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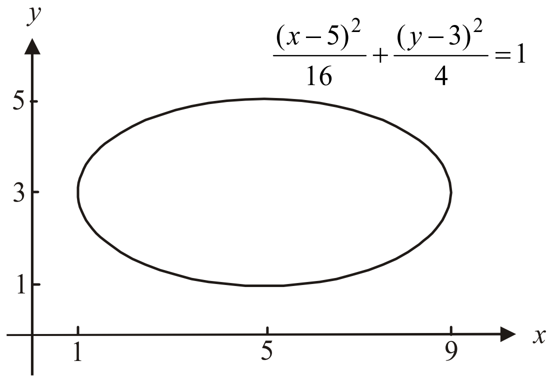
SPECIALIST MATHS TRIAL EXAMINATION 2 SOLUTIONS 2011

SECTION 1 – Multiple-choice answers

1.	A	7.	E	13.	D	19.	E
2.	C	8.	A	14.	E	20.	E
3.	C	9.	C	15.	C	21.	B
4.	A	10.	D	16.	B	22.	C
5.	D	11.	E	17.	B		
6.	C	12.	D	18.	E		

SECTION 1- Multiple-choice solutions

Question 1



There are no x or y-intercepts.
The answer is A.

Question 2

The hyperbola has its centre at $(-2,0)$ and the gradients of its asymptotes are $\pm\frac{3}{2}$.

So, $b=3$ and $a=2$.
The general equation

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$$

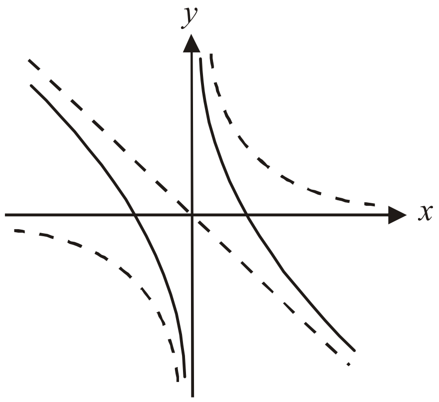
becomes
$$\frac{(x+2)^2}{4}-\frac{y^2}{9}=1$$

The answer is C.

Question 3

$$y=\frac{ax^2+b}{x}$$
$$y=ax+\frac{b}{x}$$

Method 1 – addition of ordinates



The answer is C.

Method 2
Since $a<0$, there will be an asymptote with a negative gradient i.e. $y=ax$.
This eliminates options A and B.
As $x\to\infty,\ \frac{b}{x}\to 0^+$ (from above) since $b>0$ and so $y\to ax^+$ (from above).
This eliminates option D.
As $x\to-\infty,\ \frac{b}{x}\to 0^-$ (from below) since $b>0$ and so $y\to ax^-$ (from below).
This eliminates option E.
The answer is C.

Question 4

Let $z=a+i$.
If $z=x+yi$, $\text{Arg}(z)=\tan^{-1}\left(\frac{y}{x}\right)$.

So, $\tan^{-1}\left(\frac{1}{a}\right)=\frac{5\pi}{6}$

$$\tan\left(\frac{5\pi}{6}\right)=\frac{1}{a}$$

$$\frac{-1}{\sqrt{3}}=\frac{1}{a}$$

S	A
T	C

so $a=-\sqrt{3}$
The answer is A.

Question 5

The two roots $z = 2 + i$ and $z = 2 - i$ are complex conjugates so the product of the factors $(z - 2 - i)$ and $(z - 2 + i)$ will be a quadratic with real coefficients.
This would also be the case for options A and E, so the resulting quartic would have only real coefficients.
The roots in options B and C would also mean a quadratic with real coefficients would result and hence the quartic would have only real coefficients.
The answer is D.

Question 6

If the distance from the complex number z to the complex number $-2+i$ is less than or equal to 2, then the complex number z will be in the shaded region.
That is, $|z-(-2+i)| \le 2$ or $|z+2-i| \le 2$
So we require the following set of z values:
$\{z : |z+2-i| \le 2\}$
The answer is C.

Question 7

$$u = 1+2i \qquad v = 1-\sqrt{2}i$$

$$\begin{aligned} uv &= (1+2i)(1-\sqrt{2}i) \\ &= 1-\sqrt{2}i+2i+2\sqrt{2} \\ &= 1+2\sqrt{2}+(2-\sqrt{2})i \end{aligned}$$

$\text{Re}(uv) = 1+2\sqrt{2}$
The answer is E.

Question 8

At $x = 0$, the gradient is zero.
This eliminates options B and C.
When $x = y$, the gradient equals 1 so this eliminates options D and E.
The answer is A.

Question 9

$$\frac{dy}{dx} = f(x) = \frac{1}{x^2 - 1} \qquad x_0 = 0 \qquad y_0 = 1$$

$$\begin{aligned} y_1 &= y_0 + 0.1 f(x_0) \quad \text{(formula sheet)} \\ &= 1 + 0.1 \times -1 \\ &= 0.9 \end{aligned}$$

The answer is C.

Question 10

At any time t, the thickness of the walls is $(30 - x)$ mm.

So $\dfrac{dx}{dt} = 2\%$ of $(30 - x)$

$$= \frac{2(30-x)}{100}$$

$$= \frac{30-x}{50}$$

The answer is D.

Question 11

$$\int_{\frac{\pi}{2}}^{0} (\sin(x)\cos(x))^3\,dx$$

$$= \int_{\frac{\pi}{2}}^{0} \sin^3(x)\cos^3(x)dx$$

$$= \int_{\frac{\pi}{2}}^{0} \sin^3(x)\cos^2(x)\cos(x)dx$$

$$= \int_{\frac{\pi}{2}}^{0} \sin^3(x)(1-\sin^2(x))\cos(x)dx$$

Let $u = \sin(x)$

$\dfrac{du}{dx} = \cos(x)$

$x = 0,\ \ u = 0$

$x = \dfrac{\pi}{2},\ u = 1$

$$= \int_{1}^{0} u^3(1-u^2)\frac{du}{dx}dx$$

$$= \int_{1}^{0} (u^3 - u^5)du$$

$$= \int_{0}^{1} (u^5 - u^3)du$$

The answer is E.

Question 12

Let the antiderivative function be $F(x)$.
The graph of $y = f(x)$ is the cubic function and the graph of $y = F(x)$ is the quartic function.

$$\int_{0}^{1} f(x)dx = F(1) - F(0)$$

$$= 1 - 0$$

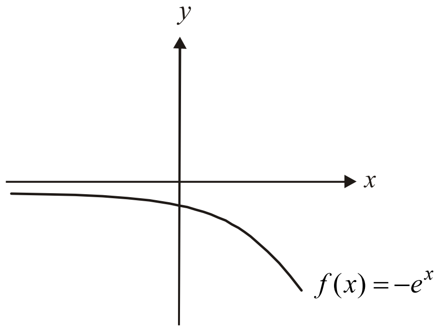
$$= 1$$

The answer is D.

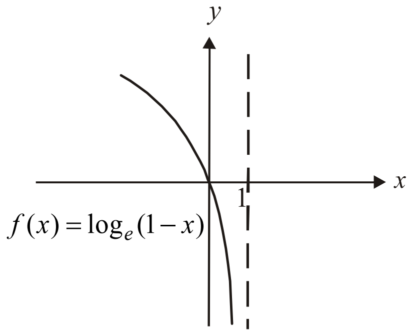
Question 13

Since $f'(x) < 0$ and $f''(x) > 0$, the graph of the function will be decreasing as x increases but the gradient increases (ie is less negative) as x increases. ("concave up")
The options are shown below.

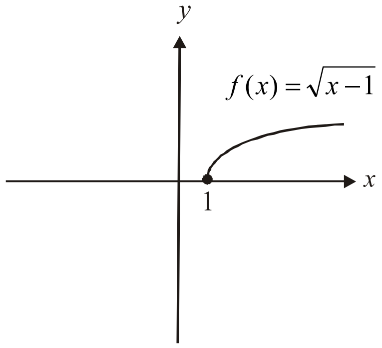
A.



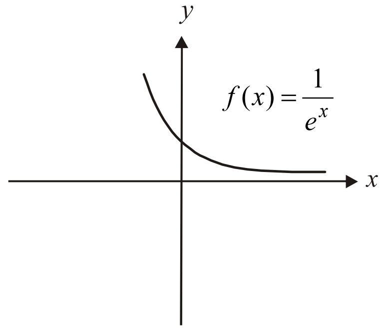
B.



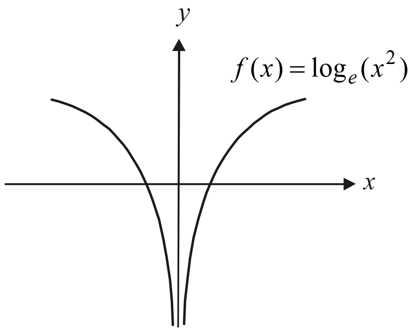
C.



D.



E.



The answer is D.

Question 14

Since $\underset{\sim}{a}$ and $\underset{\sim}{c}$ are not parallel and vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are linearly dependent, then

$\alpha(2\underset{\sim}{i}-\underset{\sim}{j}+\underset{\sim}{k})+\beta(3\underset{\sim}{i}+2\underset{\sim}{j}+\underset{\sim}{k})=\underset{\sim}{i}+x\underset{\sim}{j}-\underset{\sim}{k}$

$$\begin{array}{lrl}
\text{So,} & 2\alpha+3\beta=1 & -(1)\\
 & -\alpha+2\beta=x & -(2)\\
 & \alpha+\beta=-1 & -(3)\\
(3)\times 2 & 2\alpha+2\beta=-2 & -(4)\\
(1)-(4) & \beta=3 & \\
\text{In } (3) & \alpha+3=-1 & \\
 & \alpha=-4 & \\
(2)\text{ gives} & 4+6=x & \\
 & x=10 &
\end{array}$$

The answer is E.

Question 15

$$\underset{\sim}{v}(t)=\cos(2t)\,\underset{\sim}{i}+\sin(t)\,\underset{\sim}{j}$$

$$\underset{\sim}{r}(t)=\frac{1}{2}\sin(2t)\,\underset{\sim}{i}-\cos(t)\,\underset{\sim}{j}+\underset{\sim}{c}$$

$$\underset{\sim}{r}(0)=0\,\underset{\sim}{i}+0\,\underset{\sim}{j}$$

$$0\,\underset{\sim}{i}+0\,\underset{\sim}{j}=\frac{1}{2}\sin(0)\,\underset{\sim}{i}-\cos(0)\,\underset{\sim}{j}+\underset{\sim}{c}$$

$$0\,\underset{\sim}{i}+0\,\underset{\sim}{j}=0\,\underset{\sim}{i}-\underset{\sim}{j}+\underset{\sim}{c}$$

$$\underset{\sim}{c}=\underset{\sim}{j}$$

$$\underset{\sim}{r}(t)=\frac{1}{2}\sin(2t)\,\underset{\sim}{i}+(1-\cos(t))\,\underset{\sim}{j}$$

$$\begin{aligned}\left|\underset{\sim}{r}(\pi)\right|&=\sqrt{\frac{1}{4}\sin^2(2\pi)+(1-\cos(\pi))^2}\\&=\sqrt{0+2^2}\\&=2\end{aligned}$$

The answer is C.

Question 16

$\underset{\sim}{r}_A(t) = \underset{\sim}{i} + t^3\underset{\sim}{j}$

$\underset{\sim}{r}_B(t) = \frac{3t}{t^2+2}\underset{\sim}{i} + 8\underset{\sim}{j}$

The particles will meet iff

$$1 = \frac{3t}{t^2+2} \qquad \text{AND} \qquad t^3 = 8$$

$$t^2 + 2 = 3t \qquad\qquad t^3 - 8 = 0$$

$$t^2 - 3t + 2 = 0 \qquad\qquad (t-2)(t^2+2t+4) = 0$$

$$(t-2)(t-1) = 0 \qquad\qquad t = 2$$

$$t = 2 \text{ or } t = 1$$

The particles will meet at $t = 2$ only.
The answer is B.

Question 17

The scalar resolute of $\underset{\sim}{a}$ in the direction of $\underset{\sim}{b}$ is $\underset{\sim}{a} \bullet \hat{\underset{\sim}{b}}$

$$= (\underset{\sim}{i} - x\underset{\sim}{j} + \underset{\sim}{k}) \bullet \frac{1}{3}(\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k})$$

$$= \frac{1 - 2x + 2}{3}$$

$$= \frac{3 - 2x}{3}$$

$$\text{So } \frac{1}{3} = \frac{3-2x}{3}$$

$$x = 1$$

The answer is B.

Question 18

$\underset{\sim}{r}(t) = \left(\sqrt{t} + 1\right)\underset{\sim}{i} + (4 - t)\underset{\sim}{j}, \quad t > 0$

$\underset{\sim}{v}(t) = \frac{1}{2}t^{-\frac{1}{2}}\underset{\sim}{i} - \underset{\sim}{j}$

$\quad = \frac{1}{2\sqrt{t}}\underset{\sim}{i} - \underset{\sim}{j}$

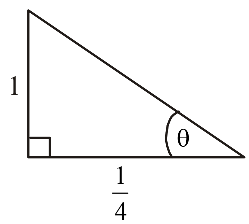
At $t = 4$,

$\underset{\sim}{v}(4) = \frac{1}{4}\underset{\sim}{i} - \underset{\sim}{j}$

$$\tan\theta = 1 \div \frac{1}{4} \qquad \text{or} \qquad \tan\theta = {}^{-}1 \div \frac{1}{4}$$

$$= 4 \qquad\qquad = -4$$

$$\theta = 75.9638...^\circ \qquad\qquad \theta = 104.036...^\circ$$



The acute angle is 75.9638...°

The nearest answer is 76°.
The answer is E.

Question 19

$$\hat{\underset{\sim}{S}} = \frac{1}{\sqrt{2}}(\underset{\sim}{i} + \underset{\sim}{j}) \qquad \text{since } |\underset{\sim}{i} + \underset{\sim}{j}| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\underset{\sim}{S} = \sqrt{2} \times \frac{1}{\sqrt{2}}(\underset{\sim}{i} + \underset{\sim}{j})$$

$$\underset{\sim}{S} = \underset{\sim}{i} + \underset{\sim}{j}$$

$$\underset{\sim}{T} = 2\underset{\sim}{j}$$

$$\underset{\sim}{S} + \underset{\sim}{T} = \underset{\sim}{i} + 3\underset{\sim}{j}$$

$$|\underset{\sim}{S} + \underset{\sim}{T}| = \sqrt{1+9}$$

$$= \sqrt{10}$$

The answer is E.

Question 20

Initially momentum is 8kgms^{-1}.

So $8 = 2 \times u$

$u = 4$

where u is the initial velocity

Five seconds later the momentum is 28kgms^{-1}.

So $28 = 2 \times v$

$v = 14$

where v is the final velocity

Since acceleration is constant,

and $u = 4, v = 14, t = 5$ and s is unknown,

$$s = \frac{1}{2}(u + v)t$$

$$= \left(\frac{4+14}{2}\right) \times 5$$

$$= 45\text{m}$$

The answer is E.

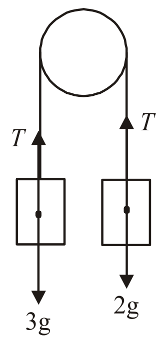
Question 21

Around the 3kg mass:

$3g - T = 3a \qquad -(1)$

Around the 2kg mass:

$T - 2g = 2a \quad -(2)$



(1) + (2) gives

$$g = 5a$$
$$a = \frac{g}{5}$$

The answer is B.

Question 22

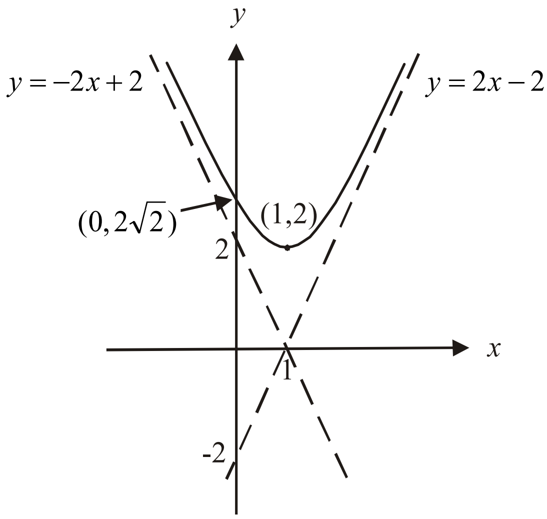
$$a = f(v)$$
$$v\frac{dv}{dx} = f(v) \qquad \text{(formula sheet)}$$
$$\frac{dv}{dx} = \frac{f(v)}{v}$$
$$\frac{dx}{dv} = \frac{v}{f(v)}$$
$$x = \int \frac{v}{f(v)}dv$$
$$x_1 - x_0 = \int_{v_0}^{v_1} \frac{v}{f(v)}dv$$
$$x_1 = \int_{v_0}^{v_1} \frac{v}{f(v)}dv + x_0$$

The answer is C.

SECTION 2

Question 1

a.



y-intercept occurs when $x=0$

$$\frac{y^2}{4}-(-1)^2=1$$

$$\frac{y^2}{4}=2$$

$$y=2\sqrt{2}$$

asymptotes

$$x-1=\pm\frac{1}{2}y$$

$$2(x-1)=\pm y$$

$$y=2x-2 \text{ or } y=-2x+2$$

(1 mark) -correct asymptotes
(1 mark) - correct shape and turning point
(1 mark) - correct y-intercept and branch

b. **i.** $\frac{y^2}{4}-(x-1)^2=1$

Method 1 – by hand

$$\frac{2y}{4}\frac{dy}{dx}-2(x-1)=0$$

$$\frac{2y}{4}\frac{dy}{dx}=2(x-1)$$

$$\frac{dy}{dx}=\frac{4(x-1)}{y}$$

(1 mark)

Since $\frac{y^2}{4}-(x-1)^2=1$

$$y^2=4(1+(x-1)^2)$$

$$y^2=4+4(x-1)^2$$

$$y=\pm\sqrt{4+4(x-1)^2}$$

$$y=\pm 2\sqrt{1+(x-1)^2}$$

but $y>0$ so $y=2\sqrt{1+(x-1)^2}$

$$\frac{dy}{dx}=\frac{2(x-1)}{\sqrt{1+(x-1)^2}}$$

or $\frac{dy}{dx}=\frac{2(x-1)}{\sqrt{x^2-2x+2}}$

(1 mark)

Method 2 – using CAS

$$\frac{y^2}{4}-(x-1)^2=1$$

$$\frac{dy}{dx}=\frac{4(x-1)}{y}$$

(1 mark)

Solve $\frac{y^2}{4}-(x-1)^2=1$ for y

$$y=\pm 2\sqrt{x^2-2x+2}$$

but $y>0$ so $y=2\sqrt{x^2-2x+2}$

$$\frac{dy}{dx}=\frac{2(x-1)}{\sqrt{x^2-2x+2}}$$

(1 mark)

ii. If $\frac{dy}{dx} = \pm 2$,

$\frac{2(x-1)}{\sqrt{x^2-2x+2}} = \pm 2$ **(1 mark)**

$$(x-1) = \pm\sqrt{x^2-2x+2}$$
$$(x-1)^2 = (x^2-2x+1)-1+2$$
$$(x-1)^2 = (x-1)^2+1$$

Clearly this is not feasible so $\frac{dy}{dx} \neq \pm 2$

(1 mark)

c. i. $\frac{d^2y}{dx^2} = \frac{2}{(x^2-2x+2)^{\frac{3}{2}}}$

(1 mark)

ii. At a point of inflection

$\frac{d^2y}{dx^2} = 0$

(Note the converse does not hold i.e. if $\frac{d^2y}{dx^2} = 0$ at $x = x_0$ there may or may not be a point of inflection at $x = x_0$.)

Since $\frac{2}{(x^2-2x+2)^{\frac{3}{2}}} \neq 0$ for $x \in R$, there cannot be a point of inflection on the graph of the function $\frac{y^2}{4} - (x-1)^2 = 1$.

(1 mark)

d. i.

$$\text{volume} = \pi\int_0^3 y^2\,dx$$
$$= 4\pi\int_0^3 (x^2-2x+2)dx$$

(1 mark)

since $\frac{y^2}{4} - (x-1)^2 = 1$

$$y^2 = 4(1+(x-1)^2)$$
$$= 4+4(x^2-2x+1)$$
$$= 4x^2-8x+8$$
$$= 4(x^2-2x+2)$$

ii. $4\pi\int_0^3 (x^2-2x+2)dx = 24\pi$ units^3

(1 mark)
Total 11 marks

Question 2

a.

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$
$$= -2\underset{\sim}{i} - \underset{\sim}{j} - 2\underset{\sim}{k} - \underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}$$
$$= -3\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}$$

(1 mark)

b.

$$\overrightarrow{BO} = \underset{\sim}{i} - 2\underset{\sim}{j} - \underset{\sim}{k}$$

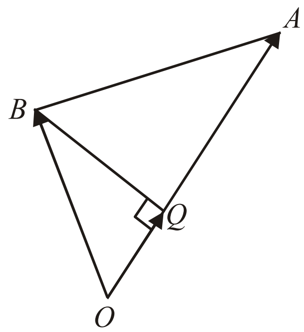
$$\overrightarrow{BA} = 3\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$$

$$\overrightarrow{BO} \bullet \overrightarrow{BA} = |\overrightarrow{BO}||\overrightarrow{BA}|\cos(\theta) \quad \text{where } \theta = \angle OBA$$
$$3 + 2 - 1 = \sqrt{1+4+1} \times \sqrt{9+1+1}\cos(\theta)$$
$$\cos(\theta) = \frac{4}{\sqrt{66}}$$

(1 mark)

c. Draw a diagram.

$\overrightarrow{OQ}$ is the component of $\overrightarrow{OB}$ parallel to $\overrightarrow{OA}$.



$$\overrightarrow{OQ} = (\overrightarrow{OB} \bullet \hat{\overrightarrow{OA}})\hat{\overrightarrow{OA}} \qquad \text{where} \qquad \hat{\overrightarrow{OA}} = \frac{1}{\sqrt{4+1+4}}(2\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k})$$
$$= \frac{1}{3}(2\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k})$$

$$= \{(-\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}) \bullet \frac{1}{3}(2\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k})\}\frac{1}{3}(2\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k})$$
$$= \frac{-2+2+2}{9}(2\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k})$$
$$= \frac{2}{9}(2\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k})$$

(1 mark) – recognition that component parallel to $\overrightarrow{OA}$ required
(1 mark) – correct unit vector
(1 mark) – correct simplification

d. Since ΔOBQ is a right-angled triangle,

$$\text{area} = \frac{1}{2} \times |\overrightarrow{OQ}||\overrightarrow{BQ}|$$

Now $\overrightarrow{BQ} = \overrightarrow{BO} + \overrightarrow{OQ}$

$$= \underset{\sim}{i}\left(1 + \frac{4}{9}\right) + \underset{\sim}{j}\left(-2 + \frac{2}{9}\right) + \underset{\sim}{k}\left(-1 + \frac{4}{9}\right)$$

$$= \frac{1}{9}(13\underset{\sim}{i} - 16\underset{\sim}{j} - 5\underset{\sim}{k})$$

$$|\overrightarrow{BQ}| = \frac{\sqrt{450}}{9}$$

$$= \frac{5\sqrt{2}}{3}$$

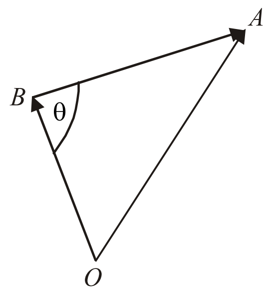
$$\text{area} = \frac{1}{2} \times |\overrightarrow{OQ}||\overrightarrow{BQ}|$$ **(1 mark)**

$$= \frac{1}{2} \times \frac{2}{9} \times \sqrt{4+1+4} \times \frac{5\sqrt{2}}{3}$$

$$= \frac{5\sqrt{2}}{9} \text{ square units}$$ **(1 mark)**

e. From part **b.**,

$$\cos\theta = \frac{4}{\sqrt{66}} \quad \text{where } \theta = \angle OBA$$



so $\sin^2\theta = 1 - \cos^2\theta$

$$= 1 - \frac{16}{66}$$

$$\sin\theta = \sqrt{\frac{50}{66}} \quad \text{since } \theta \text{ is a first quadrant angle}$$ **(1 mark)**

$$\text{Area of } \Delta OBA = \frac{1}{2}|\overrightarrow{OB}|\,|\overrightarrow{BA}|\sin\theta$$

$$= \frac{1}{2} \times \sqrt{6} \times \sqrt{11} \times \sqrt{\frac{50}{66}}$$

$$= \frac{\sqrt{50}}{2}$$

$$= \frac{5\sqrt{2}}{2}$$ **(1 mark)**

Area of ΔOBQ : Area of ΔOBA

is $\frac{5\sqrt{2}}{9} : \frac{5\sqrt{2}}{2}$

$\frac{1}{9} : \frac{1}{2}$

$2 : 9$

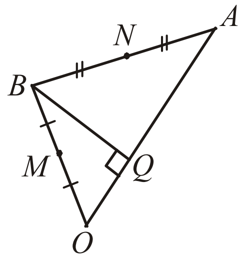
So $r = 2$ and $s = 9$.

(1 mark)

f. To prove $\overrightarrow{MN} = k\,\overrightarrow{OA}$ where k is a constant.

$$
\begin{aligned}
LHS &= \overrightarrow{MN} \\
&= \overrightarrow{MB} + \overrightarrow{BN} \\
&= \frac{1}{2}(\overrightarrow{OB}) + \frac{1}{2}(\overrightarrow{BA}) \\
&= \frac{1}{2}(-\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}) + \frac{1}{2}(3\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}) \\
&= \underset{\sim}{i} + \frac{1}{2}\underset{\sim}{j} + \underset{\sim}{k}
\end{aligned}
$$

(1 mark)

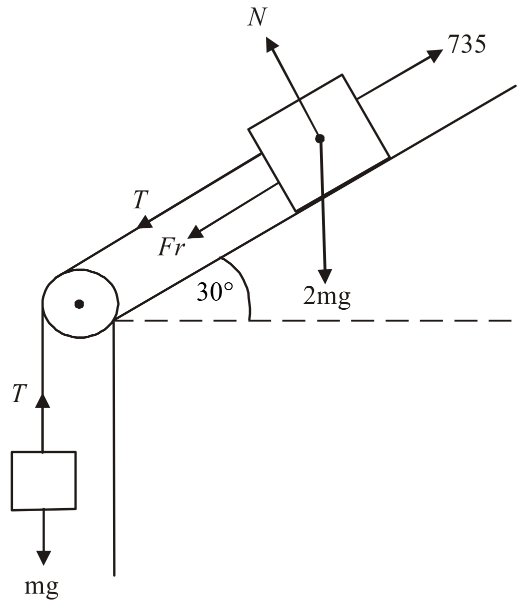


$$
\begin{aligned}
RHS &= k\,\overrightarrow{OA} \\
&= k(2\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}) \\
&= LHS \text{ where } k = \frac{1}{2}
\end{aligned}
$$

(1 mark)
Total 12 marks

Question 3

a.



(1 mark) – 3 correct forces
(1 mark) – remainder of correct forces

b. Around the load:

$T = mg \qquad -(1)$ **(1 mark)**

Around the machine:

$735 = 2mg\sin(30^\circ) + T + Fr$ and $N = 2mg\cos 30^\circ$

$735 = mg + T + \mu N$ **(1 mark)** $\quad = 2mg \times \dfrac{\sqrt{3}}{2}$

$735 = mg + mg + \dfrac{1}{\sqrt{3}} \times \sqrt{3}mg$ $\quad = \sqrt{3}mg$ **(1 mark)**

$$
\begin{aligned}
735 &= 3mg \\
m &= \frac{735}{3g} \\
&= 25
\end{aligned}
$$

So the mass of the machine is $2m = 50$kg.

(1 mark)

c. Note that the load has mass 25kg from part **b.**

Around the load:

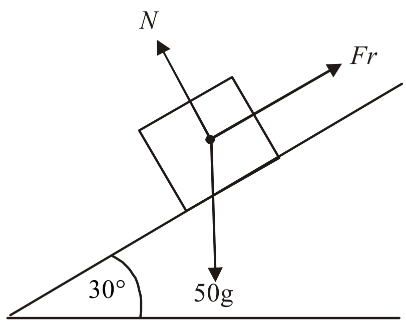
$$T - 25g = 25a$$
$$T = 25a + 25g \quad -(1)$$ **(1 mark)**

Around the machine:

$$1000 - 50g\sin(30^\circ) - T - Fr = 50a \quad \text{and} \quad N = 50g\cos 30^\circ$$
$$1000 - 25g - (25a + 25g) - \mu N = 50a \quad \textbf{(1 mark)} \qquad = 50g \times \frac{\sqrt{3}}{2}$$
$$1000 - 50g - 25a - 25\sqrt{3}g \times \frac{1}{\sqrt{3}} = 50a \qquad = 25\sqrt{3}g \quad \textbf{(1 mark)}$$
$$1000 - 75g = 75a$$
$$a = \frac{1000 - 75g}{75}$$
$$= 3.5333...$$
$$= 3.53\text{ms}^{-2} \quad \text{(to 2 decimal places)}$$

(1 mark)

d. i.



(1 mark)

ii. The weight force down the plane is $50g\sin(30°) = 25g$. **(1 mark)**

Maximum friction $= \mu N = \frac{1}{\sqrt{3}}(25\sqrt{3}g) = 25g$.

So the friction can just balance the force acting down the plane on the machine. Therefore the machine is in limiting equilibrium (stationary) and on the point of moving down the plane.

(1 mark)
Total 13 marks

Question 4

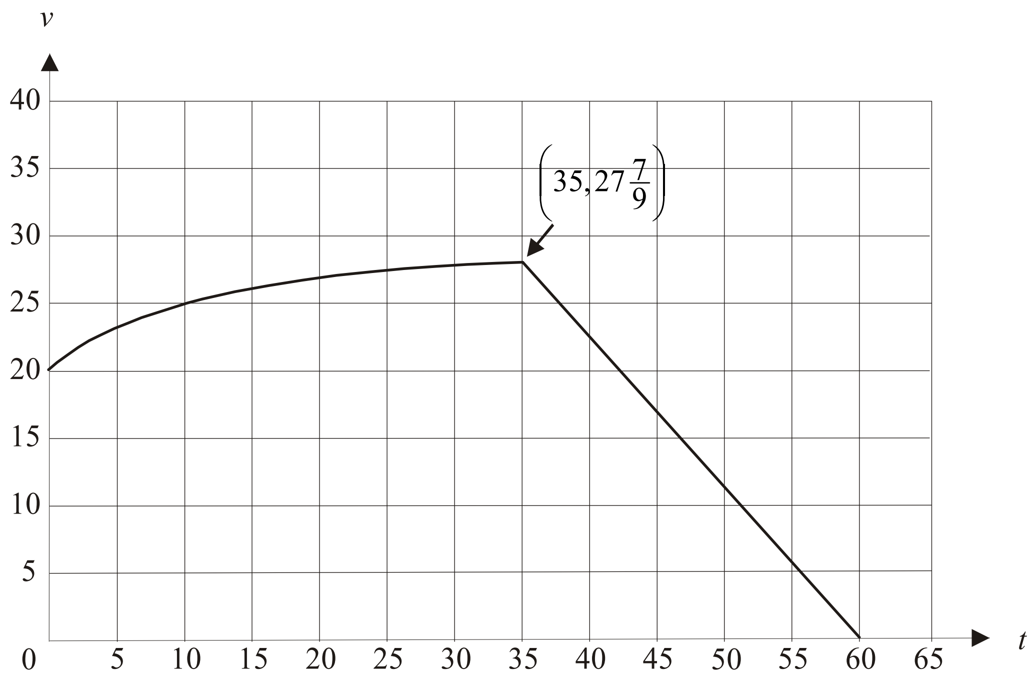
a. $$v(0)=\frac{-100}{10}+30$$
$$=20\text{ms}^{-1}$$

(1 mark)

b. For $t\in[0,35]$ the graph of the velocity function has a horizontal asymptote of $v=30$.
For $t\in(35,60]$, the velocity function is a linear function with a negative gradient and hence is a decreasing function.

(1 mark)

c.



(1 mark) – correct first branch
(1 mark) – correct second branch

d. <u>Method 1</u>

$$\text{distance}=\int_0^{35}\left(\frac{-100}{t+10}+30\right)dt+\int_{35}^{60}\left(\frac{-10t+600}{9}\right)dt$$ **(1 mark)**

$$=1246.8\text{m (correct to 1 decimal place)}$$

(1 mark)

<u>Method 2</u>

$$\text{distance}=\int_0^{35}\left(\frac{-100}{t+10}+30\right)dt+\frac{1}{2}\times v(35)\times 25$$ **(1 mark)**

$$=1246.8\text{m(correct to 1 decimal place)}$$

(1 mark)

e. $$\text{average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$
$$= \frac{1246.81...}{60}$$
$$= 20.8\text{ms}^{-1} \quad \text{(correct to 1 decimal place)}$$

(1 mark)

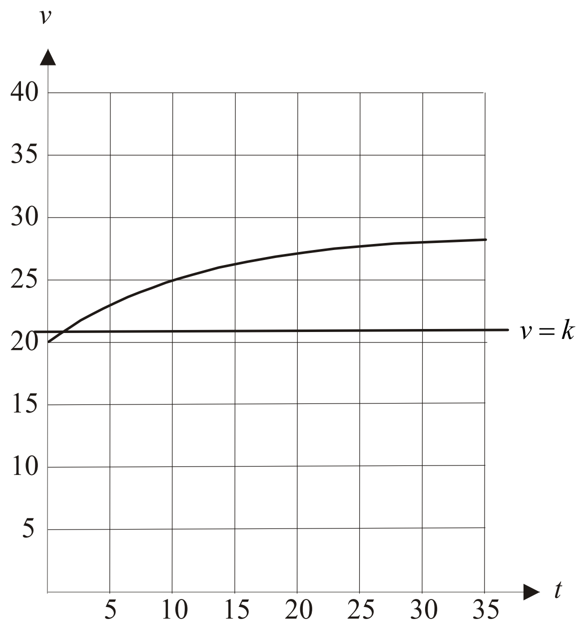
f. From part **e.**, the average speed of the car for $t \in [0,60]$ is 20.8ms^{-1}. Since the truck has been travelling at a constant speed for the 60 seconds, that constant speed must be 20.8ms^{-1}.
So $k = 20.8$ (correct to 1 decimal place)

(1 mark)

g. **i.** $v(0) = 20\text{ms}^{-1}$ for the car (from part **a.**).
Since the truck passed the car at $t = 0, k > 20$.
If the truck and the car have covered the same distance at $t = 60$ then $k = 20.8$ (from part **f.**). So if the car comes to rest before the truck passes it then $k \in (20, 20.8)$

(1 mark)

ii. Since $k \in (20, 20.8)$, we see from the graph that the car will pass the truck between $t = 0$ and $t = 35$. That is, the area under the graphs will be equal when $t = T$, which will occur soon after $t = 0$.



$$\int_0^T \left(\frac{-100}{t+10} + 30\right) dt = kT$$

(1 mark) – correct left side
(1 mark) – correct right-hand side
Total 11 marks

Question 5

a. $u = \sqrt{3} + i$, $\bar{u} = \sqrt{3} - i$

<u>Method 1</u> – using CAS

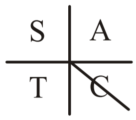
$\bar{u} = 2\text{cis}\left(\frac{-\pi}{6}\right)$

(1 mark)

<u>Method 2</u> – by hand

$|\bar{u}| = \sqrt{3+1} = 2$

$\text{Arg}(\bar{u}) = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

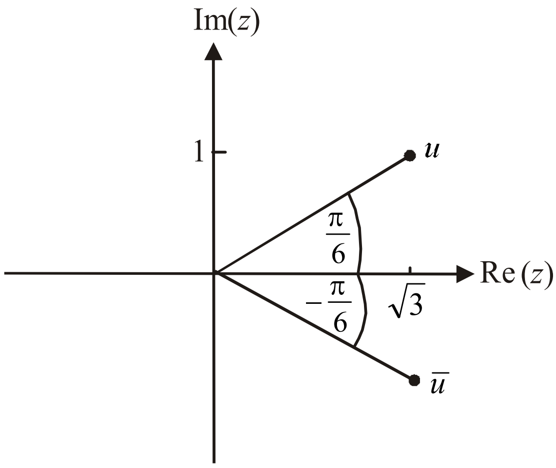


$= -\frac{\pi}{6}$

$\bar{u} = 2\text{cis}\left(-\frac{\pi}{6}\right)$

(1 mark)

b. **i.** Show u and $\bar{u}$ on an Argand diagram.



There are no solutions for which $|\text{Arg}(z)| < \frac{\pi}{6}$ that is, $-\frac{\pi}{6} < \text{Arg}(z) < \frac{\pi}{6}$.

So there are no solutions between u and $\bar{u}$.

They are spaced $2 \times \frac{\pi}{6} = \frac{\pi}{3}$ apart.

The solutions to the equation $z^n = -64$ are spaced evenly around a circle with centre at the origin. Since they are spaced $\frac{\pi}{3}$ apart, there will be six of them.

$\left(2\pi \div \frac{\pi}{3} = 6\right)$

So $n = 6$.

(1 mark)

ii. The four other solutions are

$$2\text{cis}\left(\frac{\pi}{6}+\frac{\pi}{3}\right)=2\text{cis}\left(\frac{\pi}{2}\right)$$ **(1 mark)**

$$2\text{cis}\left(\frac{\pi}{6}+\frac{2\pi}{3}\right)=2\text{cis}\left(\frac{5\pi}{6}\right)$$ **(1 mark)**

$$2\text{cis}\left(-\frac{\pi}{6}-\frac{\pi}{3}\right)=2\text{cis}\left(-\frac{\pi}{2}\right)$$ **(1 mark)**

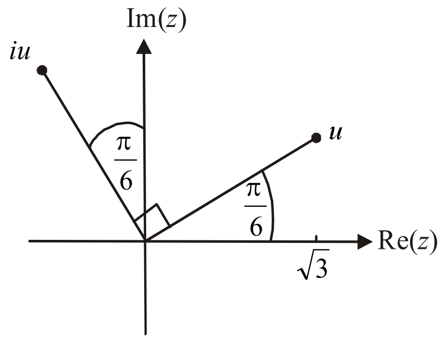
$$2\text{cis}\left(-\frac{\pi}{6}-\frac{2\pi}{3}\right)=2\text{cis}\left(-\frac{5\pi}{6}\right)$$ **(1 mark)**

c. Method 1

$$ui=2\text{cis}\left(\frac{\pi}{2}+\frac{\pi}{6}\right)$$

$$=2\text{cis}\left(\frac{2\pi}{3}\right)$$ **(1 mark)**

since multiplying a complex number by i is equivalent to rotating it anticlockwise by $\frac{\pi}{2}$.



Method 2

$$i=1\text{cis}\left(\frac{\pi}{2}\right)$$

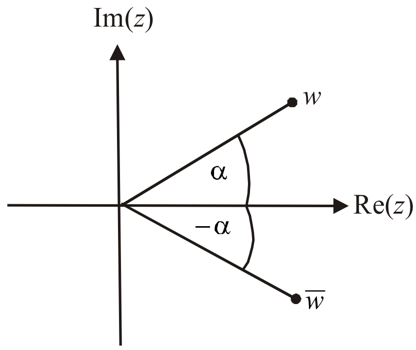
So $ui=2\text{cis}\left(\frac{\pi}{6}\right)\times\text{cis}\left(\frac{\pi}{2}\right)$

$$=2\text{cis}\left(\frac{\pi}{6}+\frac{\pi}{2}\right)$$

$$=2\text{cis}\left(\frac{2\pi}{3}\right)$$ **(1 mark)**

Also, $w=\text{cis}(\alpha)$

$\overline{w}=\text{cis}(-\alpha)$ **(1 mark)**



To show:

$$\frac{ui}{\overline{w}}=2\text{cis}\left(\frac{2\pi}{3}+\alpha\right)$$

$$\text{LHS}=\frac{ui}{\overline{w}}$$

$$=\frac{2\text{cis}\left(\frac{2\pi}{3}\right)}{\text{cis}(-\alpha)}$$

$$=2\text{cis}\left(\frac{2\pi}{3}-{}^{-}\alpha\right)$$

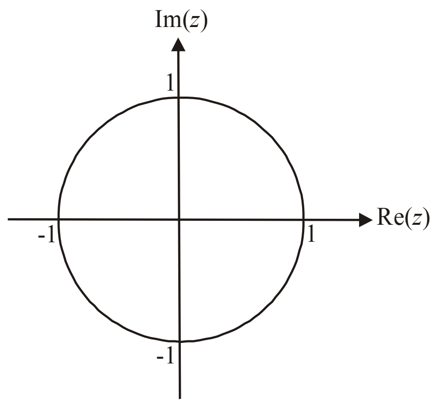
$$=2\text{cis}\left(\frac{2\pi}{3}+\alpha\right)$$

$=\text{RHS}$ **(1 mark)**

as required.

d. $S = \{z : z = w\}$
$= \{z : z = \text{cis}\alpha\}$

Since $\alpha \in R$ from part **c.**, the set of points representing the complex numbers cis(α) form a circle with a centre at the origin and a radius of 1 unit.



(1 mark) – correct shape
(1 mark) – correct radius and centre
Total 11 marks