The Mathematical Association of Victoria
Trial Exam 2011

SPECIALIST MATHEMATICS

STUDENT NAME ____

Written Examination 1

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

NOTES
Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are not permitted to bring into the examination room: notes of any kind, a calculator of any type, blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 13 pages with 3 detachable sheets of miscellaneous formulas at the back.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the back of this book during reading time.
- Write your student name/number in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$

Question 1

An object of mass 4 kg is pushed along a rough horizontal surface by a force of size 20 Newton acting in the horizontal direction. The object moves to the right with an acceleration of $2\ \text{m/s}^2$.

a. In the space below draw a diagram that shows all the forces acting on the object. You must clearly label these forces.

1 mark

b. Find in terms of g the coefficient of friction between the object and the surface.

2 marks

TURN OVER

Question 2

The velocity of a particle is given by $\dot{\underset{\sim}{r}} = -\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$. Find the vector resolute of this velocity in the direction perpendicular to the vector $3\underset{\sim}{i} - 2\underset{\sim}{j} - 6\underset{\sim}{k}$.

3 marks

Question 3

Consider the curve with equation $(x+1)^2 + \frac{(y+2)^2}{4} = 1$.

Find the gradient of the normal to the curve at the point where $y = \sqrt{3} - 2$ and $x > -1$.

3 marks

TURN OVER

Question 4

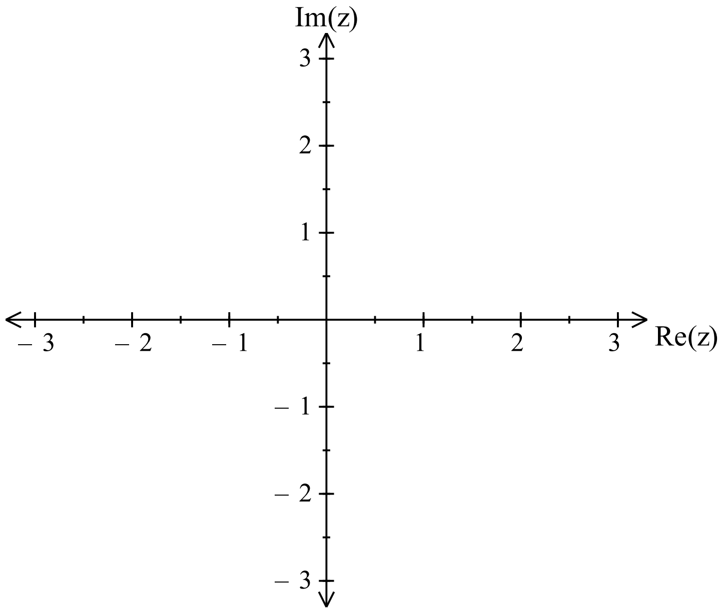
Consider $f(z) = z^3 + 2iz^2 - 2z - i,\ z \in C$.

Given that $f(-i) = 0$, find all roots of $f(z)$.

3 marks

Question 5

a. On the Argand diagram below sketch the set of points $S = \{z : (z - \sqrt{2})(\bar{z} - \sqrt{2}) = 2,\ z \in C\}$.



2 marks

b. The set S contains two points that satisfy $|z| = \left|z - 3\sqrt{2}\right|$.

Find these two points in polar form.

3 marks

TURN OVER

Question 6

A mass has velocity v m/s given by $v = \frac{1}{x} + \frac{1}{2}$ where x m is the displacement of the mass from the origin after t seconds.

a. Find the acceleration of the mass in terms of x.

1 mark

b. Given that $x = 2$ when $t = 0$, find t as a function of x.

3 marks

Question 7

Consider the function $f:\left(0,\frac{1}{4}\right)\rightarrow R,\ f(x)=\arccos\left(\frac{1}{2\sqrt{x}}\right)$.

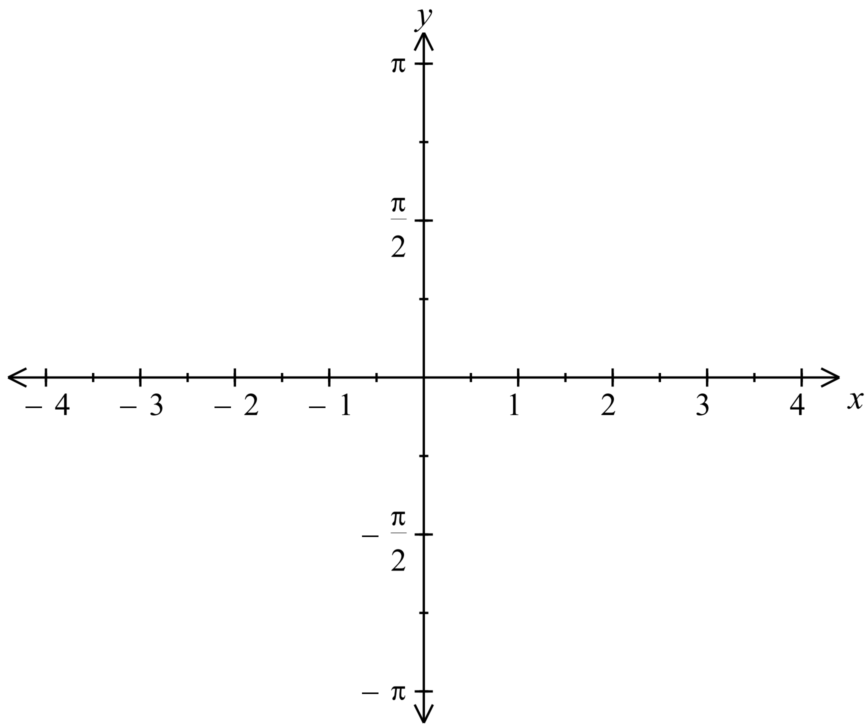
Find $f'(x)$ in the form $\frac{a}{(bx+c)\sqrt{dx+e}}$ where a, b, c, d and e are integers.

3 marks

TURN OVER

Question 8

a. On the axes below sketch the graph of the curve $y = 2\arcsin\left(\frac{x}{3}\right)$. State the coordinates of the point of inflection and label the coordinates of all endpoints.



2 marks

b. The part of the curve in **part a.** for which $\frac{3}{2} \le x \le 3$ is rotated about the y-axis to form a solid of revolution.

Find the volume of this solid. Give your answer in the form $\frac{(a\sqrt{b} + c\pi)\pi}{d}$ where a, b, c and d are integers.

4 marks

TURN OVER

Question 9

a. Express $\frac{x^2}{(x+1)^2}$ in partial fraction form.

2 marks

b. Hence solve the differential equation $\frac{dx}{dt} = \left(1 + \frac{1}{x}\right)^2$ for t in terms of x given that $x = 0$ when $t = 0$.

3 marks

Question 10

The path of a particle is given by $\underset{\sim}{r} = \sin(t)\,\underset{\sim}{i} - \cos(2t)\,\underset{\sim}{j}$, $0 \le t \le \frac{5\pi}{4}$.

a. Find the Cartesian equation of the path of the particle, stating the domain and range.

3 marks

b. Find a vector in the direction of motion of the particle at $t = \frac{\pi}{4}$.

2 marks

END OF QUESTION AND ANSWER BOOK

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$

hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$

$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$

$\sin(x - y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

$\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$

$\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$

$\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\,\text{cis}\,\theta$

$|z| = \sqrt{x^2 + y^2} = r$ $\qquad -\pi < \text{Arg}\, z \le \pi$

$z_1 z_2 = r_1 r_2\,\text{cis}(\theta_1 + \theta_2)$ $\qquad \dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\text{cis}(\theta_1 - \theta_2)$

$z^n = r^n\,\text{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$\dfrac{d}{dx}\left(x^n\right) = nx^{n-1}$ $\qquad \int x^n\,dx = \dfrac{1}{n+1}x^{n+1} + c, n \ne -1$

$\dfrac{d}{dx}\left(e^{ax}\right) = ae^{ax}$ $\qquad \int e^{ax}\,dx = \dfrac{1}{a}e^{ax} + c$

$\dfrac{d}{dx}\left(\log_e(x)\right) = \dfrac{1}{x}$ $\qquad \int \dfrac{1}{x}\,dx = \log_e|x| + c$

$\dfrac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$ $\qquad \int \sin(ax)\,dx = -\dfrac{1}{a}\cos(ax) + c$

$\dfrac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$ $\qquad \int \cos(ax)\,dx = \dfrac{1}{a}\sin(ax) + c$

$\dfrac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$ $\qquad \int \sec^2(ax)\,dx = \dfrac{1}{a}\tan(ax) + c$

$\dfrac{d}{dx}\left(\sin^{-1}(x)\right) = \dfrac{1}{\sqrt{1-x^2}}$ $\qquad \int \dfrac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\dfrac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}\left(\cos^{-1}(x)\right) = \dfrac{-1}{\sqrt{1-x^2}}$ $\qquad \int \dfrac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\dfrac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}\left(\tan^{-1}(x)\right) = \dfrac{1}{1+x^2}$ $\qquad \int \dfrac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\dfrac{x}{a}\right) + c$

product rule: $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

quotient rule: $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

chain rule: $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

Euler's method: If $\dfrac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\qquad s = ut + \dfrac{1}{2}at^2$ $\qquad v^2 = u^2 + 2as$ $\qquad s = \dfrac{1}{2}(u + v)t$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \le \mu N$

END OF FORMULA SHEET