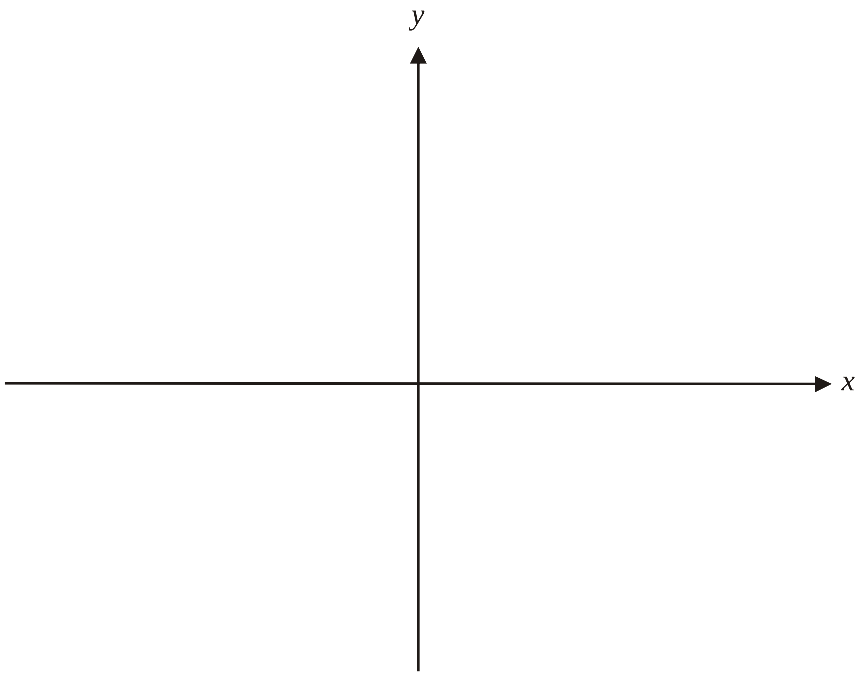
Question 2

Find the gradient of the relation $6x^2 - 3y^3 + 2x^2y^2 = 5$ at the point in the second quadrant where $y = 1$.

3 marks

Question 3

Sketch the graph of the relation $\frac{(x-1)^2}{4}-\frac{y^2}{25}=1$ on the axes below. State the equations of all asymptotes and label clearly any intercepts.



4 marks

Question 4

In the right angled triangle *ABC*, the hypotenuse is $\overrightarrow{AC} = -4\underset{\sim}{i} + \underset{\sim}{j}$.

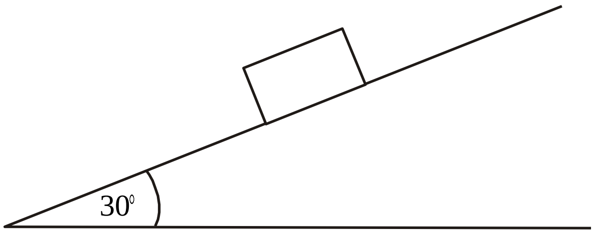
If $\overrightarrow{BC}$ is parallel to the vector $-3\underset{\sim}{i} + 2\underset{\sim}{j}$, find $\overrightarrow{AB}$.

3 marks

Question 5

A container of mass 20kg rests on a rough surface inclined at an angle of 30° to the horizontal.

a. Clearly label the 3 forces, including the normal force *N* and the friction force *F* acting on the container, on the diagram below.



1 mark

A worker applies a pushing force of 200 newtons up the slope to the container which causes it to accelerate up the slope at $0\cdot 1\text{m/s}^2$.

b. What is the coefficient of friction between the container and the surface?

2 marks

c. What minimum force *P* must the worker exceed in order to cause the container to move up the slope?

2 marks

Question 6

a. Find an antiderivative of $\sin^2\left(\frac{3x}{2}\right)$.

1 mark

b. Evaluate $\int_0^1 \frac{5x}{\sqrt{1+x^2}}\,dx$.

3 marks

Question 7

Find the volume generated when the region enclosed by the curve with equation $y = \log_e(3x - 2)$, the *y*-axis and the lines $y = 0$ and $y = 1$ is rotated about the *y*-axis to form a solid of revolution.

3 marks

Question 8

Let $u = \sqrt{2}\text{cis}\left(\frac{\pi}{4}\right)$ and $w = 2\text{cis}\left(\frac{\pi}{6}\right)$.

a. Express u in Cartesian form.

1 mark

b. Express $\frac{u}{w}$ in polar form.

1 mark

c. Hence evaluate $\sin\left(\frac{\pi}{12}\right)$.

2 marks

d. Given that u is a root of the equation $z^3 + z^2 - 4z + 6 = 0,\ z \in C$, find the other roots of the equation.

2 marks

Question 9

A particle moves in a straight line from rest. At time *t* seconds its displacement is *x* metres from a fixed point, its velocity is *v* m/s and its acceleration is *a* m/s ^{2} where

$$a = v^2 + 3v + 2, \ v \geq 0.$$

a. Find the acceleration when $t = 0$.

1 mark

b. Find an expression for *v* in terms of *t*.

4 marks

Question 10

a. Show that $\cos\left(\frac{\pi}{8}\right)=\frac{\sqrt{2+\sqrt{2}}}{2}$.

3 marks

b. Hence find the area enclosed by the graph of $y=\sin^{-1}x$, the y-axis and the line $y=\frac{\pi}{8}$.

2 marks

Specialist Mathematics Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc\sin A$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$ $\qquad$ $\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ $\qquad$ $\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ $\qquad$ $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$ $\qquad$ $\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$ $\qquad$ $\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\operatorname{cis}\theta$

$|z| = \sqrt{x^2 + y^2} = r$ $\qquad$ $-\pi < \operatorname{Arg} z \le \pi$

$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$ $\qquad$ $\frac{z_1}{z_2} = \frac{r_1}{r_2}\operatorname{cis}(\theta_1 - \theta_2)$

$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$$

$$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$$

$$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$$

$$\int \frac{1}{x}\,dx = \log_e|x| + c$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$$

$$\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$$

$$\int \frac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$$

$$\int \frac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Euler's method: If $\frac{dy}{dx} = f(x),\ x_0 = a$ and $y_0 = b$,
then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at \qquad s = ut + \frac{1}{2}at^2 \qquad v^2 = u^2 + 2as \qquad s = \frac{1}{2}(u+v)t$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$ $\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \le \mu N$