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SPECIALIST MATHEMATICS

TRIAL EXAMINATION 2

2011

Reading Time: 15 minutes
Writing time: 2 hours

Instructions to students

This exam consists of Section 1 and Section 2.
Section 1 consists of 22 multiple-choice questions and should be answered on the detachable answer sheet on page 27 of this exam. This section of the paper is worth 22 marks.
Section 2 consists of 5 extended-answer questions, all of which should be answered in the spaces provided. Section 2 begins on page 11 of this exam. This section of the paper is worth 58 marks.
There is a total of 80 marks available.
Where more than one mark is allocated to a question, appropriate working must be shown.
Where an exact value is required to a question a decimal approximation will not be accepted.
Unless otherwise stated, diagrams in this exam are not drawn to scale.
The acceleration due to gravity should be taken to have magnitude g m/s^2 where $g = 9.8$
Students may bring one bound reference into the exam.
Students may bring an approved graphics or CAS calculator into the exam.
Formula sheets can be found on pages 24 - 26 of this exam.

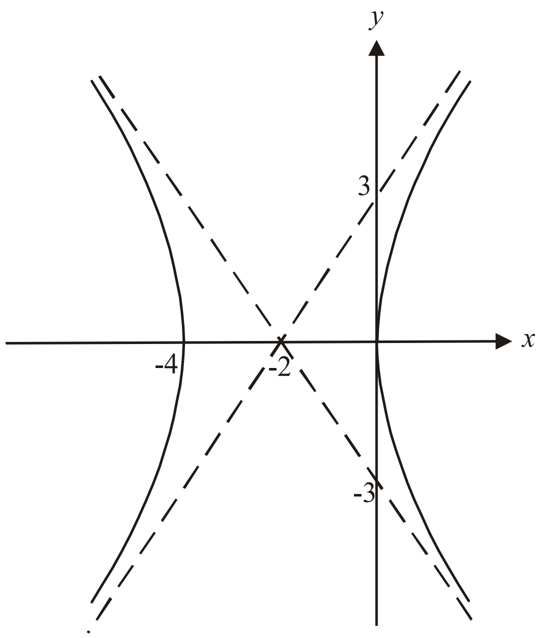
SECTION 1

Question 1

An ellipse has the equation $\frac{(x-5)^2}{16}+\frac{(y-3)^2}{4}=1$.
The graph of this ellipse has

A. no x-intercepts and no y-intercepts
B. no x-intercepts and one y-intercept
C. one x-intercept and no y-intercepts
D. one x-intercept and two y-intercepts
E. two x-intercepts and two y-intercepts

Question 2



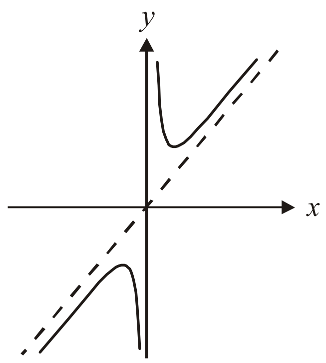
The graph shown above could have the equation

A. $\frac{x^2}{4}-\frac{(y+2)^2}{9}=1$
B. $\frac{x^2}{9}-\frac{(y+2)^2}{9}=1$
C. $\frac{(x+2)^2}{4}-\frac{y^2}{9}=1$
D. $\frac{(x+2)^2}{9}-\frac{y^2}{4}=1$
E. $\frac{(x+2)^2}{9}-\frac{(y-3)^2}{4}=1$

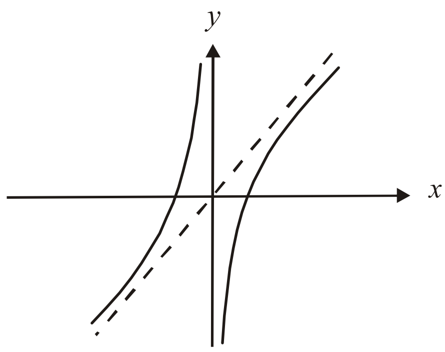
Question 3

Consider the function $y = \frac{ax^2 + b}{x}$ where a and b are integers and $a < 0$ and $b > 0$. A possible graph of this function could be

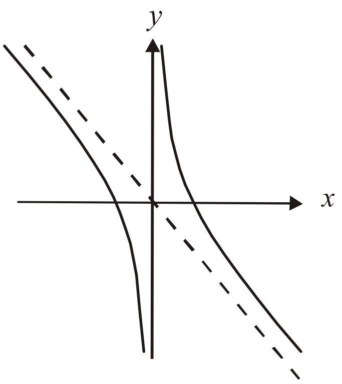
A.



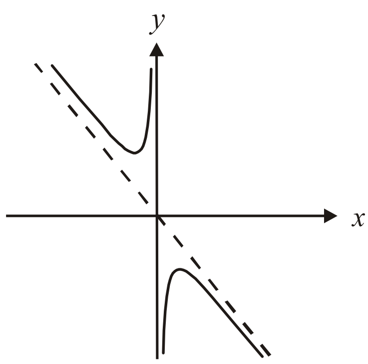
B.



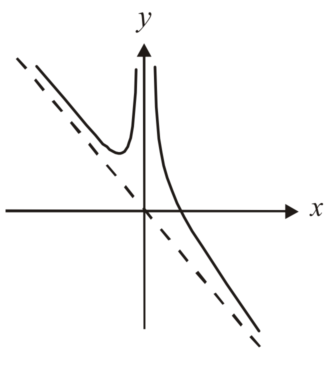
C.



D.



E.



Question 4

Given that $\text{Arg}(a+i)=\dfrac{5\pi}{6}$, where $a \in R$, then a is equal to

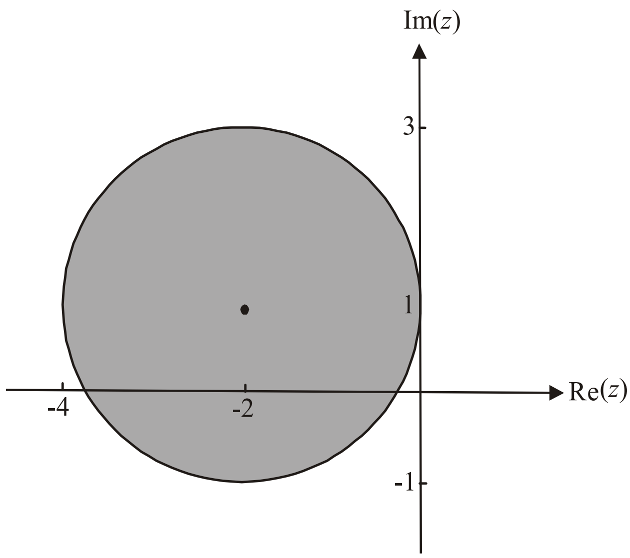
A. $-\sqrt{3}$
B. $-\dfrac{1}{\sqrt{3}}$
C. -1
D. $\dfrac{1}{\sqrt{3}}$
E. $\sqrt{3}$

Question 5

The polynomial $P(z)$, where $z \in C$, is of degree 4 and some of its coefficients are complex.
Two of the roots of the equation $P(z)=0$ are $z=2+i$ and $z=2-i$.
The other two roots could be

A. $z=i$ and $z=-i$
B. $z=1$ and $z=2$
C. $z=\sqrt{2}$ and $z=-\sqrt{2}$
D. $z=1$ and $z=2i$
E. $z=1+\sqrt{2}i$ and $z=1-\sqrt{2}i$

Question 6



The shaded region on the Argand diagram above could be defined by

A. $\{z: |z+2-i| \le 1\}$
B. $\{z: |z-1+2i| \ge 1\}$
C. $\{z: |z+2-i| \le 2\}$
D. $\{z: |z-2+i| \le 2\}$
E. $\{z: \text{Re}(z+z-i)^2+\text{Im}(z+z-i)^2 \le 2\}$

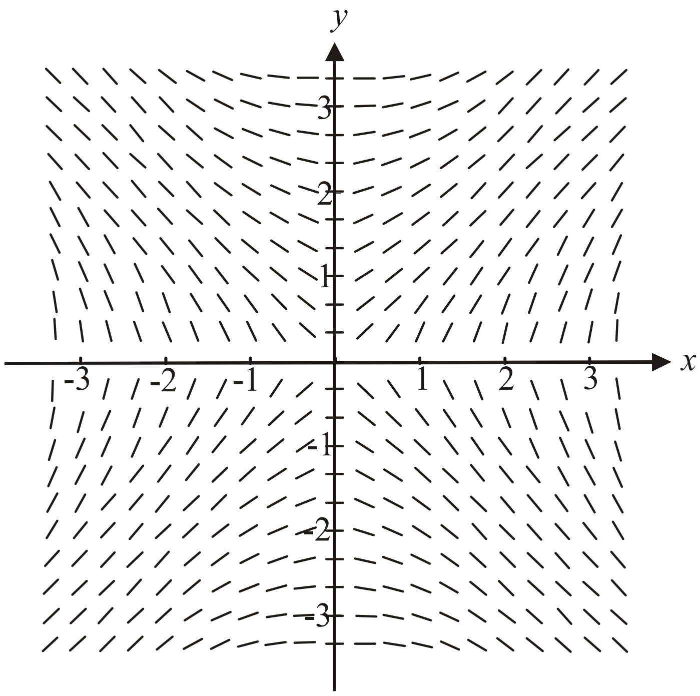
Question 7

Let $u = 1 + 2i$ and $v = 1 - \sqrt{2}i$.

The real part of uv is

A. 1
B. $2\sqrt{2}$
C. $2 - \sqrt{2}$
D. $3 + \sqrt{2}$
E. $1 + 2\sqrt{2}$

Question 8



The direction field shown above could be that of the differential equation given by

A. $\dfrac{dy}{dx} = \dfrac{x}{y}$
B. $\dfrac{dy}{dx} = \dfrac{y}{x}$
C. $\dfrac{dy}{dx} = \dfrac{1}{x}$
D. $\dfrac{dy}{dx} = \dfrac{x}{y^2}$
E. $\dfrac{dy}{dx} = \dfrac{x}{y+1}$

Question 9

For the differential equation $\frac{dy}{dx} = \frac{1}{x^2 - 1}$, where $x_0 = 0$ and $y_0 = 1$, Euler's method is used with a step size of 0.1, to find the value of y_1.
That value is

A. -0.01
B. 0.1
C. 0.9
D. 0.99
E. 1.1

Question 10

The metal walls of a tank are corroding at a rate equal to 2% of their remaining thickness. Initially the thickness of the walls was 30mm and after *t* years, *x* mm has been corroded. A differential equation which models this situation is

A. $\frac{dx}{dt} = \frac{x}{50}$
B. $\frac{dx}{dt} = 30 - \frac{x}{50}$
C. $\frac{dx}{dt} = 50 - \frac{x}{30}$
D. $\frac{dx}{dt} = \frac{30 - x}{50}$
E. $\frac{dx}{dt} = \frac{x - 30}{50}$

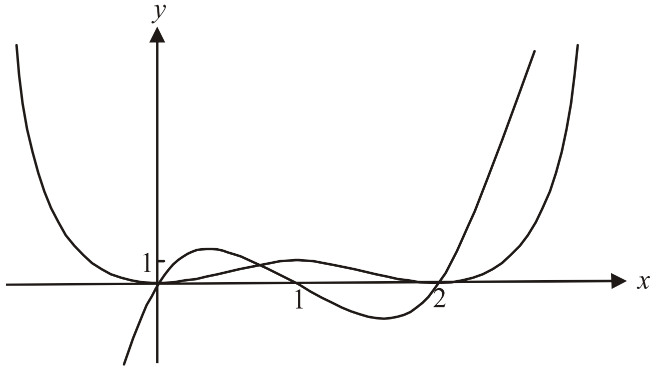
Question 11

Using a suitable substitution $\int_{\frac{\pi}{2}}^{0} (\sin(x)\cos(x))^3 \, dx$ can be expressed as

A. $-\int_{\frac{\pi}{2}}^{0} u^3 \, du$
B. $\int_{1}^{0} u^3 \, du$
C. $-\int_{1}^{0} (u^3 - u^5) \, du$
D. $-\int_{\frac{\pi}{2}}^{0} (u^3 - u^5) \, du$
E. $\int_{0}^{1} (u^5 - u^3) \, du$

Question 12

The graph of the function *f*, together with one of its antiderivative functions is shown below.



The value of $\int_0^1 f(x)dx$ is closest to

A. -2
B. -1
C. 0
D. 1
E. 2

Question 13

For a particular function *f*, $f'(x) < 0$ and $f''(x) > 0$ over its maximal domain.
The rule of the function *f* could be

A. $f(x) = -e^x$
B. $f(x) = \log_e(1-x)$
C. $f(x) = \sqrt{x-1}$
D. $f(x) = \dfrac{1}{e^x}$
E. $f(x) = \log_e(x^2)$

Question 14

A set of linearly dependent vectors are given by

$$\underset{\sim}{a} = 2\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$$

$$\underset{\sim}{b} = \underset{\sim}{i} + x\underset{\sim}{j} - \underset{\sim}{k}$$

$$\text{and} \quad \underset{\sim}{c} = 3\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}$$

where x is a real constant.

The value of x is

A. -1
B. 1
C. 2
D. 5
E. 10

Question 15

The velocity, in ms^{-1}, of a particle initially located at the origin O, is given by

$$\underset{\sim}{v}(t) = \cos(2t)\underset{\sim}{i} + \sin(t)\underset{\sim}{j}$$

where t is time in seconds.

The distance, in metres, of the particle from O at time $t = \pi$ seconds is

A. 0
B. 1
C. 2
D. $\sqrt{3}$
E. $\sqrt{5}$

Question 16

The position of two particles A and B at time t seconds, $t \geq 0$, relative to a fixed origin O is given respectively by

$$\underset{\sim}{r}_A(t) = \underset{\sim}{i} + t^3\underset{\sim}{j}$$

$$\text{and} \quad \underset{\sim}{r}_B(t) = \frac{3t}{t^2 + 2}\underset{\sim}{i} + 8\underset{\sim}{j}$$

The particles will meet at

A. $t = 1$ only
B. $t = 2$ only
C. $t = 1$ and $t = 2$ only
D. $t = 4$ only
E. $t = 1$ and $t = 4$ only

Question 17

Let $\underset{\sim}{a} = \underset{\sim}{i} - x\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{b} = \underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}$.

If the scalar resolute of $\underset{\sim}{a}$ in the direction of $\underset{\sim}{b}$ is $\frac{1}{3}$, then the value of x is

A. -1
B. 1
C. -1 or 1
D. -2 or 2
E. 3

Question 18

The path of a particle is described by $\underset{\sim}{r}(t) = \left(\sqrt{t} + 1\right)\underset{\sim}{i} + (4 - t)\underset{\sim}{j}$ where t represents time in seconds and $t > 0$.

At $t = 4$, the particle crosses the x-axis. The acute angle made with the x-axis by the path of the particle is closest to

A. 14°
B. 21°
C. 22°
D. 68°
E. 76°

Question 19

Force $\underset{\sim}{S}$ has a magnitude of $\sqrt{2}$ newtons and acts in the direction of $\underset{\sim}{i} + \underset{\sim}{j}$.

Force $\underset{\sim}{T}$ has a magnitude of 2 newtons and acts in the direction of $\underset{\sim}{j}$.

These two forces are the only forces acting on a body.
The magnitude of the total force acting on this body in newtons is

A. $\sqrt{2}$
B. $2\sqrt{2}$
C. $\sqrt{5}$
D. 5
E. $\sqrt{10}$

Question 20

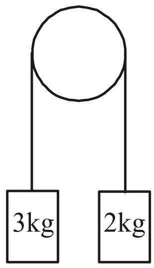
A mass of 2kg is travelling in a straight line and has a momentum of 8kgms^{-1}.
The mass accelerates at a constant rate and 5 seconds later it has a momentum of 28kgms^{-1}.
During this 5 second period the distance, in metres, travelled by the mass is

A. 10
B. 20
C. 25
D. 30
E. 45

Question 21

Two masses of 2kg and 3kg are connected by a light inextensible string that passes over a smooth, light pulley. In ms^{-2}, the acceleration of the 3kg mass downwards is

A. $\frac{-g}{5}$
B. $\frac{g}{5}$
C. $-g$
D. g
E. $5g$



Question 22

A particle moves in a straight line with its position at any time t from a fixed origin given by x, its velocity given by v and its acceleration given by a where $a = f(v)$.
Also, $v = v_0$ when $x = x_0$ and $v = v_1$ when $x = x_1$.
Therefore,

A. $x_1 = \int_{v_0}^{v_1} 1 dv + x_0$

B. $x_1 = \int_{v_0}^{v_1} \frac{1}{f(v)} dv + x_0$

C. $x_1 = \int_{v_0}^{v_1} \frac{v}{f(v)} dv + x_0$

D. $x_1 = v\int_{v_0}^{v_1} \frac{1}{f(v)} dv + x_0$

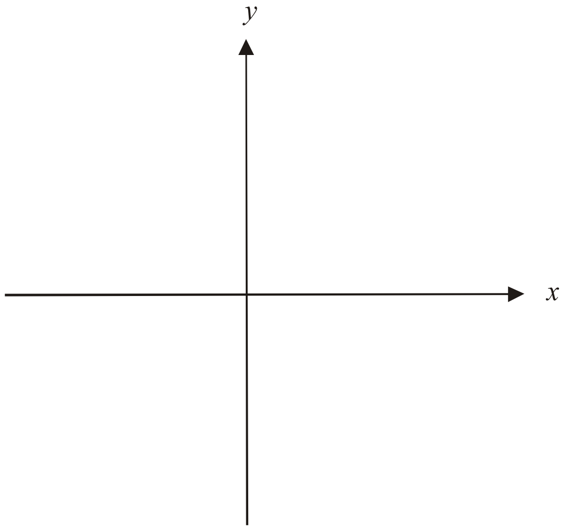
E. $x_1 = \sqrt{2\int_{v_0}^{v_1} \frac{1}{f(v)} dv + x_0{}^2}$

SECTION 2

Question 1

Consider the function with rule $\frac{y^2}{4} - (x-1)^2 = 1$ and range $y > 0$.

a. Sketch the graph of this function on the set of axes below labelling all important features.



3 marks

b. **i.** Find $\frac{dy}{dx}$ in terms of x only.

ii. Hence show that the gradient of the function cannot equal – 2 or 2.

$2+2=4$ marks

c. **i.** Find $\frac{d^2y}{dx^2}$.

ii. Hence explain why the graph of the function $\frac{y^2}{4}-(x-1)^2=1$ has no points of inflection.

$1+1=2$ marks

d. That part of the graph of the function $\frac{y^2}{4}-(x-1)^2=1$ between $x=0$ and $x=3$, is rotated about the x-axis to form a solid of revolution.

i. Write down a definite integral which can be evaluated to find the volume of this solid of revolution.

ii. Evaluate the definite integral found in part i., expressing your answer as an exact value.

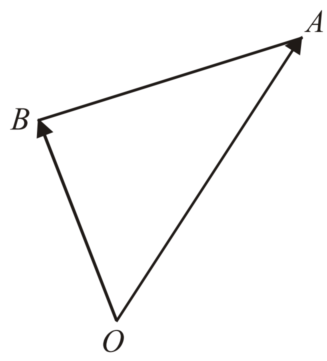
$1+1=2$ marks
Total 11 marks

Question 2

The diagram below shows triangle OAB. With O as origin,

$$\overrightarrow{OA} = 2\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}$$

$$\text{and} \quad \overrightarrow{OB} = -\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}$$



a. Find $\overrightarrow{AB}$.

1 mark

b. Find the cosine of the angle between the vectors $\overrightarrow{BO}$ and $\overrightarrow{BA}$.

1 mark

c. The point Q lies on the line segment OA so that OBQ forms a right-angled triangle. Show that $\overrightarrow{OQ} = \frac{2}{9}(2\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k})$.

3 marks

d. Find the area of triangle OBQ.

2 marks

e. Using your answer to part **b.**, and given that the ratio of the area of ΔOBQ to ΔOBA is $r:s$, find r and s.

3 marks

f. The point M is the midpoint of the line segment OB and the point N is the midpoint of the line segment AB.

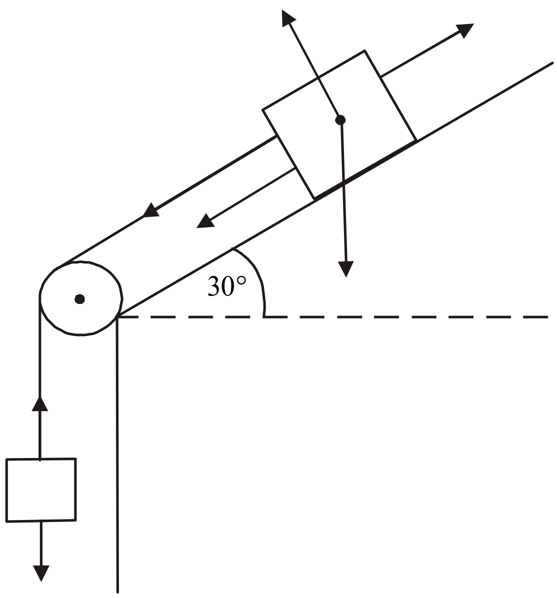
Prove that $\overrightarrow{MN}$ is parallel to $\overrightarrow{OA}$.

2 marks
Total 12 marks

Question 3

A load of mass m kg is connected by a light inextensible string passing over a smooth pulley to a machine of mass $2m$ kg. The machine sits on a rough plane which is inclined at an angle of 30^o to the horizontal. The coefficient of friction between the rough plane and the machine is $\frac{1}{\sqrt{3}}$. The machine is acted on by a pulling force acting up the plane and parallel to it with a magnitude of 735 newtons.

The machine is at the point of moving up the plane.



a. Label the forces shown on the diagram above.

2 marks

b. Show that the mass of the machine is 50kg.

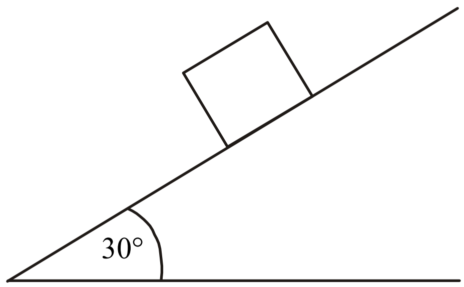
4 marks

c. The pulling force that is acting on the machine is increased to 1000 newtons. Find the acceleration of the machine up the slope. Express your answer correct to two decimal places.

4 marks

d. A power failure occurs so there is no longer a pulling force acting on the machine. At the same time, a safety mechanism detaches the string from the load so that it is no longer connected to the machine.

i. Show the forces now acting on the machine on the diagram below.



ii. Explain the motion, if any, of the machine. Use calculations to support your explanation.

$1 + 2 = 3$ marks
Totals 13 marks

Question 4

A car enters a freeway. It travels along it for a short period before decelerating due to an accident in the same lane further ahead. The car is unable to change lanes and soon comes to rest.
Its velocity v ms^{-1} at time t seconds after entering the freeway is given by

$$v(t)=\begin{cases}\dfrac{-100}{t+10}+30, & t\in[0,35]\\[2ex] \dfrac{-10t+600}{9}, & t\in(35,60]\end{cases}$$

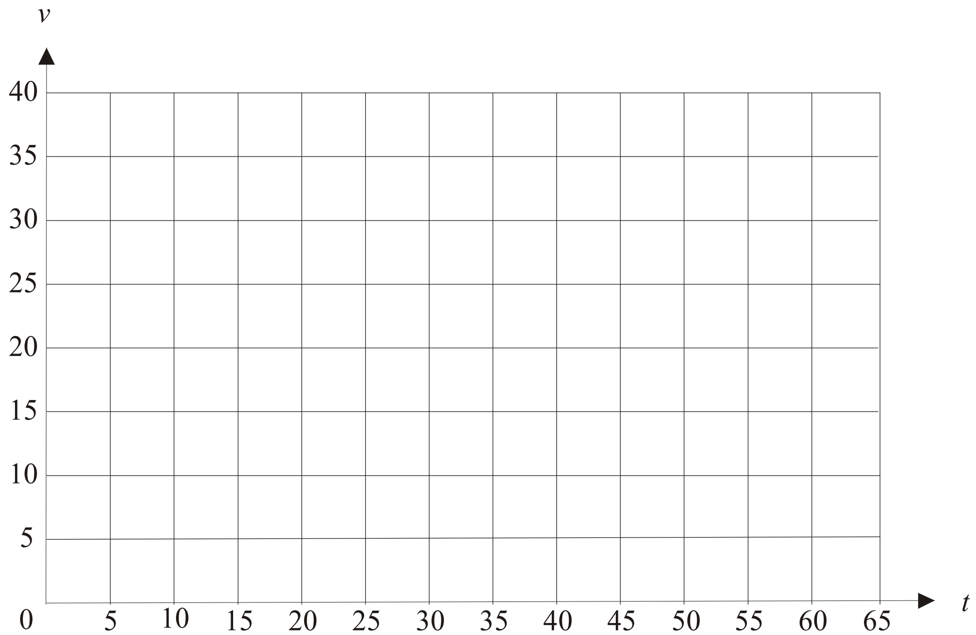
a. Find the velocity of the car, in ms^{-1}, as it enters the freeway.

1 mark

b. Explain why the car will never reach a velocity of 30ms^{-1}.

1 mark

c. On the set of axes below, draw the velocity time graph for the car for $t\in[0,60]$.



2 marks

d. Find the distance, in metres, travelled by the car from the moment it enters the freeway until it comes to rest. Express your answer correct to 1 decimal place.

2 marks

e. Find the average speed of the car. Express your answer in ms^{-1} correct to 1 decimal place.

1 mark

The instant that the car enters the freeway, a truck passes it, travelling a couple of lanes away and at a velocity of k ms^{-1} where k is a constant. The truck is unaffected by the accident and continues to travel at this constant velocity in a line parallel to that of the car along the freeway.

f. Find the value of k for which the truck passes the car at the instant the car comes to rest. Express your answer correct to 1 decimal place.

1 mark

g. Assume that the car comes to rest before the truck passes it.

i. Find the possible values of k.

ii. Let T be the time when the car passes the truck on the freeway. Write down, **but do not solve** an equation involving an integral which when solved for T gives the time when the car passes the truck.

$1+2=3$ marks
Total 11 marks

Question 5

Let $u = \sqrt{3} + i$ where $u \in C$.

a. Express $\overline{u}$ in polar form.

1 mark

b. The complex numbers u and $\overline{u}$ are both solutions to the equation $z^n = -64$ where $z \in C$ and n is a positive integer.

Given that there are no solutions for which $\left|\text{Arg}(z)\right| < \frac{\pi}{6}$,

i. find n

ii. find the other solutions. Express them in polar form.

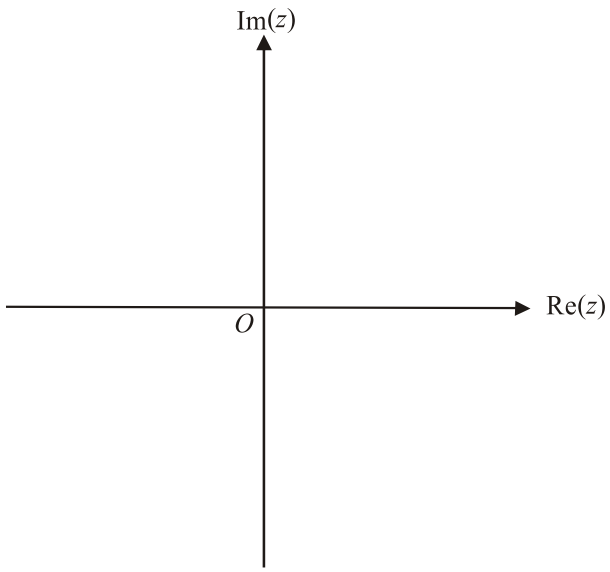
$1 + 4 = 5$ marks

c. Let $w = \text{cis}(\alpha)$, where $w \in C$ and $\alpha \in R$.

Show that $\dfrac{u\,i}{\overline{w}} = 2\,\text{cis}\left(\dfrac{2\pi}{3} + \alpha\right)$

3 marks

d. In the complex plane $S = \{z : z = w\}$ where $z \in C$. Sketch S on the argand diagram below.



2 marks
Total 11 marks

Specialist Mathematics Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc\sin A$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$ $\quad$ $\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ $\quad$ $\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ $\quad$ $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\quad$ $\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$ $\quad$ $\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\operatorname{cis}\theta$

$|z| = \sqrt{x^2+y^2} = r$ $\quad$ $-\pi < \operatorname{Arg} z \le \pi$

$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$ $\quad$ $\frac{z_1}{z_2} = \frac{r_1}{r_2}\operatorname{cis}(\theta_1 - \theta_2)$

$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$
$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$$
$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$
$$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$$
$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$$
$$\int \frac{1}{x}\,dx = \log_e|x| + c$$
$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$$
$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$$
$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$$
$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$$
$$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$$
$$\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$$
$$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$$
$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$$
$$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$$
$$\int \frac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$$
$$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$$
$$\int \frac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Euler's method: If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$,

then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\quad s = ut + \frac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \frac{1}{2}(u+v)t$

Vectors in two and three dimensions

$$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$$

$$\left|\underset{\sim}{r}\right| = \sqrt{x^2 + y^2 + z^2} = r \qquad \underset{\sim}{r_1} . \underset{\sim}{r_2} = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\underset{\sim}{\dot{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$$

Mechanics

momentum:	$\underset{\sim}{p} = m\underset{\sim}{v}$
equation of motion:	$\underset{\sim}{R} = m\underset{\sim}{a}$
friction:	$F \le \mu N$

SPECIALIST MATHEMATICS

TRIAL EXAMINATION 2

MULTIPLE- CHOICE ANSWER SHEET

STUDENT NAME:...

INSTRUCTIONS

Fill in the letter that corresponds to your choice. Example: A ▬ C D E

The answer selected is B. Only one answer should be selected.

1.	A	B	C	D	E	12.	A	B	C	D	E
2.	A	B	C	D	E	13.	A	B	C	D	E
3.	A	B	C	D	E	14.	A	B	C	D	E
4.	A	B	C	D	E	15.	A	B	C	D	E
5.	A	B	C	D	E	16.	A	B	C	D	E
6.	A	B	C	D	E	17.	A	B	C	D	E
7.	A	B	C	D	E	18.	A	B	C	D	E
8.	A	B	C	D	E	19.	A	B	C	D	E
9.	A	B	C	D	E	20.	A	B	C	D	E
10.	A	B	C	D	E	21.	A	B	C	D	E
11.	A	B	C	D	E	22.	A	B	C	D	E