Year 2010

VCE

Specialist Mathematics Solutions Trial Examination 2

KILBAHA MULTIMEDIA PUBLISHING	TEL: (03) 9817 5374
PO BOX 2227	FAX: (03) 9817 4334
KEW VIC 3101	kilbaha@gmail.com
AUSTRALIA	http://kilbaha.com.au

SECTION 1

ANSWERS

1	A	B	C	**D**	E
2	A	B	C	**D**	E
3	**A**	B	C	D	E
4	**A**	B	C	D	E
5	A	B	C	**D**	E
6	A	**B**	C	D	E
7	A	B	C	D	**E**
8	A	B	**C**	D	E
9	**A**	B	C	D	E
10	A	B	C	**D**	E
11	A	**B**	C	D	E
12	A	B	**C**	D	E
13	A	B	**C**	D	E
14	A	**B**	C	D	E
15	A	B	C	**D**	E
16	**A**	B	C	D	E
17	A	**B**	C	D	E
18	A	B	C	D	**E**
19	A	B	C	D	**E**
20	**A**	B	C	D	E
21	A	B	C	D	**E**
22	A	B	**C**	D	E

SECTION 1

Question 1 **Answer D**

The graph of $y=\dfrac{1}{3a^2+2ax-x^2}$

$y=\dfrac{-1}{x^2-2ax-3a^2}=\dfrac{-1}{(x-3a)(x+a)}$

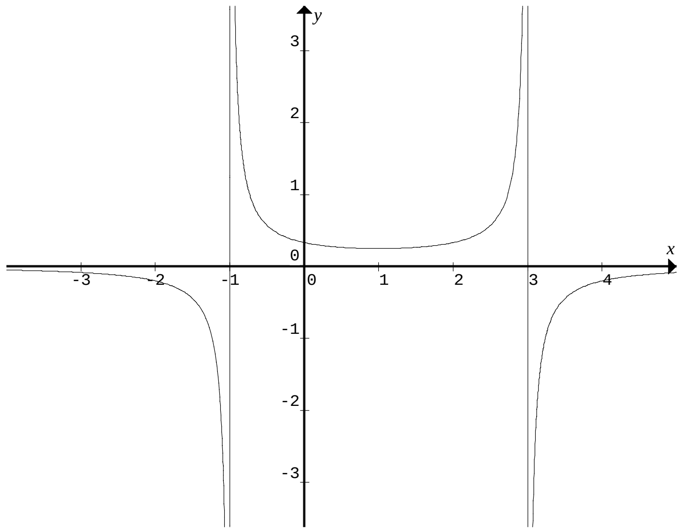
so it has vertical asymptotes at

$x=3a$ and $x=-a$

$y=\dfrac{-1}{x^2-2ax-3a^2}=\dfrac{-1}{x^2-2ax+a^2-4a^2}$

$y=\dfrac{-1}{(x-a)^2-4a^2}$ when $x=a$ $y=\dfrac{1}{4a^2}$

$\left(a,\dfrac{1}{4a^2}\right)$ is a minimum, graph with $a=1$



Question 2 **Answer D**

The hyperbola has its centre at $(-3,2)$ and the distance from the centre to the point $(-3,0)$ is 2, its equation is of the form $\dfrac{(y-2)^2}{4}-\dfrac{(x+3)^2}{a^2}=1$. The asymptotes are $\dfrac{y-2}{2}=\pm\dfrac{x+3}{a}$, one asymptote passes through the origin, $(0,0)$

$x=0 \quad y=0 \quad \Rightarrow -1=\dfrac{-3}{a} \quad \Rightarrow \quad a=3$, so the equation is $\dfrac{(y-2)^2}{4}-\dfrac{(x+3)^2}{9}=1$

Question 3 **Answer A**

$x^2-2ax+2y^2+8ay=9-10a^2$ completing the square

$x^2-2ax+a^2+2\left(y^2+4ay+4a^2\right)=9-10a^2+a^2+8a^2 \quad \Rightarrow \quad (x-a)^2+2(y+2a)^2=9-a^2$

this is an ellipse with centre $(a,-2a)$ provided that $9-a^2>0 \quad \Rightarrow \quad a^2<9$ or $|a|<3$

Question 4 **Answer A**

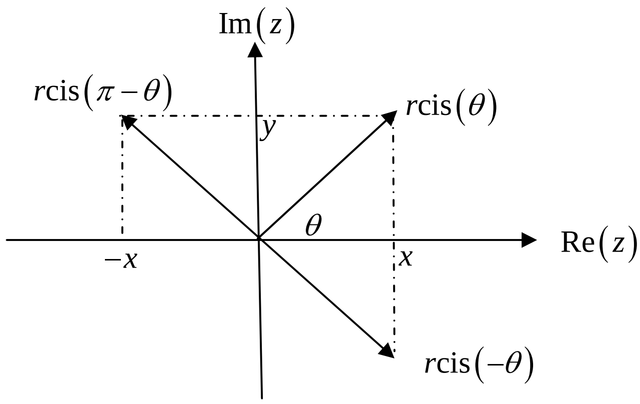
The domain and range of $y=\cos^{-1}(x)$ are $[-1,1]$ and $[0,\pi]$ respectively.

The domain of $f(x)=\dfrac{b}{\pi}\cos^{-1}\left(\dfrac{x}{b}-1\right)-b$ is $\left|\dfrac{x}{b}-1\right|\le 1 \quad \Rightarrow \quad -1\le\dfrac{x}{b}-1\le 1$

$0\le\dfrac{x}{b}\le 2 \quad \Rightarrow \quad x\in[0,2b]$ and the range is $\dfrac{b}{\pi}\times[0,\pi]-b=[-b,0]$,

so the domain and range are respectively $[0,2b]$ and $[-b,0]$

Question 5 **Answer D**



$x+yi=r\,\mathrm{cis}(\theta)=r\left(\cos(\theta)+i\sin(\theta)\right)$

$x+yi=r\cos(\theta)+ir\sin(\theta)$

$x=r\cos(\theta)$ and $y=r\sin(\theta)$, now

$-x=-r\cos(\theta)=r\cos(\pi-\theta)$ and

$y=r\sin(\theta)=r\sin(\pi-\theta)$

so $-x+yi=r\mathrm{cis}(\pi-\theta)$

Question 6 **Answer B**

$\left(\overline{z}+ai\right)\left(z-ai\right)=a^2$, let $z=x+yi$ and $\overline{z}=x-yi$

$\left(x+(a-y)i\right)\left(x+(y-a)i\right)=a^2$

$x^2+x(y-a)i+x(a-y)i+i^2(a-y)(y-a)=a^2$

$x^2+(y-a)^2=a^2$

this is a circle of radius a, with centre at $(0,a)$

Question 7 **Answer E**

$\underset{\sim}{a}=(m-2)\underset{\sim}{i}-(m+1)\underset{\sim}{j}+(m+1)\underset{\sim}{k}$

$|\underset{\sim}{a}|=\sqrt{(m-2)^2+(m+1)^2+(m+1)^2}$

$|\underset{\sim}{a}|=\sqrt{m^2-4m+4+m^2+2m+1+m^2+2m+1}$

$|\underset{\sim}{a}|=\sqrt{3m^2+6}=\sqrt{3\left(m^2+2\right)}$

Question 8 **Answer C**

$\underset{\sim}{a}=\sqrt{m}\,\underset{\sim}{i}+n\underset{\sim}{j}-n\underset{\sim}{k}$ and $\underset{\sim}{b}=-2\sqrt{m}\,\underset{\sim}{i}+4\underset{\sim}{j}-\sqrt{m}\underset{\sim}{k}$, if $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are parallel, then

$\underset{\sim}{b}=\lambda\underset{\sim}{a}$ and $\lambda=-2$, $\underset{\sim}{b}=-2\underset{\sim}{a}$, from the $\underset{\sim}{j}$ and $\underset{\sim}{k}$ components

$4=-2n$ and $-\sqrt{m}=2n \Rightarrow$ $n=-2$ and $m=16$

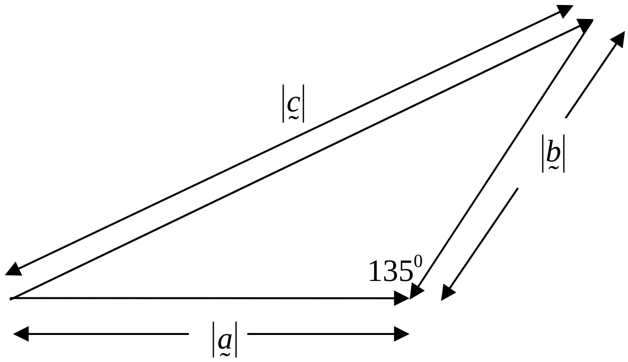
Question 9 **Answer A**

Using the Cosine Rule

$|\underset{\sim}{c}|^2=|\underset{\sim}{a}|^2+|\underset{\sim}{b}|^2-2|\underset{\sim}{a}||\underset{\sim}{b}|\cos\left(135^0\right)$

$|\underset{\sim}{c}|^2=|\underset{\sim}{a}|^2+|\underset{\sim}{b}|^2-2|\underset{\sim}{a}||\underset{\sim}{b}|\times\frac{-\sqrt{2}}{2}$

$|\underset{\sim}{c}|^2=|\underset{\sim}{a}|^2+|\underset{\sim}{b}|^2+\sqrt{2}|\underset{\sim}{a}||\underset{\sim}{b}|$



Question 10 **Answer D**

Since b, c, α and β are all real, the roots of

z^2+bz+c occur in conjugate pairs and are $u=-\alpha+\beta i$ and $\bar{u}=-\alpha-\beta i$

now $u+\bar{u}=-2\alpha$ and $u\bar{u}=\alpha^2-\beta^2 i^2=\alpha^2+\beta^2$, so that

$z^2+bz+c=z^2+2\alpha z+\alpha^2+\beta^2 \quad \Rightarrow \quad 2\alpha=b$ and $\alpha^2+\beta^2=c$

Let $v=2\alpha-2\beta i$ and $\bar{v}=2\alpha+2\beta i$, now

$v+\bar{v}=4\alpha$ and $v\bar{v}=4\alpha^2-4\beta^2 i^2=4\left(\alpha^2+\beta^2\right)$, so the quadratic is

$z^2-4\alpha z+4\left(\alpha^2+\beta^2\right)=z^2-2bz+4c$

Question 11 **Answer B**

$\underset{\sim}{s}=2\underset{\sim}{i}-3\underset{\sim}{j}+\underset{\sim}{k} \quad |\underset{\sim}{s}|=\sqrt{4+9+1}=\sqrt{14} \quad \Rightarrow \quad \hat{\underset{\sim}{s}}=\dfrac{1}{\sqrt{14}}\left(2\underset{\sim}{i}-3\underset{\sim}{j}+\underset{\sim}{k}\right)$

The vector resolute of $\underset{\sim}{r}$ in the direction of $\underset{\sim}{s}$ is equal to

$$\left(\underset{\sim}{r}.\hat{\underset{\sim}{s}}\right)\hat{\underset{\sim}{s}}=\left(\underset{\sim}{r}.\hat{\underset{\sim}{s}}\right)\frac{1}{\sqrt{14}}\left(2\underset{\sim}{i}-3\underset{\sim}{j}+\underset{\sim}{k}\right)=3\left(-2\underset{\sim}{i}+3\underset{\sim}{j}-\underset{\sim}{k}\right)=-3\left(2\underset{\sim}{i}-3\underset{\sim}{j}+\underset{\sim}{k}\right)$$

$$\Rightarrow \underset{\sim}{r}.\hat{\underset{\sim}{s}}=-3\sqrt{14}=\frac{\underset{\sim}{r}.\underset{\sim}{s}}{|\underset{\sim}{s}|}=\frac{\underset{\sim}{r}.\underset{\sim}{s}}{\sqrt{14}} \quad \Rightarrow \underset{\sim}{r}.\underset{\sim}{s}=-3\text{x}14$$

The scalar resolute of $\underset{\sim}{s}$ in the direction of $\underset{\sim}{r}$ is equal to $\underset{\sim}{s}.\hat{\underset{\sim}{r}}=\dfrac{\underset{\sim}{s}.\underset{\sim}{r}}{|\underset{\sim}{r}|}=\dfrac{-3\text{x}14}{6}=-7$

Question 12 **Answer C**

$$\ddot{\underset{\sim}{r}}(t)=8\sin^2(t)\underset{\sim}{i}+8\cos^2(t)\underset{\sim}{j}$$

$$\dot{\underset{\sim}{r}}(t)=\int 8\sin^2(t)\,dt\,\underset{\sim}{i}+\int 8\cos^2(t)\,dt\,\underset{\sim}{j}$$

$$\dot{\underset{\sim}{r}}(t)=\int 4\left(1-\cos(2t)\right)dt\,\underset{\sim}{i}+\int 4\left(1+\cos(2t)\right)dt\,\underset{\sim}{j}$$

$$\dot{\underset{\sim}{r}}(t)=2\left(2t-\sin(2t)\right)\underset{\sim}{i}+2\left(2t+\sin(2t)\right)\underset{\sim}{j}+\underset{\sim}{C}_1 \quad \text{now} \quad \dot{\underset{\sim}{r}}(0)=\underset{\sim}{0} \quad \Rightarrow \quad \underset{\sim}{C}_1=\underset{\sim}{0}$$

$$\underset{\sim}{r}(t)=\int\left(4t-2\sin(2t)\right)dt\,\underset{\sim}{i}+\int\left(4t+2\sin(2t)\right)dt\,\underset{\sim}{j}$$

$$\underset{\sim}{r}(t)=\left(2t^2+\cos(2t)\right)\underset{\sim}{i}+\left(2t^2-\cos(2t)\right)\underset{\sim}{j}+\underset{\sim}{C}_2$$

now $\underset{\sim}{r}(0)=\underset{\sim}{0} \quad \Rightarrow \underset{\sim}{i}-\underset{\sim}{j}+\underset{\sim}{C}_2=\underset{\sim}{0} \quad \Rightarrow \quad \underset{\sim}{C}_2=-\underset{\sim}{i}+\underset{\sim}{j}$

$$\underset{\sim}{r}(t)=\left(\cos(2t)+2t^2-1\right)\underset{\sim}{i}+\left(2t^2-\cos(2t)+1\right)\underset{\sim}{j}$$

Question 13 **Answer C**

$m = 5\ kg \quad \underset{\sim}{F}_1 = 10\underset{\sim}{i} \quad \left|\underset{\sim}{F}_1\right| = 10$

and $\underset{\sim}{F}_2 = -5\underset{\sim}{i} + 5\underset{\sim}{j} \quad \left|\underset{\sim}{F}_2\right| = 5\sqrt{2}$

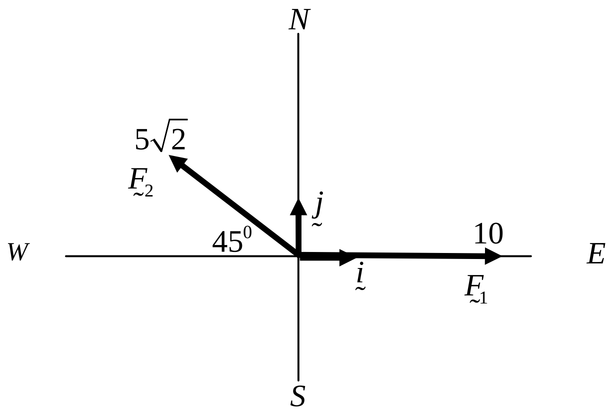
$\underset{\sim}{F}_1 + \underset{\sim}{F}_2 = 5\underset{\sim}{i} + 5\underset{\sim}{j}$

$\left|\underset{\sim}{F}_1 + \underset{\sim}{F}_2\right| = 5\sqrt{2} = ma = 5a$

$a = \sqrt{2} \quad u = 0 \quad t = 2$

using $v = u + at \quad \Rightarrow v = 2\sqrt{2}$

momentum $mv = 10\sqrt{2}$ kg ms^{-1}



Question 14 **Answer B**

$V = 16\ \text{ms}^{-1} \quad \alpha = 30^0$

The maximum height of a projectile is given by $H = \dfrac{V^2 \sin^2(\alpha)}{2g}$

$H = \dfrac{16^2 \sin^2\left(30^0\right)}{2g} = \dfrac{16^2}{2g} \text{ x } \dfrac{1}{4} = \dfrac{32}{g}$ metres

Question 15 **Answer D**

let $u = \sqrt{3x-2}$

terminals, when $x = 2 \quad u = \sqrt{4} = 2$ and when $x = 1 \ u = \sqrt{1} = 1$

$u^2 = 3x - 2 \Rightarrow 2u\dfrac{du}{dx} = 3 \quad \dfrac{dx}{du} = \dfrac{2u}{3}$ and $x = \dfrac{1}{3}\left(u^2 + 2\right)$

$$\int_1^2 \frac{1}{x\sqrt{3x-2}} \frac{dx}{du} du = \int_1^2 \frac{3}{u^2+2} \text{ x } \frac{1}{u} \text{ x } \frac{2u}{3} du = 2\int_1^2 \frac{1}{u^2+2} du$$

Question 16 **Answer A**

$y = \cos^{-1}\left(\dfrac{x}{4}\right)$ when $x = 2 \quad y = \cos^{-1}\left(\dfrac{1}{2}\right) = \dfrac{\pi}{3} \qquad P\left(2, \dfrac{\pi}{3}\right)$

$\dfrac{dy}{dx} = \dfrac{-1}{\sqrt{16-x^2}}$ when $x = 2 \quad m_T = \dfrac{-1}{\sqrt{12}} \quad m_N = \sqrt{12} = 2\sqrt{3}$

equation of the normal is $y - \dfrac{\pi}{3} = 2\sqrt{3}(x-2)$ or $y = 2\sqrt{3}\,x + \dfrac{\pi}{3} - 4\sqrt{3}$

Question 17 **Answer B**

$$m\ddot{x} = mf(x)$$

$$m\frac{d}{dx}\left(\frac{1}{2}v^2\right) = mf(x)$$

$$m\left[\frac{1}{2}v^2\right]_{v_o}^{v_1} = m\int_{x_0}^{x_1} f(x)\,dx$$

$$\frac{1}{2}mv_1^2 - \frac{1}{2}mv_0^2 = m\int_{x_0}^{x_1} f(x)\,dx$$

Question 18 **Answer E**

motion is downwards, positive direction

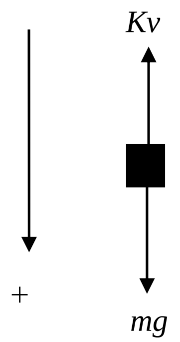
$m\ddot{y} = mg - R$ where $R = Kv = K\dot{y}$

$m\ddot{y} = mg - Kv$

$m\ddot{y} = mg - K\dot{y}$

$m\ddot{y} + K\dot{y} = mg$ let $k = \dfrac{K}{m}$

$\ddot{y} + ky = g \quad y(0) = 0 \quad \dot{y}(0) = U$



Question 19 **Answer E**

Using Euler's method, with $x_0 = 0$, $y_0 = 0$ $h = \dfrac{1}{3}$ $\dfrac{dy}{dx} = f(x,y) = e^{x+y}$

so that $x_1 = \dfrac{1}{3}$ and $x_2 = \dfrac{2}{3}$

$$y_1 = y_0 + hf(x_0, y_0) = 0 + \frac{1}{3}e^{0+0} = \frac{1}{3}$$

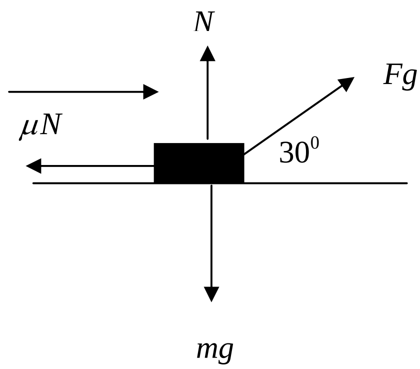
$$y_2 = y_1 + hf(x_1, y_1) = \frac{1}{3} + \frac{1}{3}e^{\frac{1}{3}+\frac{1}{3}} = \frac{1}{3}\left(1 + e^{\frac{2}{3}}\right)$$

Question 20 **Answer A**

The solution curves are of the form $y = -a\cos\left(\dfrac{\pi x}{4}\right) + c$ with $a > 0$, since the period of the solution curves is $T = \dfrac{2\pi}{\frac{\pi}{4}} = 8$, or $\dfrac{dy}{dx} = \sin\left(\dfrac{\pi x}{4}\right)$ with amplitude one.

Question 21 **Answer E**

All forces must be in newtons, $m = 3\text{kg}$ $\mu = \frac{\sqrt{3}}{2}$



resolving perpendicular to the plane

(1) $N + Fg\sin\left(30^0\right) - mg = 0$

$\Rightarrow N = 3g - \frac{Fg}{2}$

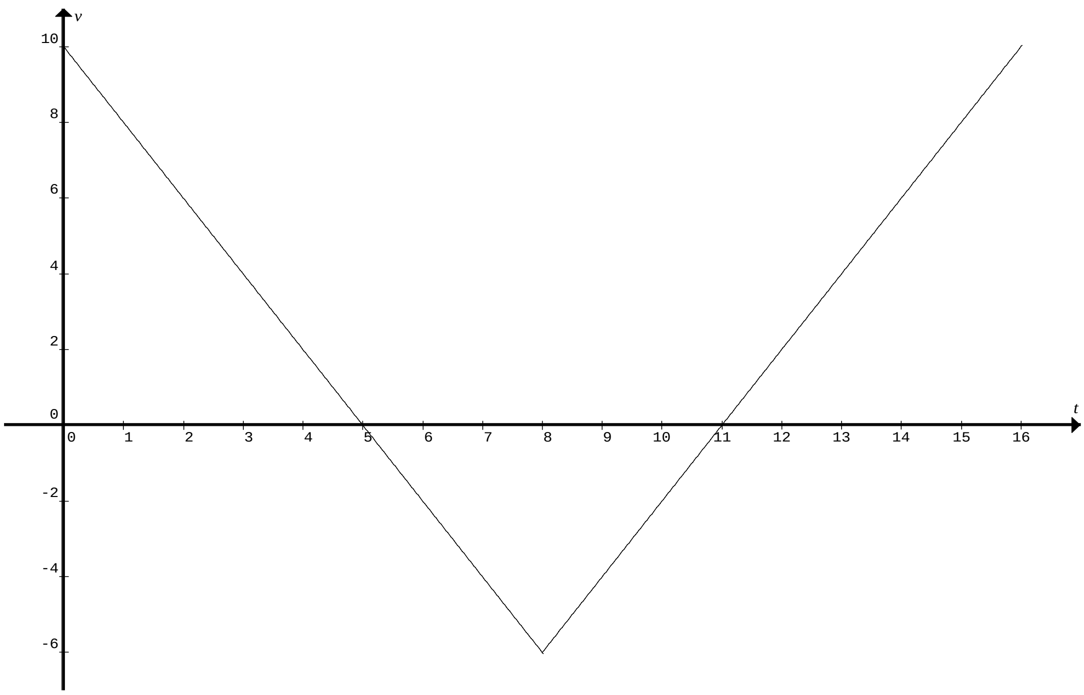
resolving parallel to the plane

(2) $Fg\cos\left(30^0\right) - \mu N = ma$

$\frac{Fg\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\left(3g - \frac{Fg}{2}\right) = 3a$

$\Rightarrow \frac{3g\sqrt{3}}{4}(F-2) = 3a$ so if $F > 2$ $\Rightarrow$ $a > 0$

Question 22 **Answer C**



The distance from the start, is the signed area of the triangles, or displacement

$\frac{1}{2}\text{x}10\text{x}5 - \frac{1}{2}\text{x}6\text{x}6 + \frac{1}{2}\text{x}5\text{x}10$

$= 25 - 18 + 25$

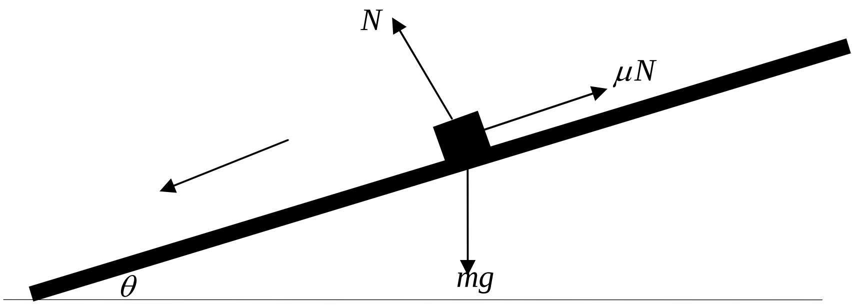
$= 32$

END OF SECTION 1 SUGGESTED ANSWERS

SECTION 2

Question 1

let the mass of the stone be *m* kg.



resolving perpendicular to the plank

$N - mg\cos(\theta) = 0 \quad \Rightarrow \quad N = mg\cos(\theta)$ M1

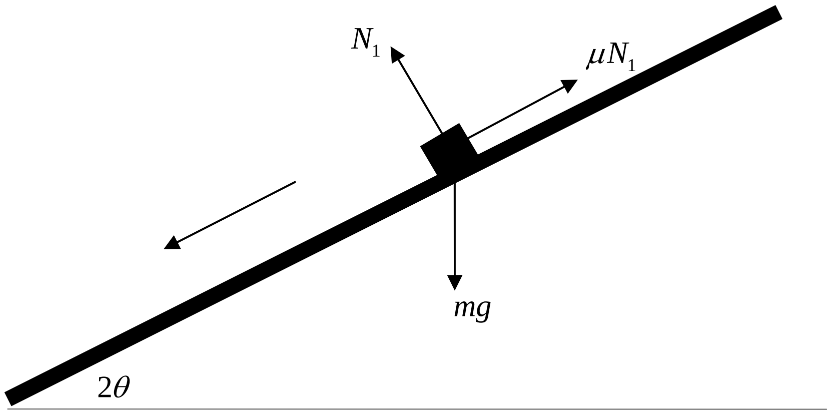
resolving parallel to the plank

$mg\sin(\theta) - \mu N = 0 \quad \Rightarrow \quad mg\sin(\theta) = \mu N$ A1

$mg\sin(\theta) = \mu mg\cos(\theta)$

$\mu = \tan(\theta)$

b.



resolving perpendicular to the plank

$N_1 - mg\cos(2\theta) = 0 \quad \Rightarrow \quad N_1 = mg\cos(2\theta)$

resolving parallel to the plank

$ma = mg\sin(2\theta) - \mu N_1$ M1

$ma = mg\sin(2\theta) - \mu mg\cos(2\theta)$ but $\mu = \tan(\theta)$

$ma = mg\left(\sin(2\theta) - \tan(\theta)\cos(2\theta)\right)$

$a = g\left(\sin(2\theta) - \dfrac{\sin(\theta)\cos(2\theta)}{\cos(\theta)}\right)$ M1

$a = g\left(\dfrac{\sin(2\theta)\cos(\theta) - \sin(\theta)\cos(2\theta)}{\cos(\theta)}\right)$

$a = \dfrac{g\sin(2\theta - \theta)}{\cos(\theta)} = g\tan(\theta)$ A1

Now using constant acceleration formulae with $u = 0 \quad v = V_1 \quad s = D$

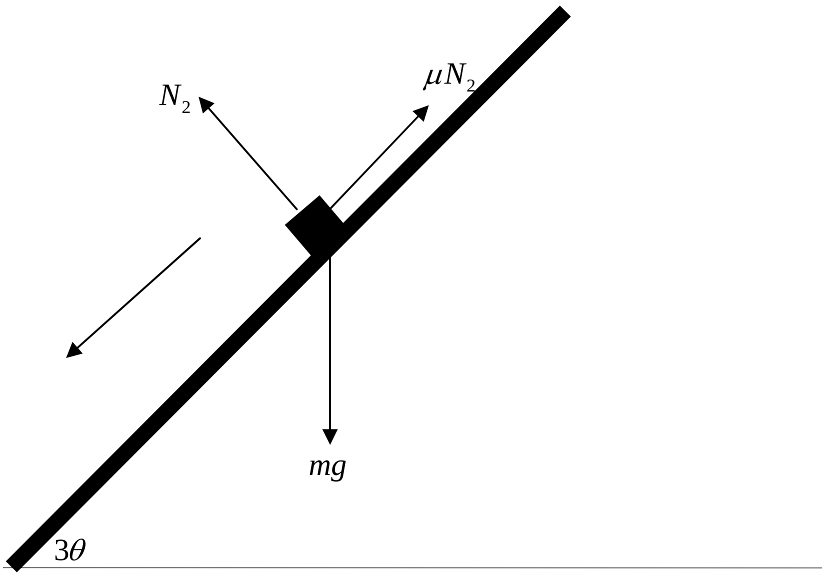
$v^2 = u^2 + 2as$ A1

$V_1 = \sqrt{2gD\tan(\theta)}$

$s = ut + \dfrac{1}{2}at^2 \quad \Rightarrow \quad t = \sqrt{\dfrac{2s}{a}}$

$T_1 = \sqrt{\dfrac{2D}{g\tan(\theta)}}$

c.



resolving perpendicular to the plank

$N_2 - mg\cos(3\theta) = 0 \quad \Rightarrow \quad N_2 = mg\cos(3\theta)$

resolving parallel to the plank

$ma = mg\sin(3\theta) - \mu N_2$

$ma = mg\sin(3\theta) - \mu mg\cos(3\theta) \quad$ but $\mu = \tan(\theta)$

$ma = mg\left(\sin(3\theta) - \tan(\theta)\cos(3\theta)\right)$

$a = g\left(\sin(3\theta) - \dfrac{\sin(\theta)\cos(3\theta)}{\cos(\theta)}\right)$ M1

$a = g\left(\dfrac{\sin(3\theta)\cos(\theta) - \sin(\theta)\cos(3\theta)}{\cos(\theta)}\right)$

$a = \dfrac{g\sin(3\theta - \theta)}{\cos(\theta)} = \dfrac{g\sin(2\theta)}{\cos(\theta)}$

$a = \dfrac{2g\sin(\theta)\cos(\theta)}{\cos(\theta)}$ A1

$a = 2g\sin(\theta)$

Now using constant acceleration formulae with $u = 0 \quad v = V_2 \quad s = D$

$v^2 = u^2 + 2as$

$V_2 = 2\sqrt{gD\sin(\theta)}$ A1

$s = ut + \frac{1}{2}at^2 \quad \Rightarrow \quad t = \sqrt{\dfrac{2s}{a}}$

$T_2 = \sqrt{\dfrac{2D}{2g\sin(\theta)}}$

$T_2 = \sqrt{\dfrac{D}{g\sin(\theta)}}$ A1

d. $\dfrac{T_2}{T_1} = \sqrt{\dfrac{D}{g\sin(\theta)}} \times \sqrt{\dfrac{g\tan(\theta)}{2D}} = \sqrt{\dfrac{Dg\sin(\theta)}{2Dg\sin(\theta)\cos(\theta)}}$ M1

$\dfrac{T_2}{T_1} = \sqrt{\dfrac{1}{2\cos(\theta)}} = \dfrac{3}{4} \quad \Rightarrow \quad \cos(\theta) = \dfrac{8}{9}$

$\theta = \cos^{-1}\left(\dfrac{8}{9}\right)$

$\theta = 27^0 16'$ A1

e. $\dfrac{V_2}{V_1}=\dfrac{2\sqrt{gD\sin(\theta)}}{\sqrt{2gD\tan(\theta)}}=\dfrac{2\sqrt{gD}\ \sqrt{\sin(\theta)\cos\theta}}{\sqrt{2gD}\ \sqrt{\sin(\theta)}}$

$\dfrac{V_2}{V_1}=\dfrac{2\sqrt{\cos(\theta)}}{\sqrt{2}}=\dfrac{2}{\sqrt{2}}\text{x}\sqrt{\dfrac{8}{9}}$

$\dfrac{V_2}{V_1}=\dfrac{4}{3}$ A1

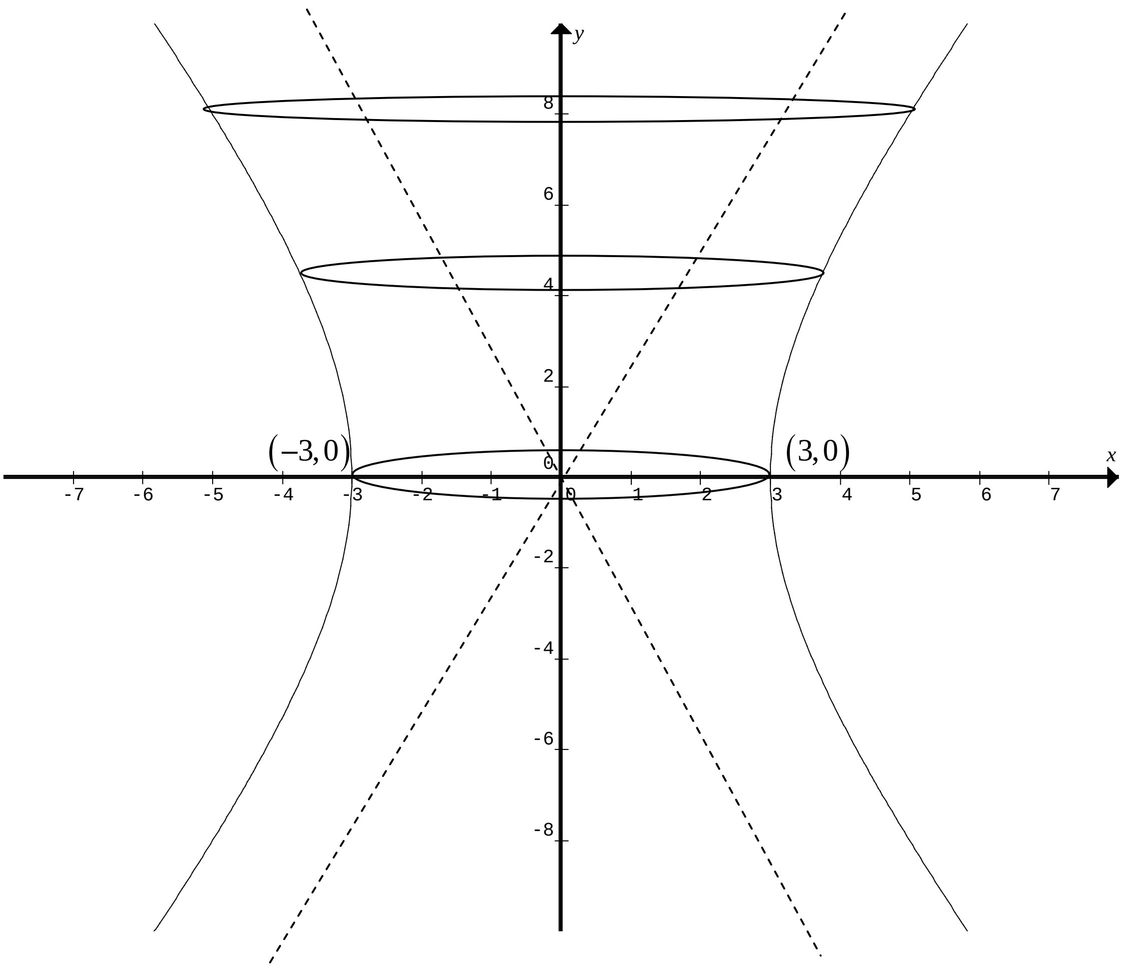
Question 2

a. $\dfrac{x^2}{9}-\dfrac{y^2}{36}=1$ crosses the x-axis at $y=0 \Rightarrow \quad x^2=9 \quad x=\pm 3$

$(3,0)\ \ (-3,0)$ A1

the asymptotes are $y=\pm 2x$

correct graph, shape of vase, intercepts G1



i. when $x = 5$, $y = H$ $\quad \dfrac{H^2}{36} = \dfrac{25}{9} - 1 = \dfrac{16}{9} \quad \Rightarrow H = \sqrt{64}$

$H = 8$ cm A1

ii. $V = \pi \int_a^b x^2 \, dy$

$\dfrac{x^2}{9} = 1 + \dfrac{y^2}{36} \quad \Rightarrow \quad x^2 = \dfrac{9}{36}\left(36 + y^2\right)$ M1

$V = \dfrac{\pi}{4} \int_0^8 \left(y^2 + 36\right) dy \quad \text{cm}^3$ A1

iii. $V = \dfrac{\pi}{4} \int_0^h \left(y^2 + 36\right) dy \quad 0 \le h \le 8$

$V = \dfrac{\pi}{4} \left[\dfrac{1}{3} y^3 + 36y \right]_0^h$ M1

$V = \dfrac{\pi}{4} \left(\dfrac{1}{3} h^3 + 36h \right) = \dfrac{\pi}{4} \left(\dfrac{h^3 + 108h}{3} \right)$

A1

$V = \dfrac{\pi h}{12} \left(h^2 + 108\right)$ for $0 \le h \le 8$

iv. $\dfrac{dV}{dt} = 18\sqrt{h} \quad \text{cm}^3/\text{s}$

$\dfrac{dV}{dh} = \dfrac{\pi}{4}\left(h^2 + 36\right)$ A1

$\dfrac{dh}{dt} = \dfrac{dh}{dV}\dfrac{dV}{dt} = \dfrac{72\sqrt{h}}{\pi\left(h^2 + 36\right)}$ M1

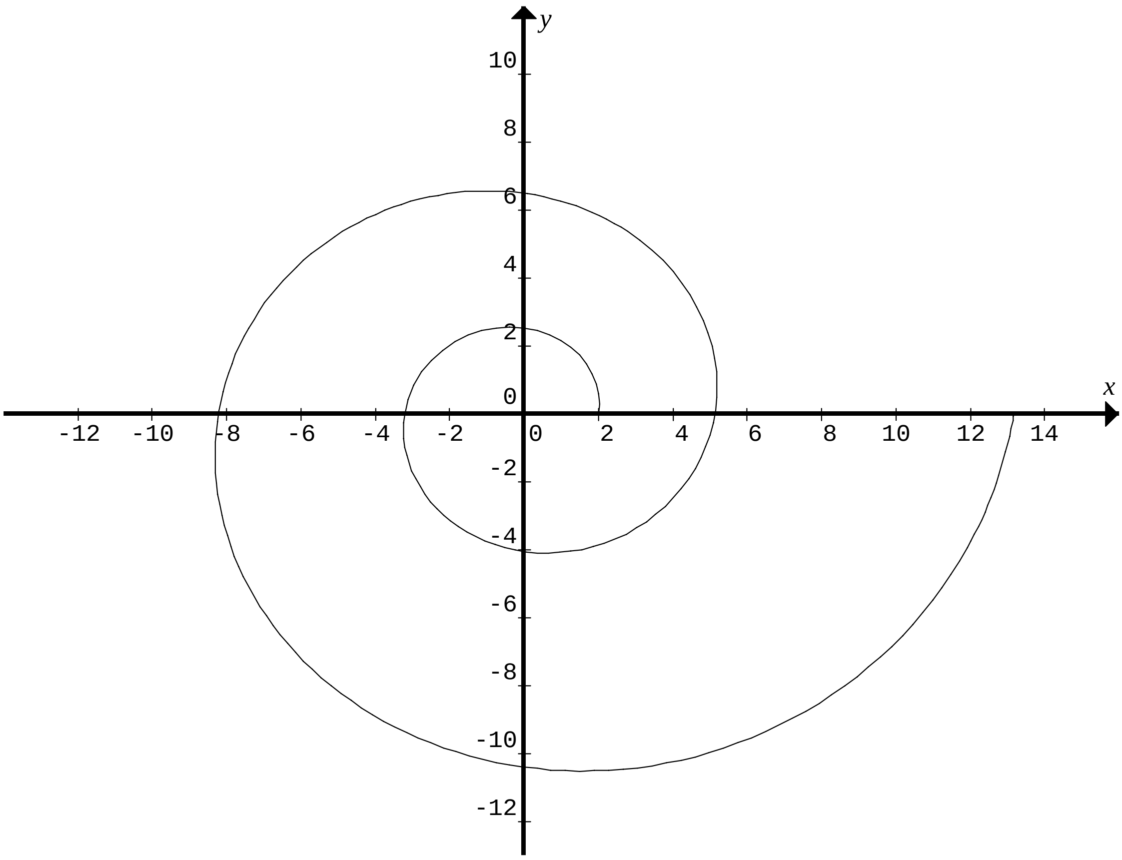
$\dfrac{dt}{dh} = \dfrac{\pi\left(h^2 + 36\right)}{72\sqrt{h}}$ M1

$t = \dfrac{\pi}{72} \int_0^8 \dfrac{h^2 + 36}{\sqrt{h}} \, dh$

$t = \dfrac{122\pi\sqrt{2}}{45}$ sec A1

Question 3

a. correct graph shape and correct *x* and *y* intercepts, correct cycle G2



b.

$x(t) = 2e^{0.3t}\cos(2t)$

$\dot{x} = \dfrac{dx}{dt} = 2\left(0.3e^{0.3t}\cos(2t) - 2e^{0.3t}\sin(2t)\right)$

$\dot{x} = \dfrac{dx}{dt} = 2e^{0.3t}\left(0.3\cos(2t) - 2\sin(2t)\right)$ M1

$y(t) = 2e^{0.3t}\sin(2t)$

$\dot{y} = \dfrac{dy}{dt} = 2\left(0.3e^{0.3t}\sin(2t) + 2e^{0.3t}\cos(2t)\right)$ A1

$\dot{y} = \dfrac{dy}{dt} = 2e^{0.3t}\left(0.3\sin(2t) + 2\cos(2t)\right)$

Now $\frac{dy}{dx}=\tan(\alpha)=\frac{dy}{dt}.\frac{dt}{dx}=\frac{\dot{y}}{\dot{x}}$

$$\tan(\alpha)=\frac{2e^{0.3t}\left(0.3\sin(2t)+2\cos(2t)\right)}{2e^{0.3t}\left(0.3\cos(2t)-2\sin(2t)\right)}$$

M1

$$\tan(\alpha)=\frac{0.3\sin(2t)+2\cos(2t)}{0.3\cos(2t)-2\sin(2t)}$$ divide by $\cos(2t)$

$$\tan(\alpha)=\frac{0.3\tan(2t)+2}{0.3-2\tan(2t)}$$

c. In the triangle $2t+(180-\alpha)+\beta=180$

$\beta=\alpha-2t$

A1

$$\tan(\beta)=\tan(\alpha-2t)$$

$$\tan(\beta)=\frac{\tan(\alpha)-\tan(2t)}{1+\tan(\alpha)\tan(2t)}$$

M1

$$\tan(\beta)=\frac{\left(\frac{0.3\tan(2t)+2}{0.3-2\tan(2t)}\right)-\tan(2t)}{1+\left(\frac{0.3\tan(2t)+2}{0.3-2\tan(2t)}\right)\tan(2t)}$$

$$\tan(\beta)=\frac{\frac{0.3\tan(2t)+2-\tan(2t)\left(0.3-2\tan(2t)\right)}{0.3-2\tan(2t)}}{\frac{0.3-2\tan(2t)+\tan(2t)\left(0.3\tan(2t)+2\right)}{0.3-2\tan(2t)}}$$

$$\tan(\beta)=\frac{0.3\tan(2t)+2-0.3\tan(2t)+2\tan^2(2t)}{0.3-2\tan(2t)+0.3\tan^2(2t)+2\tan(2t)}$$

$$\tan(\beta)=\frac{2+2\tan^2(2t)}{0.3+0.3\tan^2(2t)}=\frac{2\left(1+\tan^2(2t)\right)}{0.3\left(1+\tan^2(2t)\right)}$$

$$\tan(\beta)=\frac{20}{3}$$ A1

d. Now the speed $|\dot{r}(t)| = \sqrt{\dot{x}^2 + \dot{y}^2}$

$\dot{x} = 2e^{0.3t}\left(0.3\cos(2t) - 2\sin(2t)\right)$

$\dot{x}^2 = \left(2e^{0.3t}\left(0.3\cos(2t) - 2\sin(2t)\right)\right)^2$ M1

$\dot{x}^2 = 4e^{0.6t}\left(0.3^2\cos^2(2t) - 1.2\sin(2t)\cos(2t) + 4\sin^2(2t)\right)$

$\dot{y} = 2e^{0.3t}\left(0.3\sin(2t) + 2\cos(2t)\right)$

$\dot{y}^2 = \left(2e^{0.3t}\left(0.3\sin(2t) + 2\cos(2t)\right)\right)^2$ M1

$\dot{y}^2 = 4e^{0.6t}\left(0.3^2\sin^2(2t) + 1.2\cos(2t)\sin(2t) + 4\cos^2(2t)\right)$

$\dot{x}^2 + \dot{y}^2 = 4e^{0.6t}\left(0.3^2\left(\sin^2(2t) + \cos^2(2t)\right) + 4\left(\sin^2(2t) + \cos^2(2t)\right)\right)$

$\dot{x}^2 + \dot{y}^2 = 4e^{0.6t}\left(4 + 0.3^2\right)$

Now the speed $|\dot{r}(t)| = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{4e^{0.6t}\left(0.3^2 + 4\right)} = 2\sqrt{4 + 0.3^2}\,e^{0.3t}$

$a = 2\sqrt{4.09} \qquad k = 0.3$ A1

e. the total distance is $s = \int_a^b |\dot{r}(t)|\,dt$

from **d.** it follows that

$$s = \int_0^{2\pi} 2\sqrt{4 + 0.3^2}\,e^{0.3t}\,dt$$

$$s = \frac{2\sqrt{4 + 0.3^2}}{0.3}\left[e^{0.3t}\right]_0^{2\pi}$$ A1

$$s = \frac{20\sqrt{4.09}}{3}\left[e^{0.6\pi} - 1\right]$$

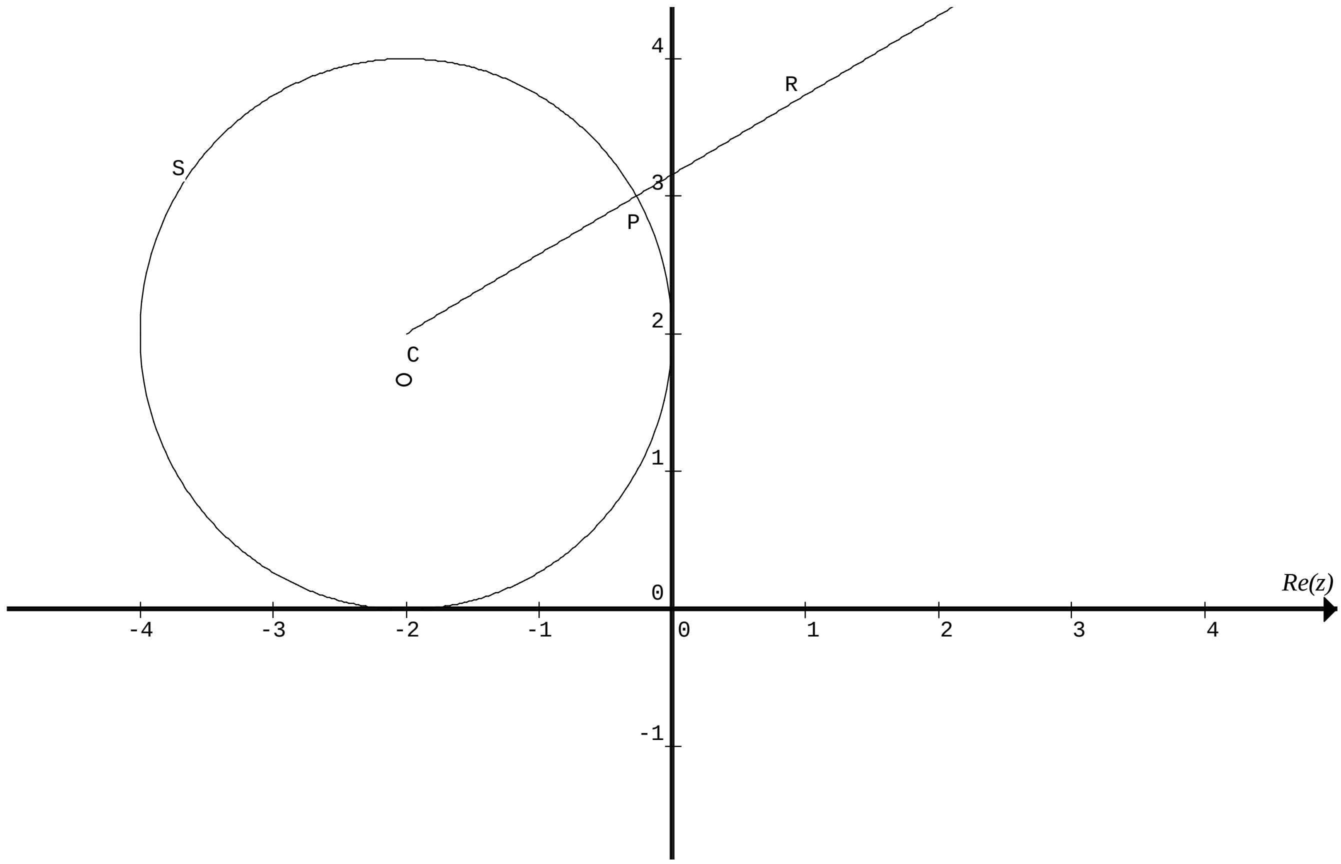
$s = 75.314$ A1

Question 4

a. S is a circle, $(x+2)^2+(y-2)^2=4$ centre at $(-2,2)$ radius 2 A1

R is a ray, starting from $(-2,2)$**,** an open circle**,** since the point is not included, making an angle of 30^0 with the line $y=2$. A1

correct circle and ray on the diagram A1



b. maximum value is the furthest point from the origin *O*, let $C(-2,2)$ M1

now the distance from $d(OC)=2\sqrt{2}$ and the radius of the circle is 2, so

$z \in S \qquad |z|_{\max}=2\sqrt{2}+2=2\left(1+\sqrt{2}\right)$ A1

c. $R \quad \text{Arg}(z+2-2i) = \text{Arg}((x+2)+(y-2)i) = \frac{\pi}{6} \quad \Rightarrow \tan^{-1}\left(\frac{y-2}{x+2}\right) = \frac{\pi}{6}$

$y-2 = \frac{1}{\sqrt{3}}(x+2) \quad \text{for} \quad x > -2$

$x+2 = \sqrt{3}(y-2)$ substitute into $(x+2)^2 + (y-2)^2 = 4$ M1

$4(y-2)^2 = 4 \quad \Rightarrow \quad y-2 = \pm 1 \quad \text{but} \quad y > 2$

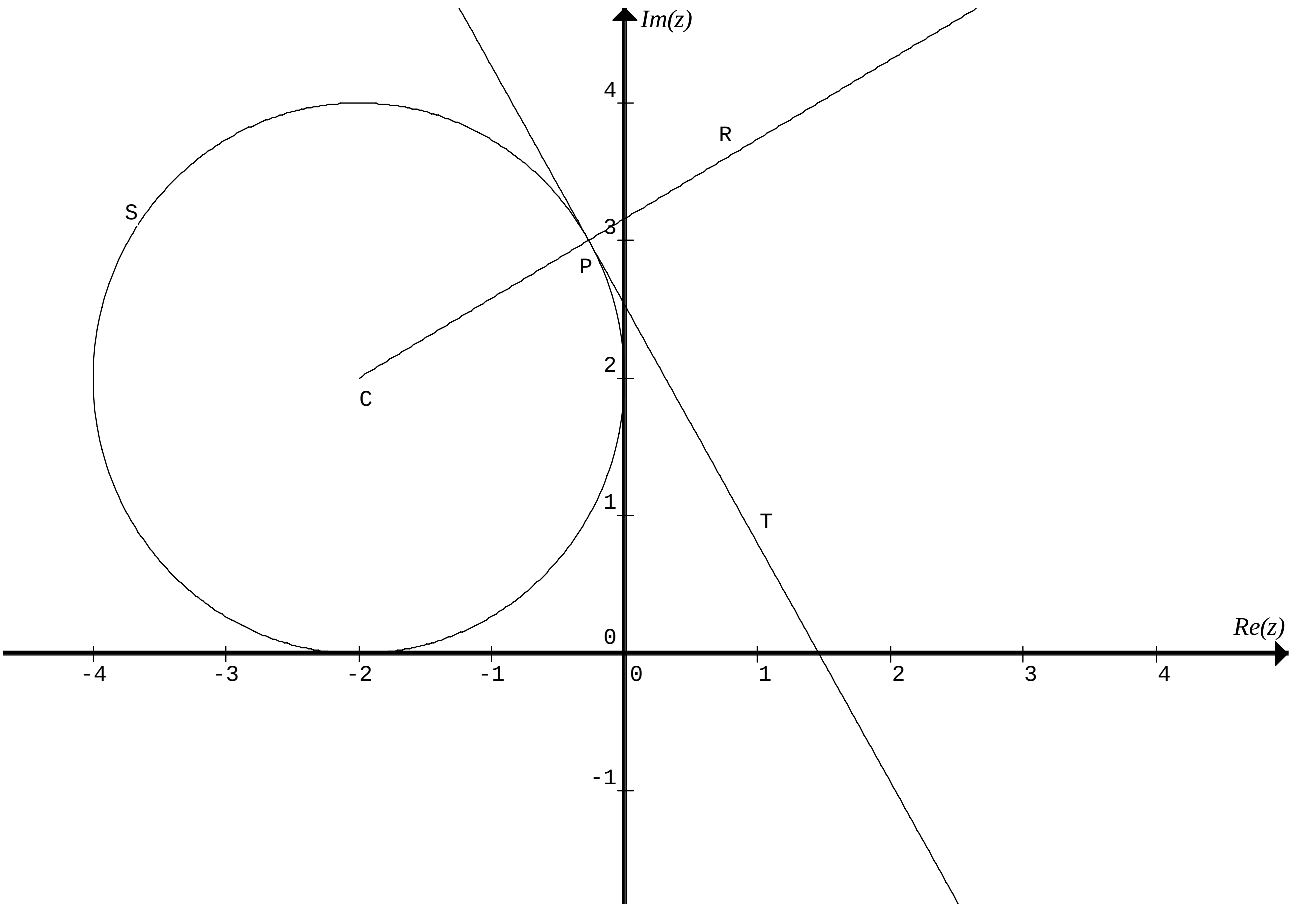
so $b = 3$ A1

$a + 2 = \sqrt{3}$

$a = \sqrt{3} - 2$ A1

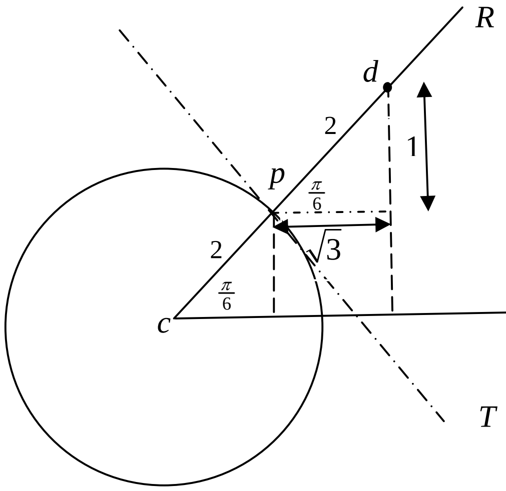
$p = \sqrt{3} - 2 + 3i$

d.



The point *d*, is on the ray *R*, and the distance $d(cp) = d(pd) = 2$

$|z+2-2i| = |z-d|$, since $p = \sqrt{3} - 2 + 3i$ M1



$\text{Im}(d) = 4$ (one up from *p*), and so $\text{Re}(d) = 2\sqrt{3} - 2$ ($\sqrt{3}$ across from *p*)

$d = 2\sqrt{3} - 2 + 4i$ A1

e. $|z+2-2i| = \left|z - 2\sqrt{3} + 2 - 4i\right|$ let $z = x + yi$

$|x+2+(y-2)i| = \left|x - 2\sqrt{3} + 2 + (y-4)i\right|$

$\sqrt{(x+2)^2 + (y-2)^2} = \sqrt{\left(x+2-2\sqrt{3}\right)^2 + (y-4)^2}$

$(x+2)^2 + y^2 - 4y + 4 = (x+2)^2 - 4\sqrt{3}(x+2) + 12 + y^2 - 8y + 16$ M1

$4y = -4\sqrt{3}x - 8\sqrt{3} + 24$

$y = -\sqrt{3}\,x + 6 - 2\sqrt{3}$ $\text{Im}(z) = y$ and $\text{Re}(z) = x$

$\text{Im}(z) = m\,\text{Re}(z) + k$

$m = -\sqrt{3}$ $k = 6 - 2\sqrt{3}$ A1

Alternatively using geometry, $P\left(\sqrt{3}-2, 3\right)$ $C(-2, 2)$

the gradient of $m(PC) = \dfrac{y_p - y_c}{x_p - x_c} = \dfrac{3-2}{\sqrt{3}-2+2} = \dfrac{1}{\sqrt{3}}$

so the gradient of the line perpendicular is $m = -\sqrt{3}$

and the line through *P*, is $y - 3 = -\sqrt{3}\left(x - \left(\sqrt{3}-2\right)\right)$

$y = -\sqrt{3}\,x + 6 - 2\sqrt{3}$ $\Rightarrow$ $k = 6 - 2\sqrt{3}$

Question 5

a. let $y = 14\log_e\left(14-\sqrt{x}\right)+\sqrt{x}$ for $0 < x < b$

$\frac{dy}{dx} = \frac{-14}{2\sqrt{x}}\left(\frac{1}{14-\sqrt{x}}\right)+\frac{1}{2\sqrt{x}}$

$\frac{dy}{dx} = \frac{-7}{\sqrt{x}\left(14-\sqrt{x}\right)}+\frac{1}{2\sqrt{x}}$

$\frac{dy}{dx} = \frac{-14+\left(14-\sqrt{x}\right)}{2\sqrt{x}\left(14-\sqrt{x}\right)}$ M1

$\frac{dy}{dx} = \frac{-\sqrt{x}}{2\sqrt{x}\left(14-\sqrt{x}\right)}$

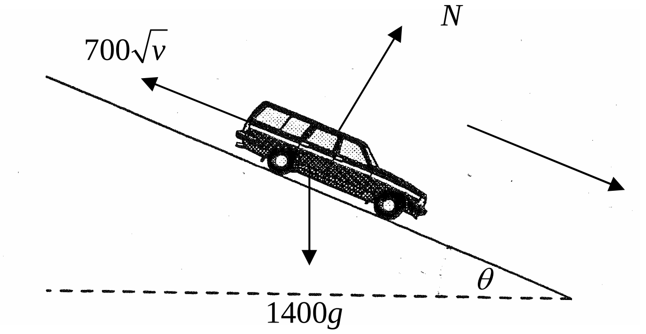
$\frac{dy}{dx} = \frac{-1}{2\left(14-\sqrt{x}\right)} = \frac{1}{2\left(\sqrt{x}-14\right)}$

so that $\frac{d}{dx}\left[14\log_e\left(14-\sqrt{x}\right)+\sqrt{x}\right] = \frac{1}{2\left(\sqrt{x}-14\right)}$

since $\sqrt{x} > 0$ and $14-\sqrt{x} > 0$ it follows that

$0 < x < 196 \quad \Rightarrow \quad b = 196$ A1

b.i.



$ma = mg\sin(\theta) - R$ and correct forces A1

$m = 1400$ kg $\quad R = 700\sqrt{v} \quad \theta = \sin^{-1}\left(\frac{5}{7}\right) \Rightarrow \sin(\theta) = \frac{5}{7}$

$1400a = 1400 \times 9.8 \times \frac{5}{7} - 700\sqrt{v}$

A1

$a = 7 - \frac{\sqrt{v}}{2}$

ii. use $a = \ddot{x} = \frac{dv}{dt} = 7 - \frac{\sqrt{v}}{2} = \frac{14-\sqrt{v}}{2}$ inverting gives

$$\frac{dt}{dv} = \frac{2}{14-\sqrt{v}}$$

$$t = \int_0^{16} \left(\frac{2}{14-\sqrt{v}} \right) dv$$ from **a.**

$$t = -4\left[14\log_e\left(14-\sqrt{v}\right) + \sqrt{v} \right]_0^{16}$$ M1

$$t = -4\left[\left(14\log_e(10) + 4\right) - 14\log_e(14) \right]$$

$$t = 56\log_e\left(\frac{7}{5}\right) - 16 \text{ sec}$$ A1

iii. use $\ddot{x} = v\frac{dv}{dx} = \frac{14-\sqrt{v}}{2}$ inverting gives

$$\frac{dx}{dv} = \frac{2v}{14-\sqrt{v}}$$ A1

$$x = \int_0^{V} \left(\frac{2v}{14-\sqrt{v}} \right) dv$$ now when $x = 20$ $V = ?$ M1

$$20 = \int_0^{V} \left(\frac{2v}{14-\sqrt{v}} \right) dv$$ since $0 < V < 196$

solving gives $V = 14.76 \text{ m/s}$ A1

END OF SECTION 2 SUGGESTED ANSWERS