Neap:

Trial Examination 2010

VCE Specialist Mathematics Units 3 & 4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of Booklet

Section	Number of questions	Number of questions to be answered	Number of marks
1	22	22	22
2	5	5	58
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

Question and answer booklet of 19 pages with a detachable sheet of miscellaneous formulas in the centrefold.

Answer sheet for multiple-choice questions.

Instructions

Detach the formula sheet from the centre of this booklet during reading time.
Write **your name** and your **teacher's name** in the space provided above on this page and in the space provided on the answer sheet for multiple-choice questions.
All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2010 VCE Specialist Mathematics Units 3 & 4 Written Examination 2.

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

The graph of the function with rule $f(x) = ax + \frac{b}{x^2}$ where $a > 0$ and $b < 0$ has

A. two asymptotes and a local maximum at $x = \left(\frac{2b}{a}\right)^{\frac{1}{3}}$

B. two asymptotes and a local minimum at $x = \left(\frac{2b}{a}\right)^{\frac{1}{3}}$

C. one asymptote and a local maximum at $x = \left(\frac{2b}{a}\right)^{\frac{1}{3}}$

D. one asymptote and a local minimum at $x = \left(\frac{2b}{a}\right)^{\frac{1}{3}}$

E. two asymptotes and a local maximum at $x = 2\left(\frac{b}{a}\right)^{\frac{1}{3}}$

Question 2

The set of values of m for which the equation $\frac{x^2}{9-m} + \frac{y^2}{m-4} = 1$ defines an ellipse is

A. $4 \le m \le 9$

B. $2 < m < 3$

C. $m < 4$ or $m > 9$

D. $4 < m < 9$

E. $m \ne 4, 9$

Question 3

The graph of $y = \sec(3t)$ for $0 \le t \le \pi$ has vertical asymptotes at

A. $t = \frac{\pi}{2}$

B. $t = \frac{\pi}{6}$ and $t = \frac{\pi}{2}$

C. $t = \frac{\pi}{6}$, $t = \frac{\pi}{2}$ and $t = \frac{5\pi}{6}$

D. $t = \frac{\pi}{3}$, $t = \frac{2\pi}{3}$ and $t = \pi$

E. $t = 0$, $t = \frac{\pi}{3}$, $t = \frac{2\pi}{3}$ and $t = \pi$

Question 4

Which one of the following is **not** true for the function with rule $g(x) = 1 - 2\tan^{-1}(x)$?

A. $g(1) = 1 - \frac{\pi}{2}$

B. The maximal domain of g is $x \in R$.

C. The range of g is $(-\pi + 1, \pi + 1)$.

D. The graph of g has a stationary point of inflection at (0, 1).

E. $g'(x) < 0$ for $x \in R$

Question 5

The curve given parametrically by $x = e^t$ and $y = e^{-2t}$ for $t \geq 0$ may be expressed in cartesian form as

A. $y = \frac{1}{2x}, x \geq 1$

B. $y = \frac{1}{x^2}, x \geq 0$

C. $y = \frac{1}{2x}, x \geq 0$

D. $y = \frac{1}{\sqrt{x}}, x \geq 1$

E. $y = \frac{1}{x^2}, x \geq 1$

Question 6

If $z = \text{cis}(\theta)$ and n is a positive integer, then $z^n - \frac{1}{z^n}$ is equal to

A. $2\cos(n\theta)$

B. $2i\sin(n\theta)$

C. 0

D. $2\sin(n\theta)$

E. $2i\cos(n\theta)$

Question 7

For any complex number, z, the complex number $w = -iz$ is found by

A. rotating z through $\frac{3\pi}{2}$ in a clockwise direction about the origin.

B. rotating z through $\frac{3\pi}{2}$ in an anticlockwise direction about the origin.

C. reflecting z in the Im(z) axis.

D. reflecting z in the Re(z) axis.

E. reflecting z in the line Im(z) = Re(z).

Question 8

P(z) is a cubic polynomial with real coefficients.

If $z = ai$ is a solution of $P(z) = 0$, then *P(z)* could be

A. $z^3 + a^2$

B. $z^3 - a^2 z$

C. $z^3 + a^2 z$

D. $z^3 - a^3$

E. $z^3 + a^3$

Question 9

If $z = r\text{cis}(\theta)$, then $\left(\frac{1}{\bar{z}}\right)^2$ is equal to

A. $\frac{1}{r^2}\text{cis}(2\theta)$

B. $-\frac{1}{r^2}\text{cis}(2\theta)$

C. $r^2\text{cis}(2\theta)$

D. $-r^2\text{cis}(2\theta)$

E. $\frac{1}{r}\text{cis}(\theta)$

Question 10

If $\frac{dy}{dx} = e^{x^3}$ and $y = 2$ when $x = 0$, then the value of *y* when $x = 3$ can be found by evaluating

A. $y = \int_0^3 e^{t^3}\, dt + 2$

B. $y = \int_0^3 e^{t^3}\, dt - 2$

C. $y = \int_0^3 (e^{t^3} + 2)\, dt$

D. $y = \int_0^3 e^{t^3}\, dt$

E. $y = \int_0^3 (e^{t^3} - 2)\, dt$

Question 11

Using an appropriate substitution, $\int_0^{\frac{\pi}{6}} \tan^2(x)\sec^2(x)dx$ can be expressed as

A. $\int_0^{\frac{\pi}{6}} u^2 du$

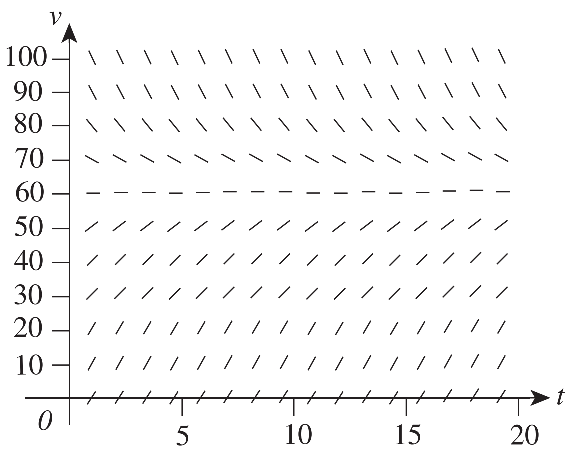
B. $\int_0^{\sqrt{3}} u^2 du$

C. $-\int_{\sqrt{3}}^{0} u^2 du$

D. $-\int_{\frac{1}{\sqrt{3}}}^{0} u^2 du$

E. $\int_0^{\frac{1}{\sqrt{3}}} \frac{1}{u^2} du$

Question 12



The direction field shown above represents a differential equation $\frac{dv}{dt} = f(v)$ that describes a model for the velocity, v m/s, at time, t seconds, of a skydiver falling from an aeroplane.

The skydiver's terminal velocity is

A. 100 m/s
B. 90 m/s
C. 60 m/s
D. 20 m/s
E. 10 m/s

Question 13

The vector component of $4\underset{\sim}{i} + \underset{\sim}{j} + 3\underset{\sim}{k}$ in the direction of $2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ is $2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$.

The vector component of $4\underset{\sim}{i} + \underset{\sim}{j} + 3\underset{\sim}{k}$ perpendicular to $2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ is

A. $-2\underset{\sim}{i} - 3\underset{\sim}{j} - 2\underset{\sim}{k}$

B. $2\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$

C. $2\underset{\sim}{i} + 3\underset{\sim}{j} + 2\underset{\sim}{k}$

D. $-6\underset{\sim}{i} + \underset{\sim}{j} - 4\underset{\sim}{k}$

E. $6\underset{\sim}{i} - \underset{\sim}{j} + 4\underset{\sim}{k}$

Question 14

Given $\underset{\sim}{u} = 2\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}$, $\underset{\sim}{v} = 2\underset{\sim}{j} + 2\underset{\sim}{k}$ and $\underset{\sim}{w} = m\underset{\sim}{i} + n\underset{\sim}{j}$, the values of m and n for which $\underset{\sim}{u} + \underset{\sim}{w}$ is parallel to $\underset{\sim}{v}$ are

A. $m = 0$ and $n = \frac{1}{2}$

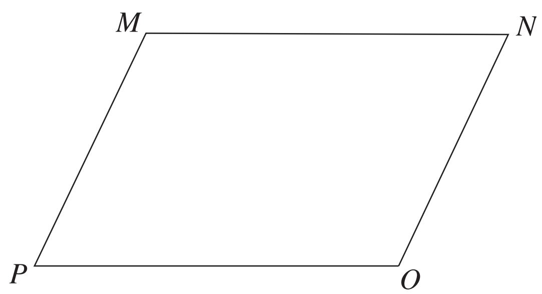
B. $m = -2$ and $n = \frac{1}{2}$

C. $m = 2$ and $n = 1$

D. $m = 2$ and $n = -1$

E. $m = -2$ and $n = -1$

Question 15



To prove that quadrilateral $MNOP$ is a rhombus, it is sufficient to show that

A. $\overrightarrow{MN} = \overrightarrow{PO}$

B. $\overrightarrow{MN} = \overrightarrow{PO}$ and $|\overrightarrow{MN}| = |\overrightarrow{MP}|$

C. $\overrightarrow{MN} \cdot \overrightarrow{MP} = 0$

D. $\overrightarrow{MO} \cdot \overrightarrow{NP} = 0$

E. $\overrightarrow{MN} = \overrightarrow{PO}$ and $\overrightarrow{MP} = \overrightarrow{NO}$

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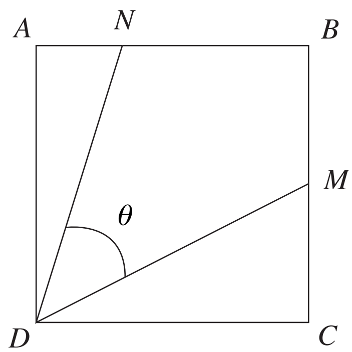
Question 16

ABCD is a square. *M* is the midpoint of *BC* and *N* divides *AB* internally in the ratio 1 : 2.

Given that $\overrightarrow{DC} = \underset{\sim}{i}$ and $\overrightarrow{DA} = \underset{\sim}{j}$, the angle θ, measured in radians, between *DM* and *DN* is



A. $\frac{\pi}{12}$

B. $\frac{\pi}{8}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{6}$

E. $\frac{\pi}{4}$

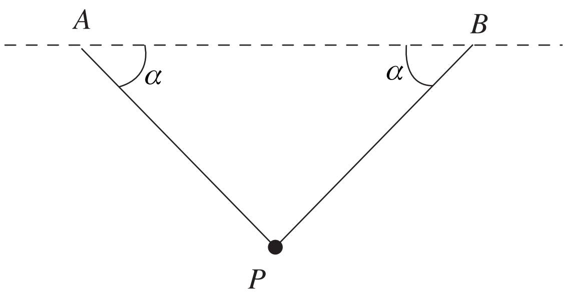
Question 17

A body of mass 10 kg is acted upon by three coplanar forces $\underset{\sim}{N}$, $\underset{\sim}{T}$ and $\underset{\sim}{R}$ where $\underset{\sim}{N} = -2\underset{\sim}{i} - \underset{\sim}{j}$, $\underset{\sim}{T} = 8\underset{\sim}{i} - 3\underset{\sim}{j}$ and $\underset{\sim}{R} = 3\underset{\sim}{i} + 16\underset{\sim}{j}$. The forces are measured in newtons.

The magnitude of the acceleration of the body, in m/s^2, is

A. 1.5
B. 3
C. 4.5
D. 6
E. 15

Question 18



A light inelastic string is attached to two points A and B which are in a horizontal line. A particle P of mass m kg is attached to the string by means of a smooth ring and hangs in equilibrium.

AP and BP each make an angle of α with the horizontal.

The tension, T Newtons, in the string is

A. $\dfrac{mg}{2\sin(\alpha)}$

B. $\dfrac{mg}{\sin(\alpha)+\cos(\alpha)}$

C. $\dfrac{mg}{\sin(\alpha)}$

D. $\dfrac{mg}{2\cos(\alpha)}$

E. $\dfrac{m}{2\sin(\alpha)}$

Question 19

A particle moving with constant acceleration has speed u m/s at A and speed v m/s at B.

The particle's speed, v_m m/s, midway between A and B is

A. $u+v$

B. $\dfrac{u+v}{2}$

C. $\dfrac{u^2+v^2}{2}$

D. $\dfrac{u-v}{2}$

E. $\sqrt{\dfrac{u^2+v^2}{2}}$

Question 20

A train of mass m kg accelerates along a straight horizontal track under the action of a constant tractive force of magnitude T newtons. Resistance to the train's motion is proportional to v^2 when the train's velocity is v m/s. The train's terminal velocity is V m/s.

Given that the train's acceleration is a m/s^2, an expression for a in terms of v, m, T and V is

A. $a = \frac{T}{mV}(V - v)$

B. $a = \frac{mV^2}{T}(V^2 - v^2)$

C. $a = \frac{T}{mV^2}(V^2 - v^2)$

D. $a = \frac{mV^2}{T}(v^2 - V^2)$

E. $a = \frac{T}{mV^2}(v^2 - V^2)$

Question 21

A body of mass 2.5 kg is travelling in a straight line. Its velocity decreases from 10 m/s to 6 m/s in a time of 2 s.

The change of momentum of the particle in kg m/s, in the direction of its motion, is

A. –20
B. –10
C. –5
D. 10
E. 15

Question 22

A particle moves in a straight line with velocity in m/s given by $v = \frac{2}{\sqrt{t^2 + 1}}, t \geq 0$.

Which one of the following statements about the particle's motion is **false**?

A. The particle has an initial acceleration of 0 m/s^2.
B. The particle's accleration is always negative.
C. The distance, x metres, travelled by the particle in the first 3 seconds of motion is given by the definite integral $x = \int_0^3 \frac{2}{\sqrt{t^2 + 1}} dt$.
D. From a non-zero initial velocity, the particle slows down, fairly slowly at first and then more rapidly.
E. The particle has an initial velocity of 2 m/s.

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

A salt solution containing 2 grams of salt per litre flows into a tank initially filled with 50 litres of water containing 10 grams of salt.

The salt solution enters the tank at 5 litres per minute, the mixture is kept uniform by stirring, and it flows out of the tank at 5 litres per minute.

There are A grams of salt in the tank after t minutes.

a. Show that $\frac{dA}{dt} = 10 - \frac{A}{10}$.

2 marks

b. **i.** Show that the time taken for the amount of salt in the tank to reach 50 grams can be represented by $t = 10\int_{10}^{50} \frac{1}{100 - A} dA$.

ii. Hence find the exact time taken for the amount of salt in the tank to reach 50 grams.

2 + 1 = 3 marks

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c. The salt solution enters the tank for a period of 15 minutes. At this time, both the inlet and outlet pipes to the tank are simultaneously closed.

Use an appropriate definite integral to find the amount of salt in the tank and explain why only one answer is physically possible. Give your answer correct to the nearest tenth of a gram.

3 marks
Total 8 marks

Question 2

A cricket ball is hit at ground level from a fixed origin O on a horizontal surface.

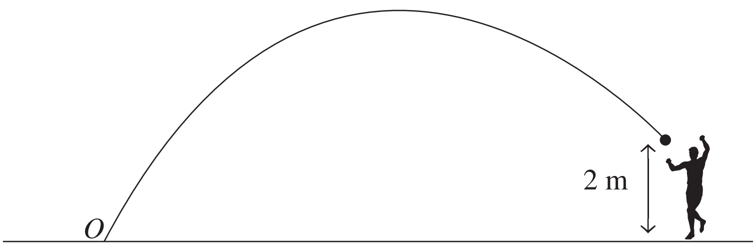
The cricket ball's initial speed is 30 m/s and is projected at an angle of 50° to the horizontal. At any time t seconds, the cricket ball's position vector as a function of time is given by

$\underset{\sim}{r}(t) = 30t\cos(50°)\underset{\sim}{i} + (30t\sin(50°) - 4.9t^2)\underset{\sim}{j}$ where the components are measured in metres.

a. Find, correct to the nearest tenth of a metre, the maximum height reached by the cricket ball.

2 marks

After reaching its maximum height, the cricket ball is caught by a fieldsman 2 metres vertically above ground level. This situation is shown in the diagram below.



b. Show that the cricket ball's time of flight, correct to one decimal place, is 4.6 seconds.

2 marks

c. Find, correct to one decimal place, the cricket ball's speed at the instant that it is caught.

3 marks

d. Find the angle, correct to the nearest tenth of a degree, between the direction of the cricket ball's motion and the positive horizontal direction at the instant that the ball is caught.

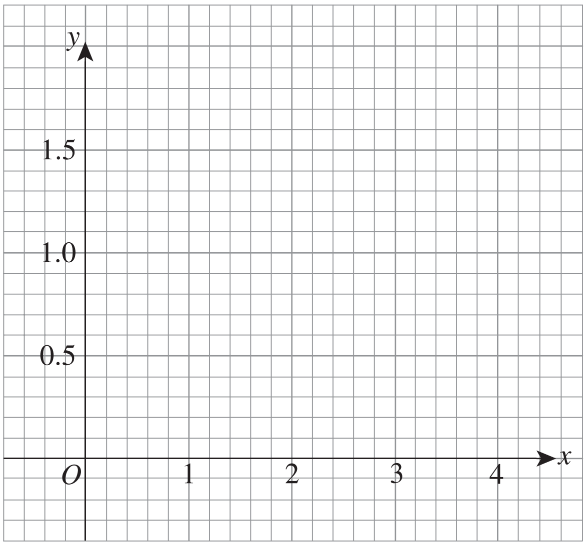
3 marks
Total 10 marks

Question 3

Consider the function $g : [0, \infty) \to R$ where $g(x) = \dfrac{1}{x^2 + 1}$.

a. On the axes below, sketch the graph of g.

Give the equations of any asymptotes and the coordinates of any intercepts.



2 marks

Point P lies on the graph of g and is where the largest negative gradient occurs.

b. **i.** Find $g''(x)$.

ii. Hence find the exact x-coordinate of point P and show that the largest negative gradient occurs at P.

1 + 4 = 5 marks

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The point Q lies on the graph of g and has coordinates $\left(1, \frac{1}{2}\right)$.

The tangent to the graph of g at point Q has equation $y = 1 - \frac{x}{2}$.

Both the graph of g and its tangent line at point Q for $0 \le x \le 1$ are rotated 360° about the y-axis to each form a volume of solid of revolution.

V_1 is the volume of solid of revolution formed by rotating the graph of g.

V_2 is the volume of solid of revolution formed by rotating the tangent line.

c. **i.** Express V_1 and V_2 as definite integrals.

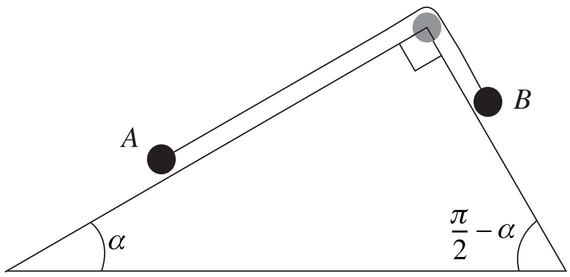
ii. Calculate V_1 and V_2 and hence show that $\log_e(2) > \frac{2}{3}$.

4 + 3 = 7 marks
Total 14 marks

Question 4

Two particles, *A* and *B*, of mass m_1 kg and m_2 kg respectively, are connected by a light inelastic string that passes over a smooth pulley at the top of a triangular wedge. Each inclined surface of the wedge is smooth.

The base angles are α and $\frac{\pi}{2} - \alpha$ respectively. The tension in the string is *T* newtons.



a. On the diagram above, clearly label all forces acting on particles *A* and *B*.

2 marks

Let a m/s^2 be the acceleration of particle *A* down its inclined plane.

b. By resolving the forces **parallel** to each inclined plane, write down an equation of motion for each particle.

2 marks

c. Show that if $\alpha > \tan^{-1}\left(\frac{m_2}{m_1}\right)$, then particle *A* will slide down the inclined plane.

3 marks

If $\alpha = \tan^{-1}\left(\frac{2m_2}{m_1}\right)$, it can be shown that $a = \frac{g\sin(\alpha)}{\tan(\alpha) + 2}$.

d. **i.** Find $\frac{da}{d\alpha}$ and hence find, correct to two decimal places, the two base angles so that a is a maximum.

ii. Hence find, correct to two decimal places, the maximum value of a.

4 + 1 = 5 marks
Total 12 marks

Question 5

In the complex plane, C is the circle with equation $|z + 5 - i| = \sqrt{2}$.

a. Show that the cartesian equation of C is given by $(x + 5)^2 + (y - 1)^2 = 2$.

2 marks

In the complex plane, L is the half-line with equation $\text{Arg}(z + 2i) = \frac{3\pi}{4}$.

b. Show that the cartesian equation of L is given by $y = -x - 2, x < 0$.

2 marks

In the complex plane, point B has coordinates (–4, 2).

c. Verify that point B lies on L **and** also lies on C.

1 mark

d. Hence, or otherwise, show that L touches C.

3 marks

The complex number u lies on C and is such that $\text{Arg}(u + 2i)$ has its maximum value.

e. Find u in exact cartesian form and find the exact maximum value of $\text{Arg}(u + 2i)$, expressing your answer in the form $\pi - \tan^{-1}\left(\frac{p}{q}\right)$ where p and q are positive integers.

6 marks

Total 14 marks

END OF QUESTION AND ANSWER BOOKLET