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***Specialist  
Mathematics***

***2011***

***Trial Examination 1***

## Instructions

Answer **all** questions. Do **not** use calculators.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working or explanation **must** be shown.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

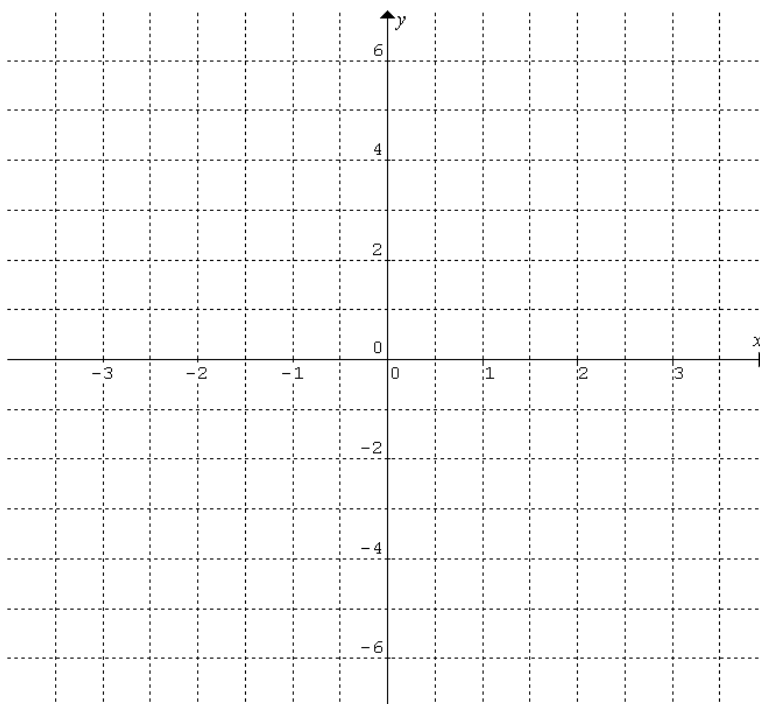
Take the **acceleration due to gravity** to have magnitude  $g \text{ ms}^{-2}$ , where  $g = 9.8$ .

### Question 1

a. Sketch the graph of  $y = \sqrt{3} \cos^{-1}\left(\frac{x}{2}\right)$ . Clearly label the end-points and axis-intercepts with exact coordinates.

(Use  $\pi \approx 3.14$  and  $\sqrt{3} \approx 1.73$  to help you to sketch)

2 marks



b. Find the exact value of  $\int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{3} \cos^{-1}\left(\frac{x}{2}\right) dx$ .

2 marks

### Question 2

Let  $A(-2,1,0)$ ,  $B(-1,2,-2)$  and  $C(0,-3,4)$  be three points in 3-dimensional space.

a. Determine whether  $A$ ,  $B$  and  $C$  are *collinear* or not.

2 marks

b. Determine whether the position vectors of  $A$ ,  $B$  and  $C$  are *coplanar* or not.

2 marks

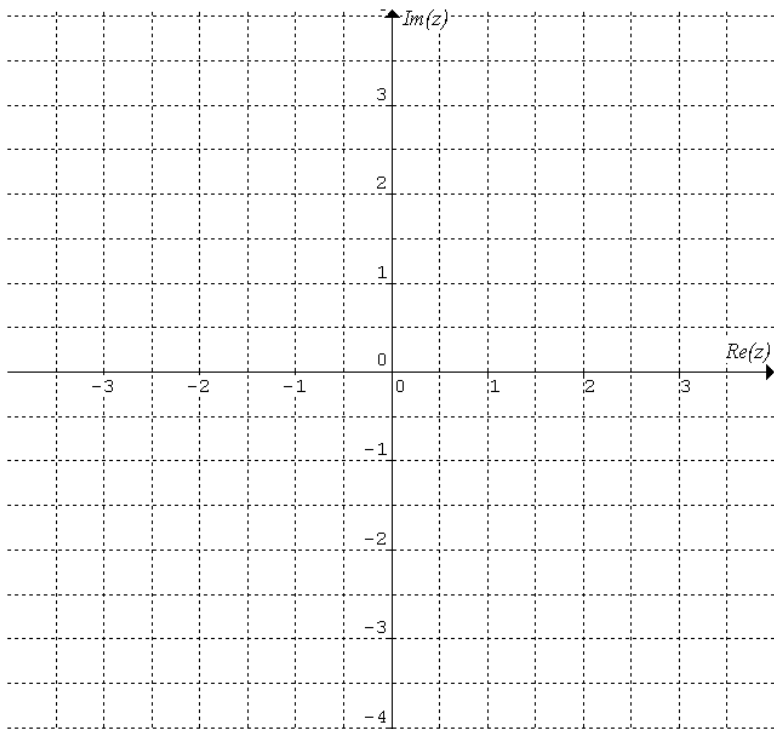
c. Find a vector that bisects the angle between the position vectors of  $B$  and  $C$ .

2 marks

### Question 3

Shade clearly the region in the complex plane for  $\{z : 1 \leq z + \bar{z} \leq 2\} \cap \{z : 1 \leq |z - 1 + i| \leq 2\}$ .

3 marks



**Question 4**

Let  $z = \cos \theta + i \sin \theta$ .

a. Find  $|iz - z|$ . 1 mark

b. Find  $\arg(iz - z)$ . 2 marks

c. Hence, or otherwise, show that  $\cos \theta - \sin \theta = \sqrt{2} \sin\left(\frac{3\pi}{4} + \theta\right)$ . 1 mark

**Question 5**

The position of a particle is given by  $\tilde{r} = \frac{t}{2} \tilde{i} + (49t - 4.9t^2) \tilde{j}$ , where time  $t \geq 0$  is measured in seconds and distance is in metres.  $\tilde{i}$  is a horizontal unit vector to the right and  $\tilde{j}$  a vertical vector pointing upward.

a. Find the initial speed (correct to the nearest whole number) of the particle. 1 mark

b. Show that the acceleration of the particle is constant. 1 mark

c. Find the horizontal displacement when the particle returns to the same level as its initial position. 1 mark

d. Find the cartesian equation of the path of the particle's motion. 1 mark

**Question 6**

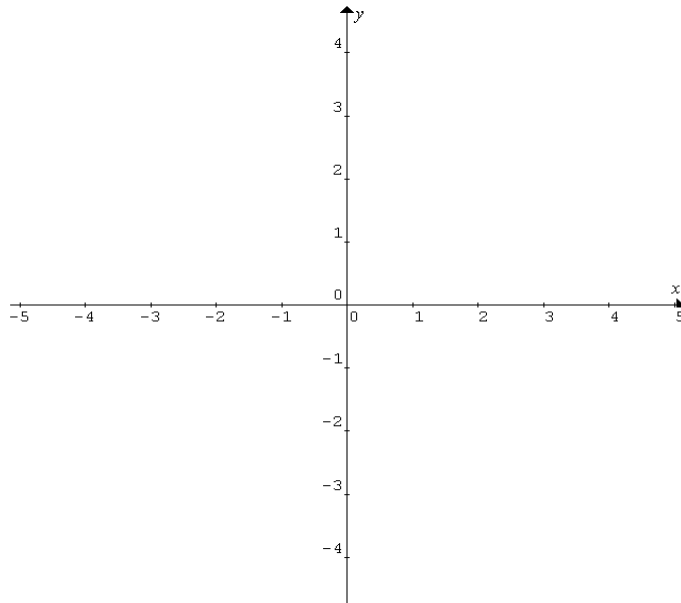
$y$  is a differentiable function of  $x$  and satisfies  $8x^3 - y^3 = 1 - 6xy^2 + 12x^2y$ .

a. Find  $\frac{dy}{dx}$ .

2 marks

b. Sketch the graph of  $8x^3 - y^3 = 1 - 6xy^2 + 12x^2y$ .

2 marks

**Question 7**

Let  $f(x) = \frac{\log_e(x^2)}{|x|}$  where  $x \in \mathbb{R} \setminus \{0\}$ .

a. Find the exact value of  $f'(-e)$ .

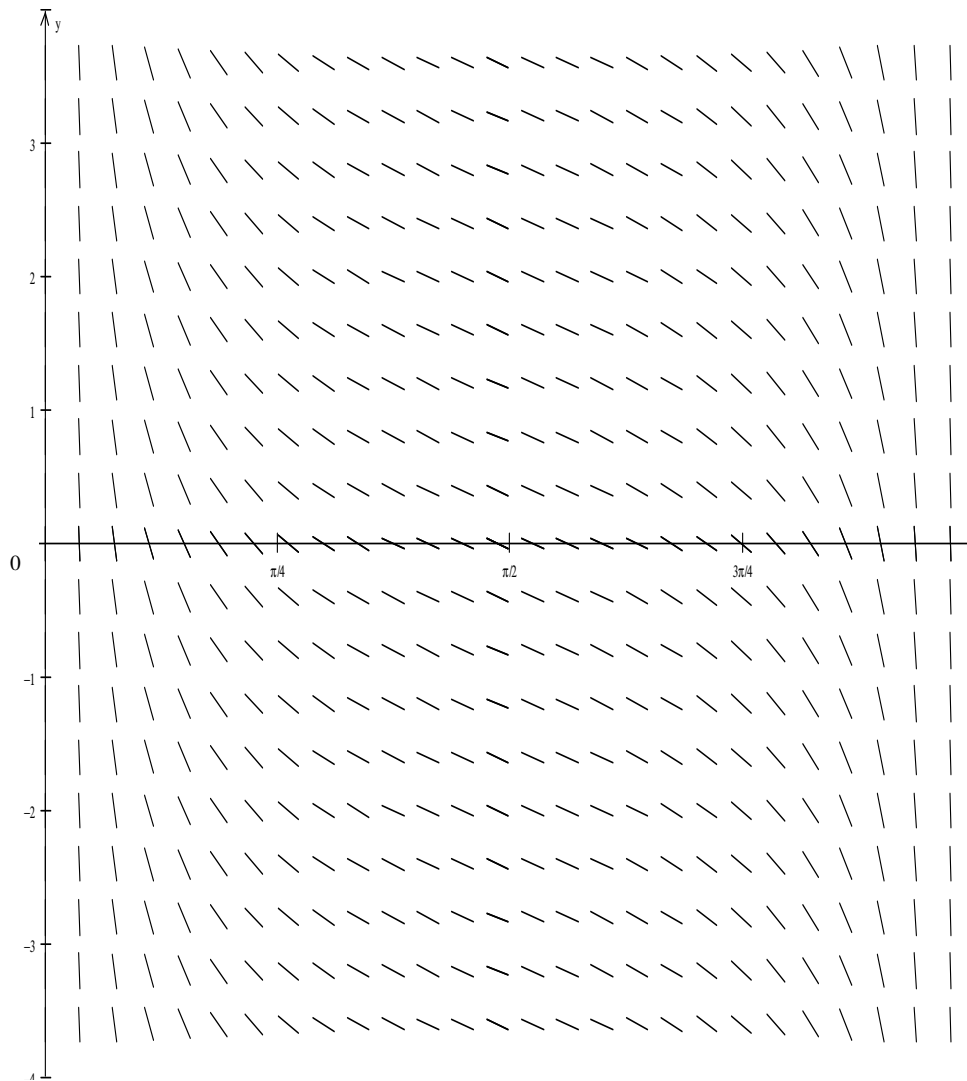
2 marks

b. Find the exact value of  $\int_{-e}^{-1} f(x) dx$ .

2 marks

### Question 8

The slope field for an unknown differential equation  $\frac{dy}{dx} = f(x)$  is shown below, where  $x = 0$  and  $x = \pi$  are asymptotes.



a. Sketch a solution curve to the differential equation  $\frac{dy}{dx} = f(x)$ .

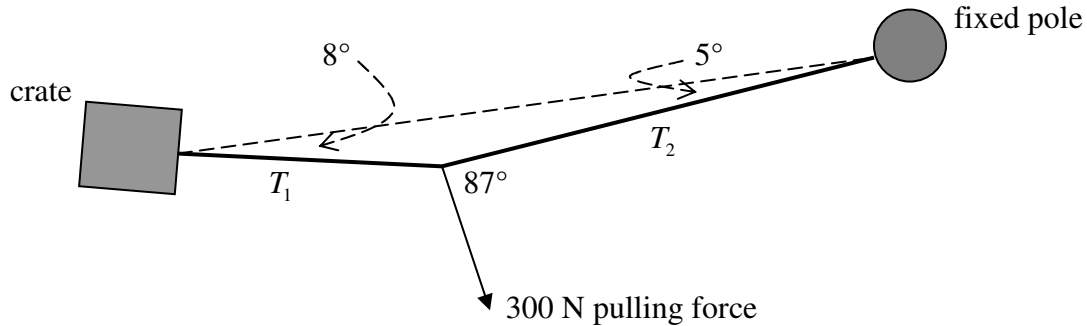
1 mark

b. Hence determine  $f(x)$  in simplest form.

2 marks

### Question 9

A strong rope connects a large crate to a fixed pole. The crate is about to move when a person pulls the rope with a force of 300 N. The following diagram shows the aerial view of the situation. Ignore the weight of the rope in your calculations.



a. Draw a clear triangle of forces to represent the pulling force and the tension forces  $T_1$  and  $T_2$ . Show the measure of each of the interior angles of the triangle of forces.

2 marks

b. Use the triangle of forces to find tension  $T_1$  to the nearest newton.

2 marks

### Question 10

A 0.2-kg particle is moving along the  $x$ -axis. Its velocity at position  $x$  is given by  $v = \pm\sqrt{10 - 8x - 2x^2}$ . Time is measured in seconds and distance in metres.

a. Find the acceleration of the particle at the origin ( $x = 0$ ).

1 mark

b. Find the resultant force on the particle at the origin.

1 mark

c. Find the maximum speed of the particle.

2 marks

**End of Exam 1**