Specialist Mathematics

2010

Trial Examination 1

Instructions

Answer **all** questions. Do **not** use calculators.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1

Find all solutions to the equation $4z^3 - i2\sqrt{3}z^2 - 3z = 0$, $z \in C$. 　　　　　　　　　3 marks

Question 2

Find $\{z : |z + 4| + |z - 3\sqrt{3}| = 11\} \cap \{z : \text{Re}(z) = 0\}$, $z \in C$. 　　　　　　　　　3 marks

Question 3

The position of a particle in circular motion is given by $\tilde{r} = -3\tilde{i} + 3\cos\dfrac{t}{10}\tilde{j} - 3\sin\dfrac{t}{10}\tilde{k}$, where time t is measured in seconds and distance is in metres.

a. What is the exact time taken to complete one turn? 　　　　　　　　　1 mark

b. Find the exact value of the particle's speed. 　　　　　　　　　2 marks
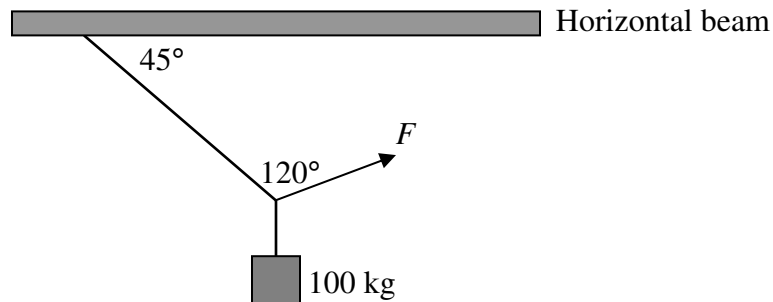
Question 4

A particle travels in a straight line with a constant acceleration of $^-3\,\text{ms}^{-2}$. It starts at $x=^-2\,\text{m}$ with a velocity of $^+8.5\,\text{ms}^{-1}$. Find the time when the particle passes $x=^+3\,\text{m}$ in the direction opposite to its initial velocity.
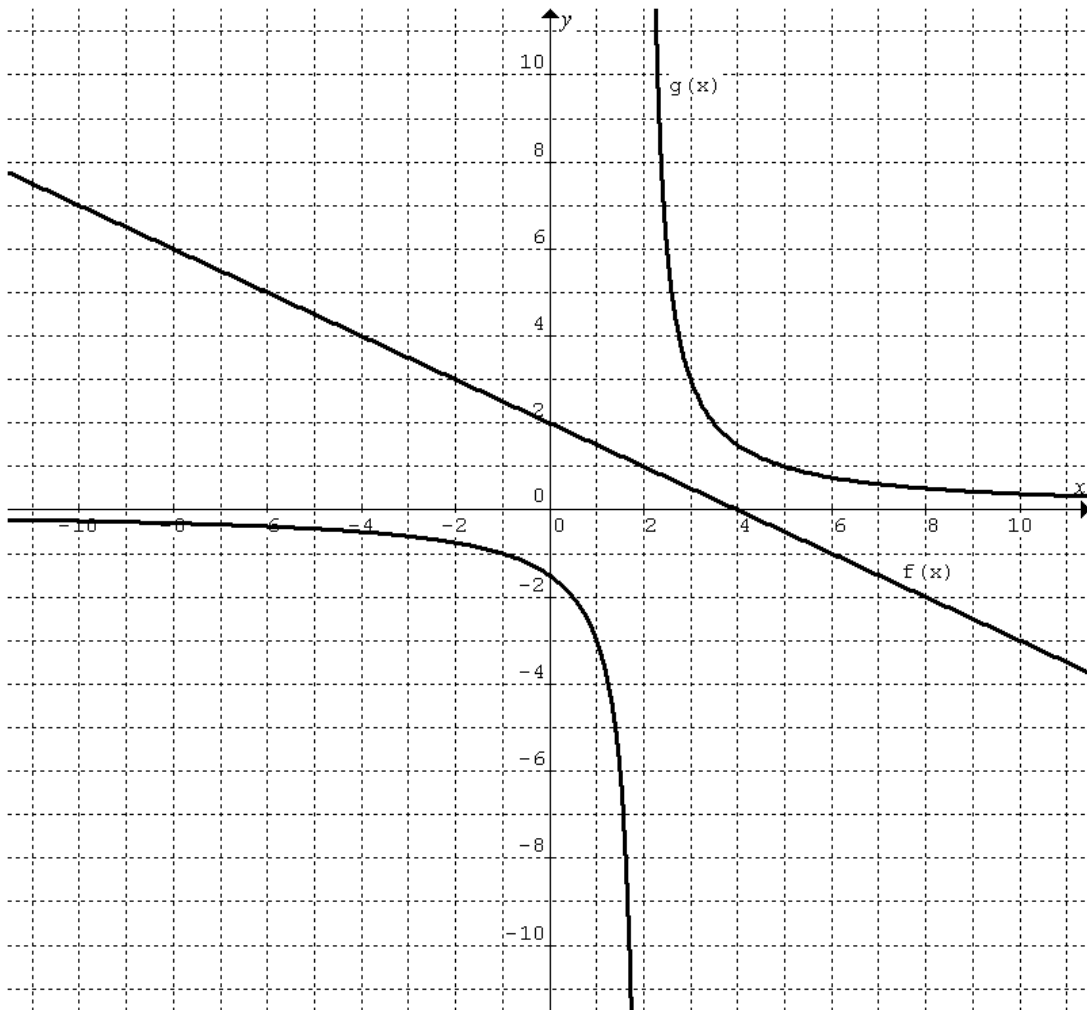
3 marks

Question 5

Find the magnitude of force F (newtons) required to keep the 100-kg mass in equilibrium.   3 marks

Question 6

The graphs of $f(x)$ and $g(x)$ are shown below.

a. On the same set of axes accurately sketch the graph of $h(x) = f(x) - g(x)$.   3 marks

b. Determine the equation of each asymptote of $h(x)$.   2 marks

Question 7

a. Prove that $\dfrac{1}{1-\sin x} = \sec x(\sec x + \tan x)$. 2 marks

b. *Hence* evaluate the exact value of $\displaystyle\int_0^{\frac{\pi}{3}} \dfrac{1}{1-\sin x}\,dx$. 3 marks
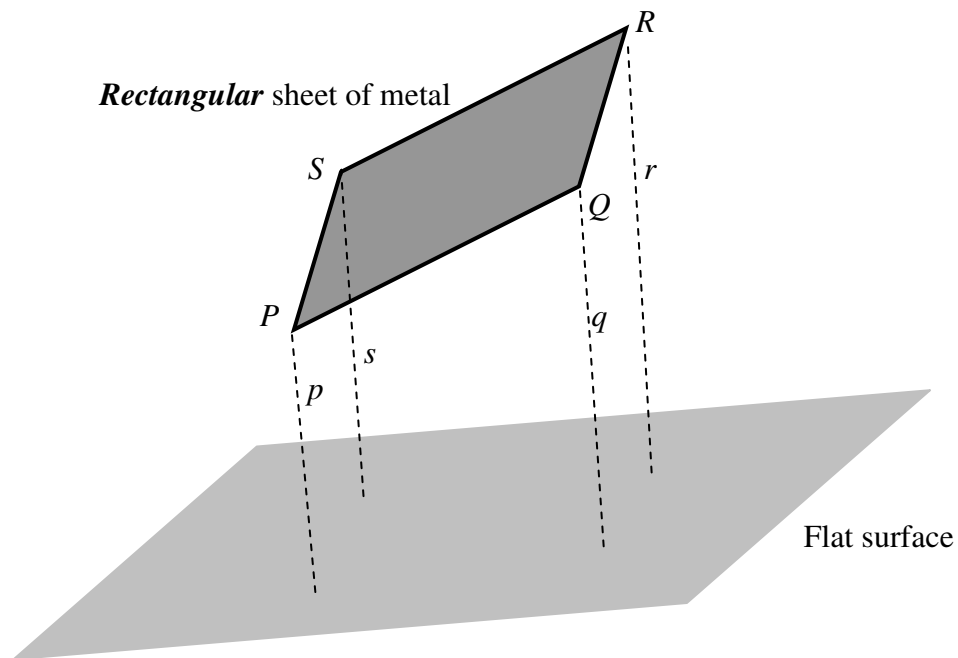
Question 8

a. Find the exact area bounded by the graph of $y = \sin^{-1} x$ and the y-axis. 3 marks

b. Hence or otherwise, find the exact area bounded by the graph of $y = 2\sin^{-1}(2x) + \pi$ and the y-axis.

2 marks

Question 9

PQRS is a rigid rectangular sheet of metal of negligible thickness.

p, *q*, *r* and *s* are respectively the *perpendicular* distances of *P*, *Q*, *R* and *S* from a flat surface.

a. Given that the diagonals of the rectangular metal sheet bisect each other, show that $\overrightarrow{OP} + \overrightarrow{OR} = \overrightarrow{OQ} + \overrightarrow{OS}$, where *O* is a point on the flat surface.
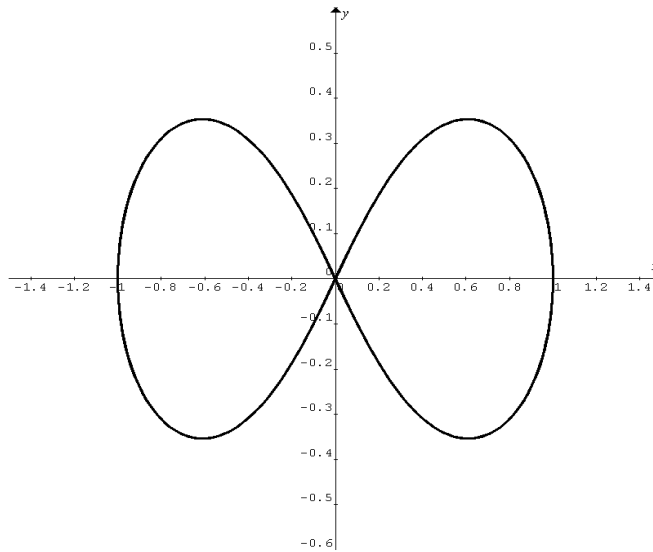
2 marks

b. *Hence* show that $p + r = q + s$.

3 marks

Question 10

The graph of $\left(x^2 + y^2\right)^2 = x^2 - y^2$ is shown below.

a. Find $\dfrac{dy}{dx}$ in terms of x and y.

3 marks

b. Find the exact coordinates of the points on the graph at which the tangents are horizontal.

2 marks

End of Exam 1