INSIGHT

YEAR 12 *Trial Exam Paper*

2011
SPECIALIST MATHEMATICS
UNIT 4

Written examination 2

QUESTION AND ANSWER BOOK

STUDENT NAME:

Reading time: 15 minutes
Writing time: 2 hours
Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
		Total	80

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, once bound reference, one approved graphics calculator or CAS (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring sheets of paper or white out liquid/tape into the examination.

Materials provided
- The question and answer book of 29 pages with a separate sheet of miscellaneous formulas.
- Answer sheet for multiple-choice questions.

Instructions
- Write your **name** in the box provided and on the multiple-choice answer sheet.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

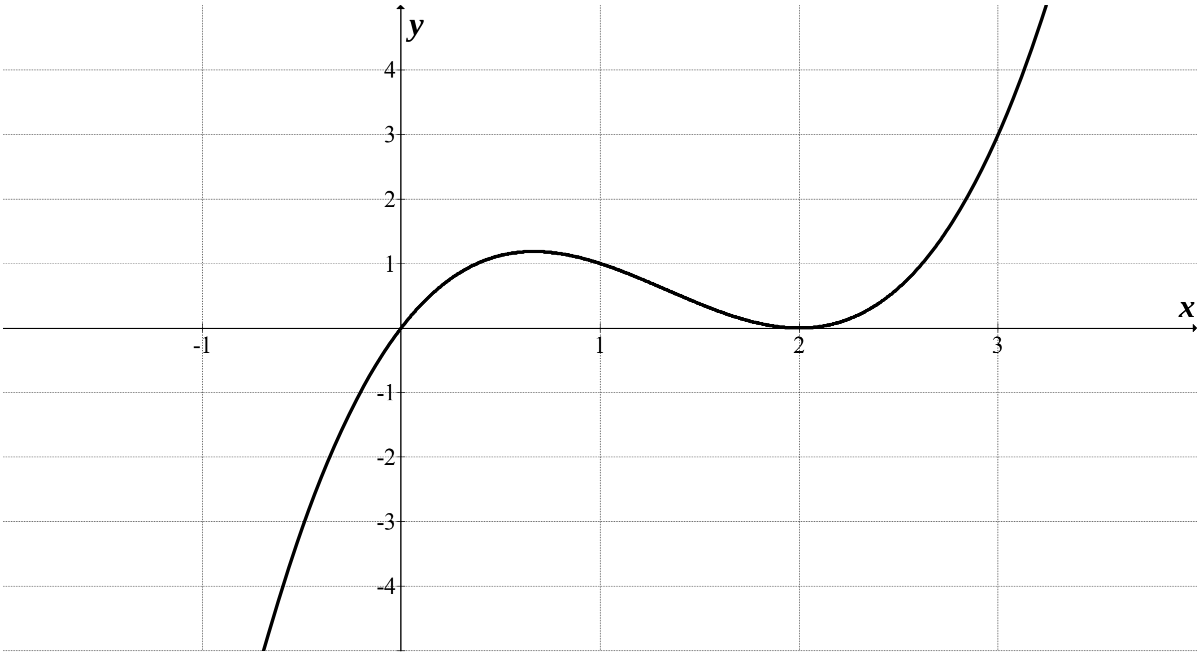
At the end of the exam
- Place the multiple-choice answer sheet inside the front cover of this book.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

Section 1

Instructions for Section 1
Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

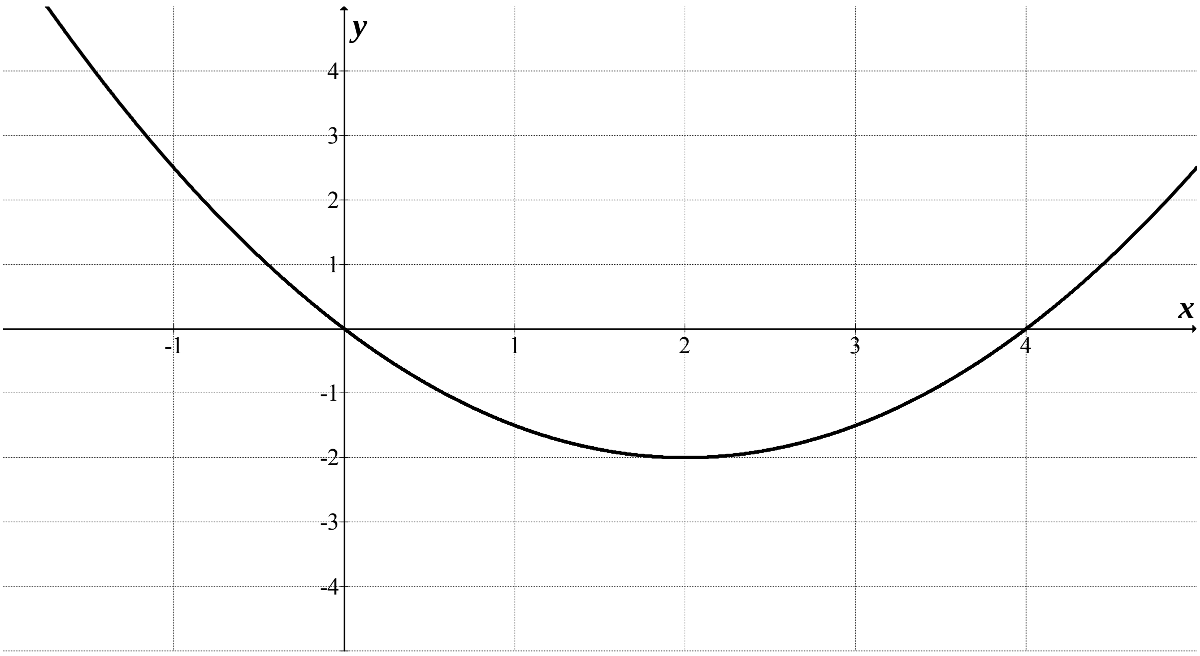
Question 1
The graph of $y = f(x)$ is shown below.



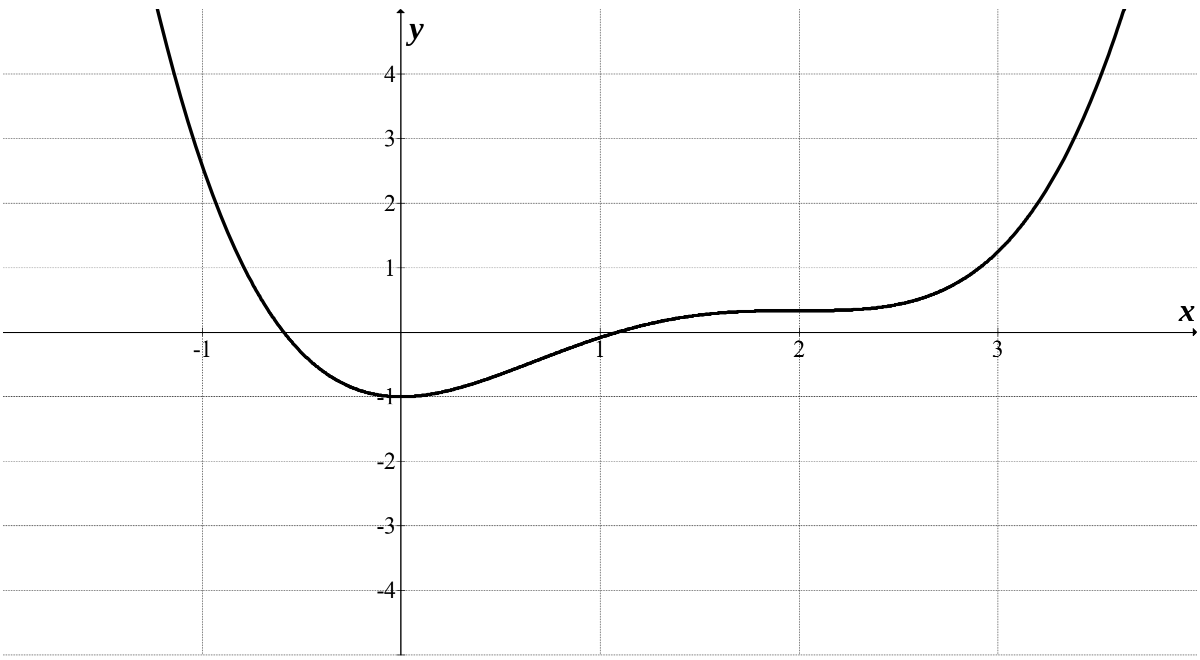
SECTION 1 – Question 1 – continued

Which of the following graphs could be its antiderivative $F(x)$?

A.



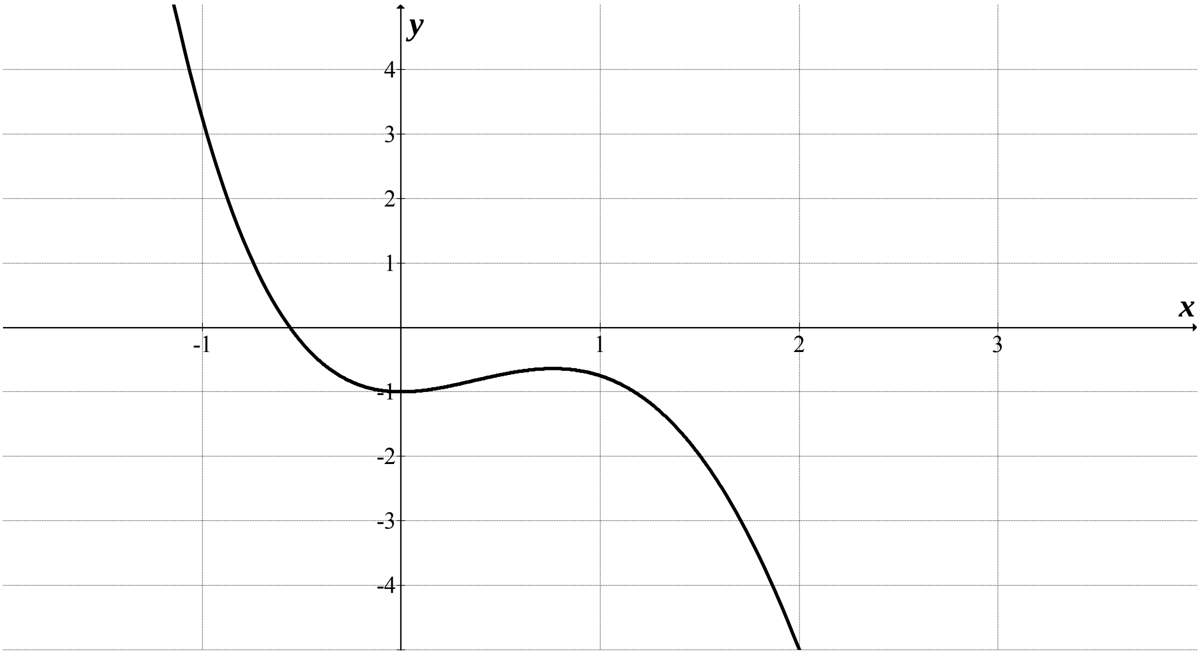
B.



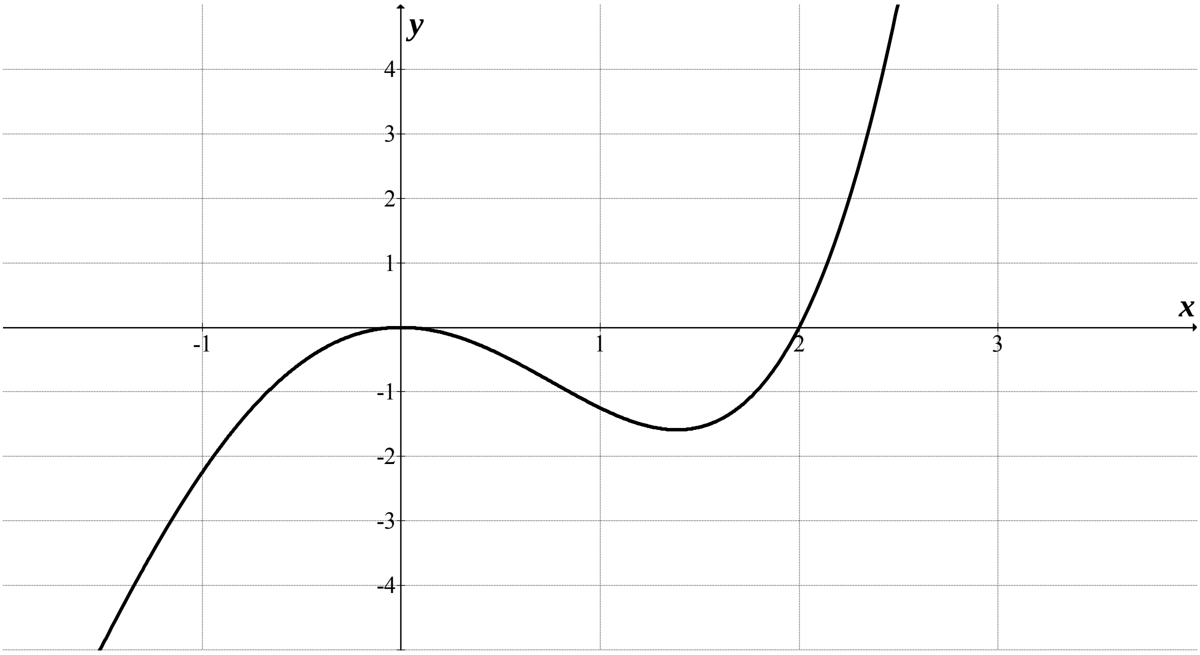
SECTION 1 – Question 1 – continued

TURN OVER

C.

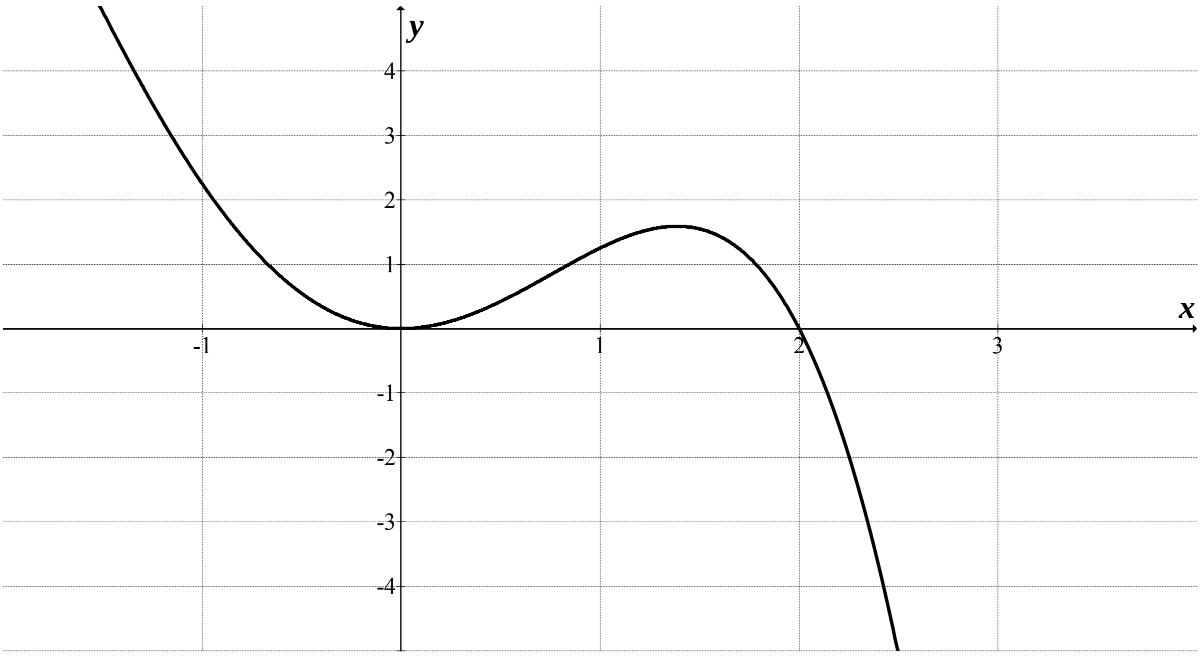


D.



SECTION 1 – Question 1 – continued

E.



Question 2

The graph of the function $f(x) = ax + \frac{b}{x^2}$, where $a < 0$ and $b > 0$, must have

A. one asymptote and a local minimum at $x = \left(\frac{b}{2a}\right)^{\frac{1}{3}}$

B. one asymptote and a local maximum at $x = \left(\frac{b}{2a}\right)^{\frac{1}{3}}$

C. two asymptotes and a local minimum at $x = \left(\frac{b}{2a}\right)^{\frac{1}{3}}$

D. two asymptotes and a local minimum at $x = \left(\frac{2b}{a}\right)^{\frac{1}{3}}$

E. two asymptotes and a local maximum at $x = \left(\frac{b}{2a}\right)^{\frac{1}{3}}$

SECTION 1 – continued
TURN OVER

Question 3

Which one of the following relations has a pair of asymptotes with an angle of $\frac{\pi}{3}$ between them?

A. $2(x-2)^2-(y+1)^2=1$

B. $3(x-2)^2-(y+1)^2=1$

C. $(x-2)^2-3(y+1)^2=3$

D. $3(x-2)^2+(y+1)^2=3$

E. $x^2-4x-2y^2+1=0$

Question 4

The position vectors of two particles, P and Q, are given by
$\underset{\sim}{p}=(5t-3)\underset{\sim}{i}+t^2\underset{\sim}{j}$ and $\underset{\sim}{q}=7\underset{\sim}{i}+(5t-6)\underset{\sim}{j},\ t\geq 0.$

The two particles will collide when

A. $t=3$

B. $t=6$

C. $t=2$ and $t=3$

D. $t=2$

E. $t=2$ and $t=6$

Question 5

The velocity of an object at time t seconds is given by $\underset{\sim}{v}(t)=2t\underset{\sim}{i}+5\underset{\sim}{j}-\frac{4}{(t+1)^2}\underset{\sim}{k}$.

When $t=1$ the position of the particle is $\underset{\sim}{r}(t)=2\underset{\sim}{i}+3\underset{\sim}{j}+\underset{\sim}{k}$.

The initial position of the object is given by

A. $\underset{\sim}{i}+2\underset{\sim}{j}-\underset{\sim}{k}$

B. $2\underset{\sim}{i}+3\underset{\sim}{j}-6\underset{\sim}{k}$

C. $\underset{\sim}{i}-2\underset{\sim}{j}-3\underset{\sim}{k}$

D. $2\underset{\sim}{i}+2\underset{\sim}{j}-\underset{\sim}{k}$

E. $\underset{\sim}{i}-2\underset{\sim}{j}+3\underset{\sim}{k}$

Question 6

The value of the constant p for which the vectors $\underset{\sim}{u} = 2\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k}$, $\underset{\sim}{v} = -\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$ and $\underset{\sim}{w} = p\underset{\sim}{i} + 3\underset{\sim}{j} - 3\underset{\sim}{k}$ are linearly dependent is

A. □1
B. 1
C. 0
D. 2
E. □2

Question 7

The vector resolute of $\underset{\sim}{a} = 2\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ that is perpendicular to $\underset{\sim}{b} = 3\underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k}$ is

A. $\frac{1}{3}(2\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k})$

B. $\frac{7}{17}(4\underset{\sim}{i} - 3\underset{\sim}{j} + 3\underset{\sim}{k})$

C. $\frac{2\sqrt{17}}{17}$

D. $\frac{2}{17}(3\underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k})$

E. $\frac{2}{\sqrt{17}}(3\underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k})$

Question 8

The graph of the function $f : [0, \infty) \to R,\ f(x) = \sec(ax),\ a > 0$ has asymptotes located at

A. $x = 0, \frac{\pi}{a}, \frac{2\pi}{a}, \frac{3\pi}{a}, ...$

B. $x = 0, \frac{\pi}{2a}, \frac{3\pi}{2a}, \frac{5\pi}{2a}, ...$

C. $x = ..., \frac{-5\pi}{2a}, \frac{-3\pi}{2a}, \frac{-\pi}{2a}$

D. $x = \frac{\pi}{2a}, \frac{3\pi}{2a}, \frac{5\pi}{2a}, ...$

E. $x = \frac{\pi}{a}, \frac{2\pi}{a}, \frac{3\pi}{a}, ...$

SECTION 1 – continued
TURN OVER

Question 9

For $\frac{-\pi}{2} < x < \frac{\pi}{2}$, the graphs of the two curves given by $y = \text{cosec}^2(x)$ and $y = 2|\cot(x)|$ intersect at

A. the point $\left(\frac{\pi}{4}, 1\right)$ only

B. the two points $\left(\frac{-\pi}{4}, 2\right)$ and $\left(\frac{\pi}{4}, 2\right)$

C. the point $\left(\frac{-\pi}{4}, -2\right)$ only

D. the point $\left(\frac{\pi}{4}, 2\right)$ only

E. the two points $\left(\frac{-\pi}{6}, 1\right)$ and $\left(\frac{\pi}{6}, 1\right)$

Question 10

Given that $z = 2\,\text{cis}\left(\frac{\pi}{3}\right)$ and $w = 5\,\text{cis}\left(\frac{-\pi}{6}\right)$, it follows that $\text{Arg}\left(z^3(\overline{w})^4\right)$ is

A. $-\pi$

B. $\frac{2\pi}{3}$

C. $\frac{5\pi}{3}$

D. $\frac{-\pi}{3}$

E. $\frac{\pi}{3}$

Question 11

If $z_1 = \sqrt{3} + i$ and $z_2 = 1 - i$, then $\frac{z_1}{z_2}$ is equal to

A. $\sqrt{2}\,\text{cis}\left(\frac{5\pi}{12}\right)$

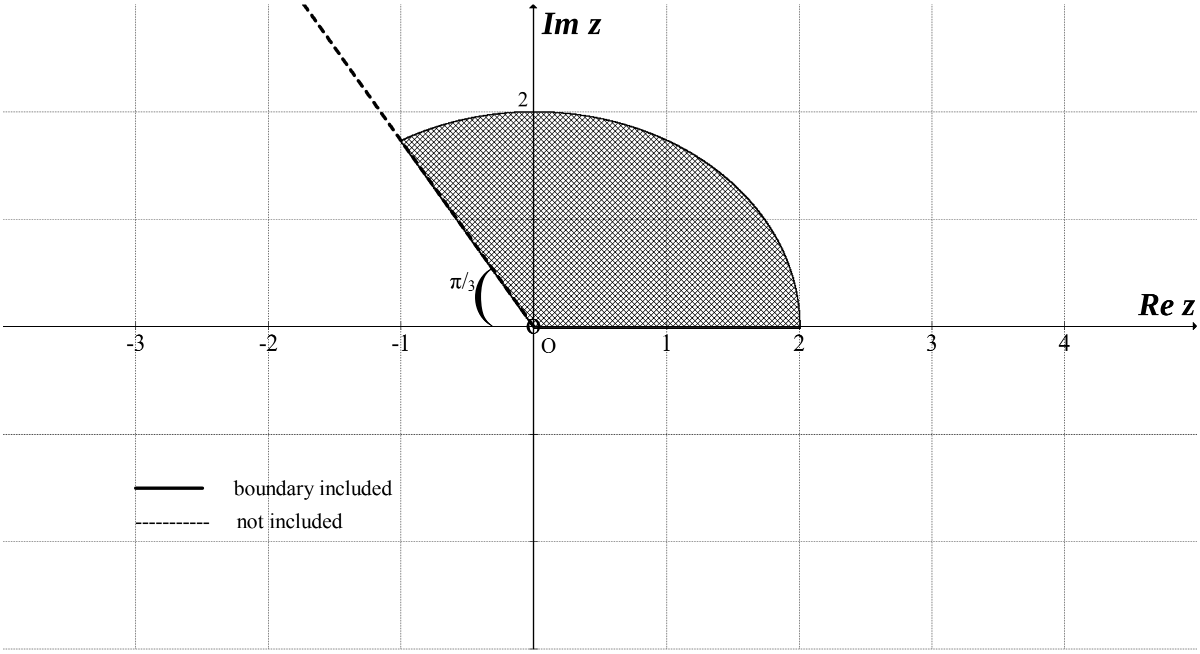
B. $\sqrt{2}\,\text{cis}\left(\frac{\pi}{12}\right)$

C. $\frac{1}{\sqrt{2}}\text{cis}\left(\frac{5\pi}{12}\right)$

D. $\frac{1}{\sqrt{2}}\text{cis}\left(\frac{\pi}{12}\right)$

SECTION 1 – continued

Question 12



The shaded region on the Argand diagram above can be described by

A. $\left\{z : Arg(z) > \frac{\pi}{3}\right\} \cup \{z : |z| \leq 2\}$

B. $\left\{z : 0 < Arg(z) < \frac{2\pi}{3}\right\} \cup \{z : |z| \leq 2\}$

C. $\left\{z : 0 < Arg(z) < \frac{2\pi}{3}\right\} \cap \{z : |z| \leq 2\}$

D. $\left\{z : 0 \leq Arg(z) < \frac{2\pi}{3}\right\} \cap \{z : |z| < 2\}$

E. $\left\{z : 0 \leq Arg(z) < \frac{2\pi}{3}\right\} \cap \{z : |z| \leq 2\}$

Question 13
On an Argand diagram, a set of points that lie on a circle of radius 4 centred at (1, 0) is

A. $\{z : (z-1)(\overline{z}-1) = 16\}$

B. $\{z : (z-1)^2 = 4\}$

C. $\{z : (z-1)(\overline{z}-1) = 4\}$

D. $\{z : z\overline{z} = 4\}$

E. $\{z : (\text{Re}(z))^2 + (\text{Im}(z))^2 = 4\}$

Question 14

If $f'(t) = e^{t^2}$ and $f(0) = 3,$ then the solution to the differential equation when $t = 2$ can be found by evaluating

A. $\int_0^2 e^{t^2}.dt + 3$

B. $\int_0^2 e^{t^2}.dt - 3$

C. $\int_0^2 2te^{t^2}.dt + 3$

D. $\int_0^2 \left(e^{t^2} + 3\right).dt$

E. $\int_0^3 e^{t^2}.dt - 2$

Question 15

Given $\frac{dy}{dx} = \frac{2x}{y+2}$ and $\left(x_0, y_0\right) = (1, 2).$

Using Euler's rule with a step size of 0.2, the value of y_2, correct to 3 decimal places, is

A. 2.100
B. 2.060
C. 2.200
D. 2.217
E. 2.211

Question 16

The temperature inside a kitchen is 22°C. A cup of soup is removed from the microwave and stirred evenly when its temperature is 70°C. The rate at which the temperature of the soup drops is proportional to the excess of its temperature above its surrounding air temperature.

If *T* is the temperature of the soup at time *t* minutes after it is removed from the microwave and *k* is a positive constant, a differential equation relating *T* and *t* is

A. $\frac{dT}{dt} = k(T - 22),\ T(0) = 70$

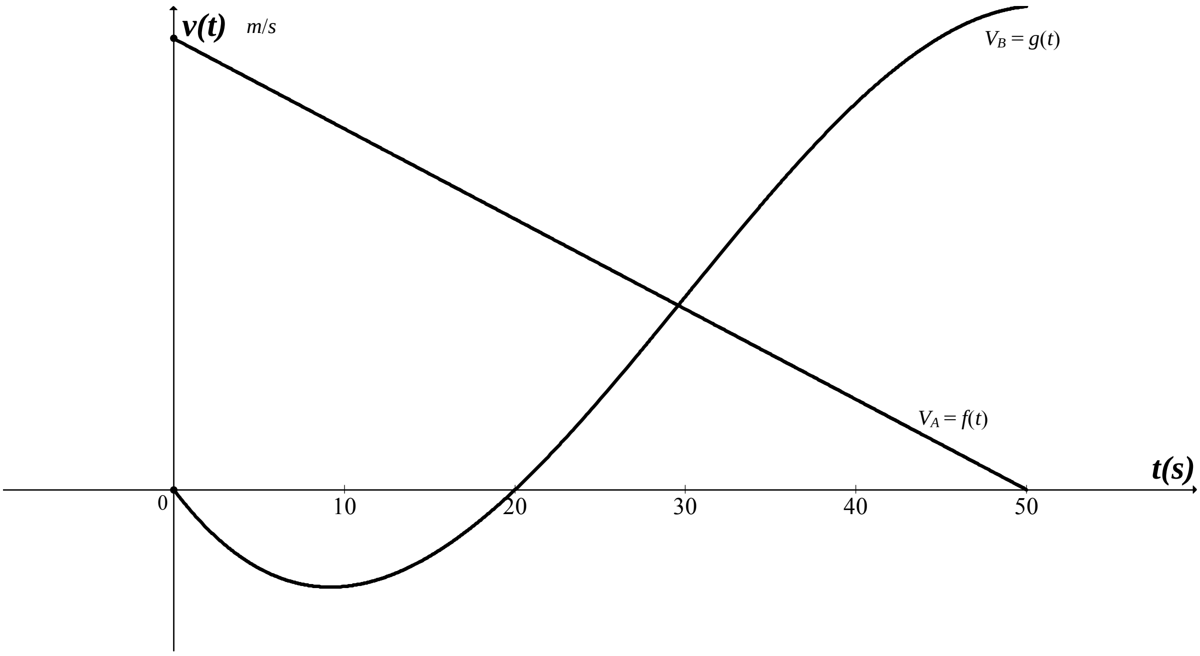
B. $\frac{dT}{dt} = k(T - 70),\ T(0) = 22$

C. $\frac{dT}{dt} = -k(T - 70),\ T(0) = 22$

D. $\frac{dT}{dt} = -kT - 22,\ T(0) = 70$

E. $\frac{dT}{dt} = -k(T - 22),\ T(0) = 70$

Question 17



Two particles, A and B, move from the same point in a straight line with their respective velocities $V_A = f(t)$ and $V_B = g(t)$ in m/s, where t is in seconds and $t \geq 0$.
The velocity□time graphs for the particles is shown above.

At $t = 50$ seconds, the distance, in metres, between the two particles is

A. $\left|\int_0^{50} g(t).dt - 50f(t)\right|$

B. $\left|\int_{20}^{50} g(t).dt - \int_0^{20} g(t).dt\right| - 50f(t)$

C. $\left|\int_{20}^{50} g(t).dt - \int_0^{20} g(t).dt - 50f(t)\right|$

D. $\left|\int_{30}^{50} g(t).dt + \int_0^{30} g(t).dt - 50f(t)\right|$

E. $\left|\int_0^{50} (g(t) - f(t)).dt\right|$

Question 18

Using an appropriate substitution, $\int_0^{\frac{\pi}{3}} \cot(x)\sec^2(x).dx$ can be expressed as

A. $\int_0^{\frac{1}{2}} \frac{du}{u}$

B. $\int_0^{\sqrt{3}} \frac{du}{u}$

C. $\int_0^{\frac{\sqrt{3}}{2}} \frac{1-u^2}{u}.du$

D. $-\int_0^{\sqrt{3}} u.du$

E. $\int_0^{\sqrt{3}} u.du$

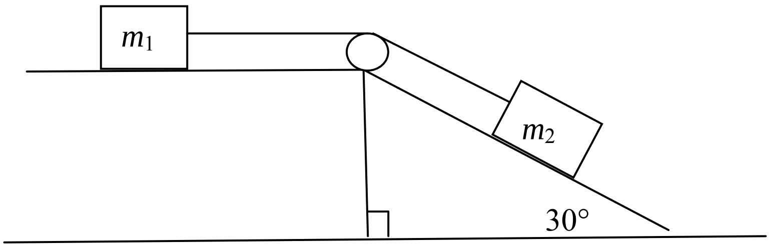
Question 19

A mass is travelling in a straight line at 6 m/s and has a momentum 24 kg m/s. A force acts on the mass in the same direction of motion so that the acceleration of the mass is 0.25 m/s^2. When it has a momentum of 32 kg m/s, the mass has travelled a distance of

A. 14 metres
B. 96 metres
C. 24 metres
D. 28 metres
E. 56 metres

Question 20

A mass of m_1 kg rests on a rough horizontal surface and is connected to a second mass m_2 kg, which is twice the mass of m_1. The second mass, m_2, rests on a rough surface inclined at an angle of 30° to the horizontal. The two masses are connected by a light, inextensible string that passes over a smooth pulley. The coefficient of friction between each mass and their surface is the same. The mass m_2 is on the verge of sliding down the plane.



The value of the coefficient of friction μ is

A. $\dfrac{1}{\sqrt{3}-2}$

B. $\dfrac{1+\sqrt{3}}{2}$

C. $\dfrac{1}{1+\sqrt{3}}$

D. $\dfrac{2}{\sqrt{3}-1}$

E. $\dfrac{1}{1-\sqrt{3}}$

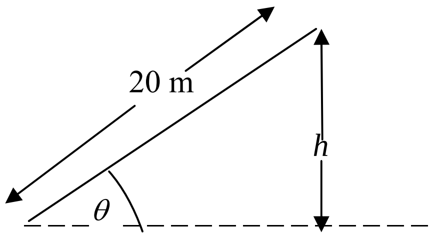
Question 21

An object moves in a straight line with velocity, in m/s, $v = \dfrac{4}{t+1}, t \geq 0$.

Which one of the following statements about the object's motion is **false**?

A. The object's acceleration is always negative.
B. The object has an initial acceleration of □4 m/s^2.
C. The distance travelled by the object in the first 3 seconds of motion is 4 $\log_e$ 4 metres.
D. The object has an initial velocity of 4 m/s.
E. The object gradually increases its non-zero initial velocity.

Question 22
A crane of length 20 metres makes an angle of θ radians with the horizontal ground as it is being raised. The height of the end of the crane above the ground is *h* metres.



The crane is being raised at a rate of 0.08 radians/second. The rate at which the height of the crane is increasing when $\theta = \dfrac{\pi}{4}$ is

A. $\dfrac{4\sqrt{2}}{5}$ **m/s**

B. $\dfrac{\sqrt{2}}{5}$ m/s

C. $\dfrac{8\sqrt{2}}{5}$ m/s

D. $\dfrac{2\sqrt{2}}{5}$ m/s

E. $\dfrac{8}{5}$ m/s

END OF SECTION 1

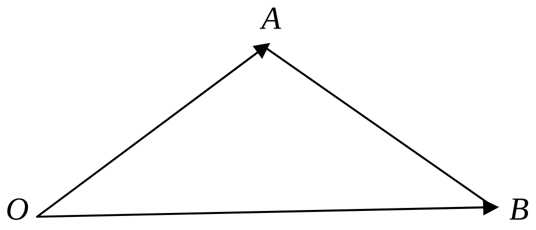
END OF SECTION 1

Section 2

Instructions for Section 2
Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

In the triangle below $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OB} = \underset{\sim}{b}$ and $|\underset{\sim}{b}| = 2|\underset{\sim}{a}|$.



a. If $\underset{\sim}{a}.\underset{\sim}{b} = a^2$, find the size of $\angle AOB$.

2 marks

b. Express $\overrightarrow{AB}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

Hence use a vector method to show that $\angle OAB$ is equal to 90°.

2 marks

SECTION 2 – Question 1 – continued
TURN OVER

c. If M is the midpoint of $\overrightarrow{OB}$, express $\overrightarrow{MA}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. **Hence** use a vector method to show that triangle OAM is equilateral.

2 marks

Three points, P, Q and R, have their respective coordinates given by (0, 1, 1), (1, 3, −1) and (2, 2, 5).

d. Find the coordinates of a point M on $\overrightarrow{PR}$ that is closest to the point Q.

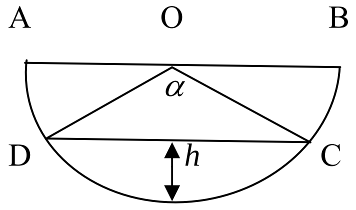
4 marks

Total 2 + 2 + 2 + 4 = 10 marks

Question 2

A relaxation bath is 1 metre long and has a semi-circular cross-section of radius 1 metre, as shown below.

AB is the diameter of the semi-circle, the angle $DOC = \alpha$ radians and *h* is the depth of water in the bath.



a. Show that the height of water in the bath is $h = 1 - \cos\left(\frac{\alpha}{2}\right)$.

1 mark

b. Show that the volume of water in the tank is $V = 0.5(\alpha - \sin\alpha)$.

1 mark

c. Water is pumped into the bath at a rate of 0.5 m^3 per minute. Find the exact rate at which the water level is rising when the depth of water in the bath is 0.5 metres.

4 marks

d. When there is 2000 litres of pure water in the bath, an Epsom salt solution of concentration 0.08 kg per litre is pumped in at a rate of 50 litres per minute. The mixture is kept uniform and at the same time the bath is drained at a rate of 50 litres per minute.

i. What is the limit to the amount of Epsom salt in the bath water?

1 mark

ii. Show that $\frac{dx}{dt} = \frac{160 - x}{40}$, where x is the amount of Epsom salt in the bath after t minutes.

1 mark

iii. **Hence**, show that $x = 160\left(1 - e^{\frac{-t}{40}}\right)$.

3 marks
1+1+3=5 marks

e. From the time the mixing began, how long will it take for the concentration of Epsom salt in the bathwater to reach 0.05 kg per litre? Give your answer to the nearest tenth of a minute.

1 mark
Total 1+1+4+5+1=12 marks

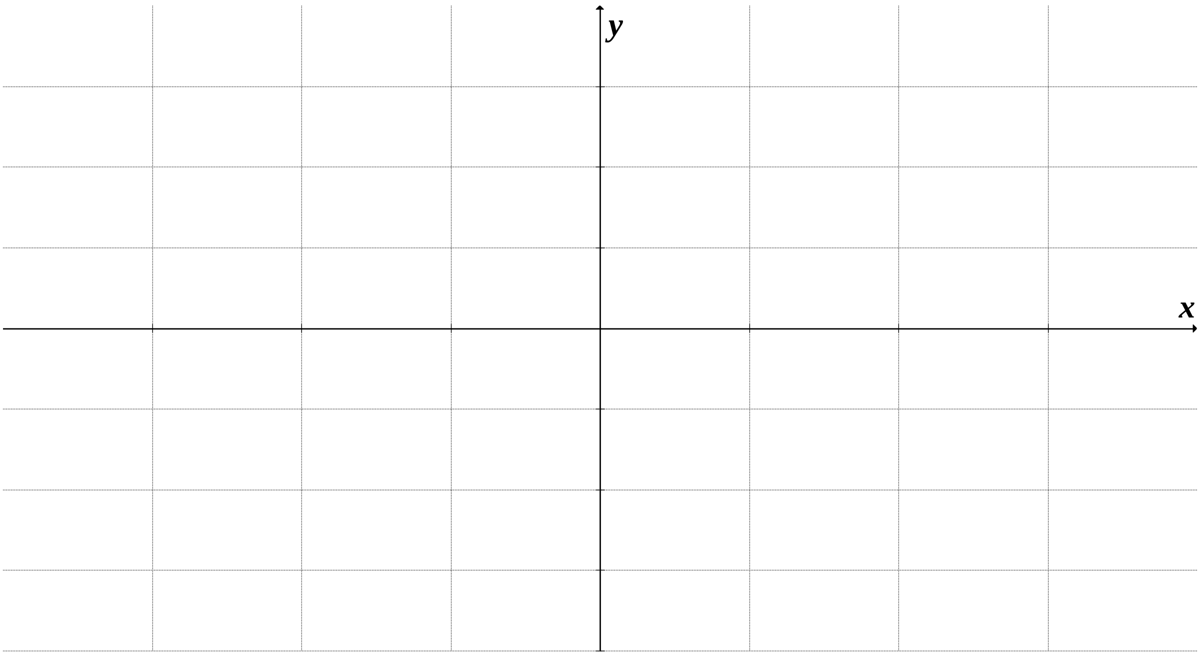
Question 3

The position vector of a particle at any time $t \geq 0$ is $\underset{\sim}{r}(t) = t\,\underset{\sim}{i} + \tan^{-1}\left(\frac{t}{2}\right)\underset{\sim}{j}$.

a. Determine the Cartesian equation of the path of the particle.

1 mark

b. Sketch the graph on the set of axes below, labelling any asymptotes and the coordinates of any end points.



2 marks

SECTION 2 – Question 3 – continued

c. **i.** On the same set of axes given in part **b**, shade the area between the graph, the x-axis and the line $x = 2$.

1 mark

ii. Use Calculus to determine the exact area between the graph, the x-axis and the line $x = 2$. Express your answer in the form $a + \log_e b$, where $a, b \in R$.

4 marks

1+4 = 5 marks

d. Use Calculus to determine the exact volume generated when the area between the graph, the x-axis and the line $x = 2$ is rotated around the y-axis.

3 marks

e. A normal line to the curve intersects with the x-axis at $x = 2\sqrt{3} + \frac{\pi}{24}$. Find the coordinates of the point on the curve through which this normal line passes.
Give your answer correct to 3 decimal places.

4 marks
Total 1 + 2 + 5 + 3 + 4 = 15 marks

Question 4

A curve in the complex plane has its rule given by

$T = \{z : |z-5| - |z-1| = 2,\ z \in C\}$

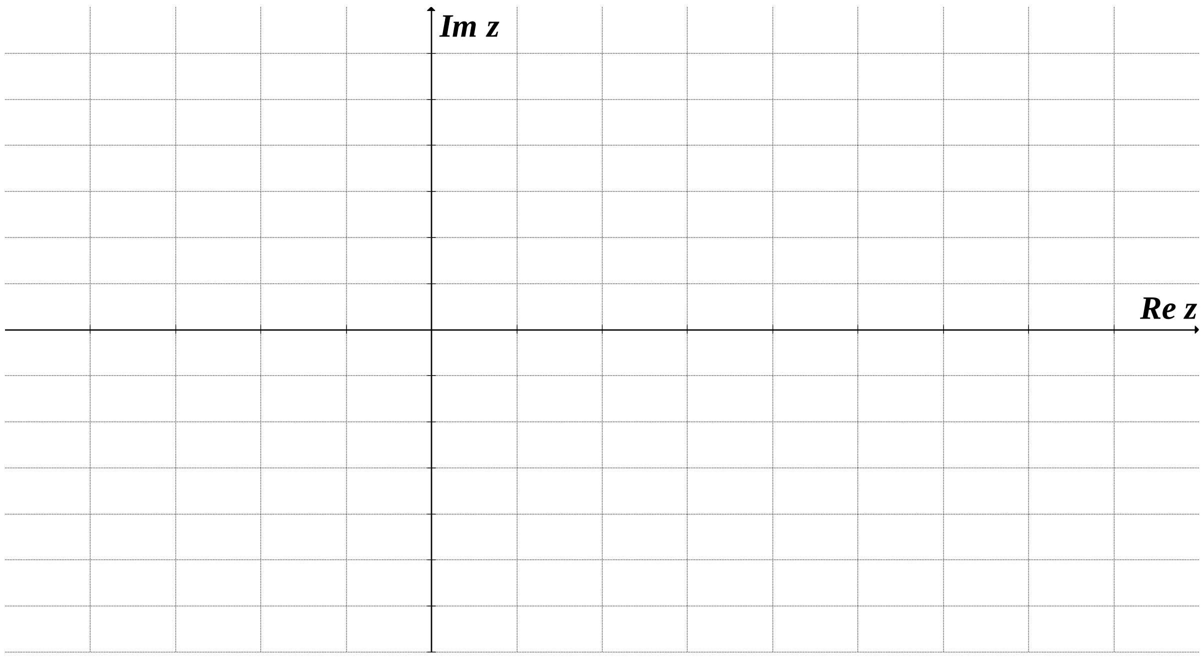
a. Show that the points $z = 2$ and $z = 1 + 3i$ belong to T.

2 marks

b. Find the exact values of b if $z = bi$ also belongs to T.

2 marks

c. Sketch the graph of T on the complex plane below. Clearly show all axes intercepts. (Do not show any asymptotes.)



2 marks

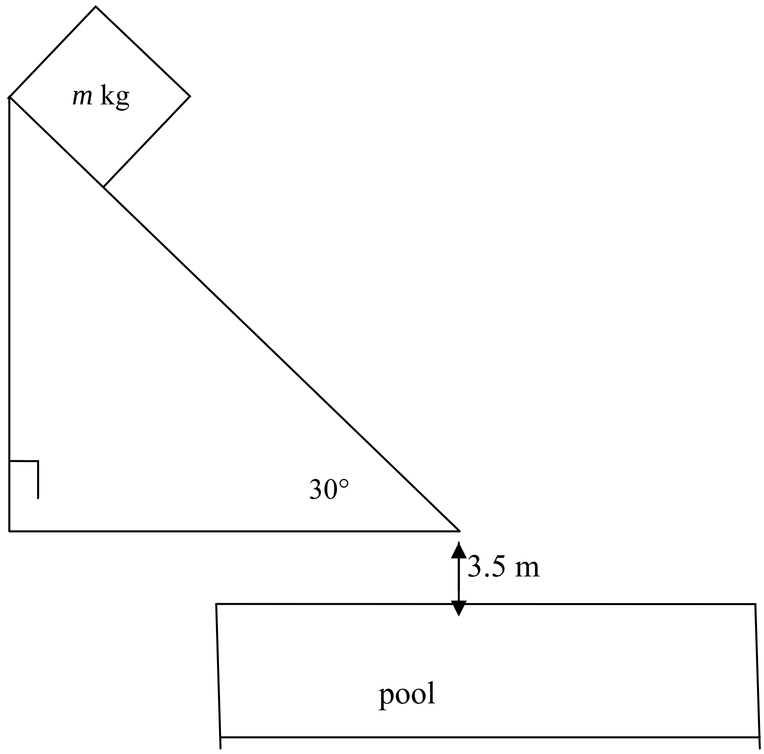
d. Express the relation defined by T in Cartesian form.

3 marks
Total 2 + 2 + 2 + 3 = 9 marks

Question 5

Daredevil Park has a waterslide into a deep pool. The 12.5 metre slide is inclined at 30° to the horizontal and the bottom of the slide is 3.5 metres above the water surface of the pool.

Albert, who is of mass *m* kg, positions himself on the slide at the top and is supported by his friend Charlie. When the applied force acting directly up the slide is $12g$ newtons, Albert is on the verge of moving down the slide. The coefficient of friction between Albert and the slide is $\frac{\sqrt{3}}{5}$. The situation is shown on the diagram below.



a. **i.** On the diagram above, label all the forces acting on Albert while he is being held at the top of the slide.

1 mark

ii. **Hence**, calculate the value of m.

3 marks
1+3=4 marks

Suddenly, Charlie releases his hold on Albert, who moves from rest down the slide, reaching the end of the slide after travelling a distance of 12.5 metres.

b. Show that the acceleration of Albert while he is on the slide is $\frac{g}{5}$ m/s^2.

3 marks

c. Show that Albert reaches the end of the slide with a speed of 7 m/s.

1 mark

Take the bottom end of the slide as the origin, $\underset{\sim}{i}$ as the unit vector horizontally to the right and $\underset{\sim}{j}$ as the unit vector vertically up.

d. Find the velocity vector $\underset{\sim}{v}(t)$ which represents the velocity of Albert *t* seconds after he leaves the slide.

2 marks

SECTION 2 – Question 5 – continued

e. Calculate Albert's speed 0.5 seconds after he reaches the bottom of the slide.
Give your answer to the nearest tenth of a second.

2 marks
Total 4 + 3 + 1 + 2 + 2 = 12 marks

END OF SECTION 2

END OF QUESTION AND ANSWER BOOK