The Mathematical Association of Victoria
Trial Examination 2011
Specialist Maths Examination 2 - SOLUTIONS

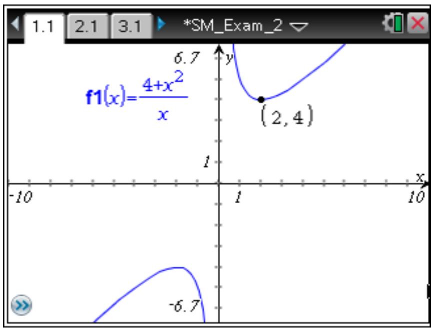
SECTION 1

Answers

1. E	**2. B**	**3. A**	**4. D**	**5. E**	**6. A**
7. C	**8. A**	**9. B**	**10. D**	**11. B**	**12. D**
13. C	**14. E**	**15. D**	**16. B**	**17. C**	**18. B**
19. B	**20. C**	**21. E**	**22. B**		

Question 1 **Answer E**

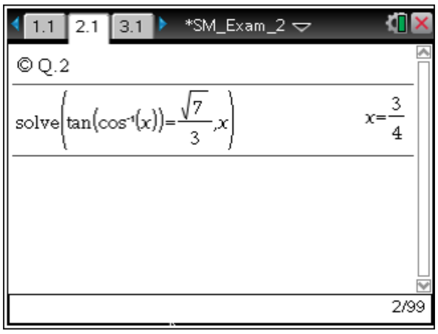
Substituting $x = 2$ for each option gives the result $f(2) = 4$ for option E only.



Question 2 **Answer B**

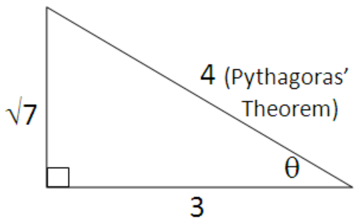
Using CAS, or knowledge of trigonometry,

$$\tan\left(\cos^{-1}\left(\frac{3}{4}\right)\right) = \frac{\sqrt{7}}{3}$$



Alternatively,

The situation can be represented geometrically on a right angled triangle.

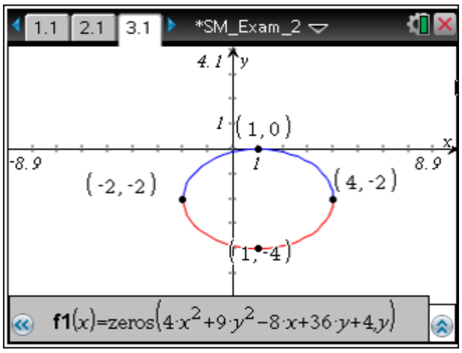


$$\frac{p}{q} = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{3}{4}$$

Question 3 **Answer A**

By graphing, using a conics application on CAS or by completing the square:

Centre is $(1, -2)$, horizontal semi-axis is 3 and vertical semi-axis is 2.



Alternatively,

$$4x^2 + 9y^2 - 8x + 36y + 4 = 0$$

$$4(x^2 - 2x + 1) + 9(y^2 + 4y + 4) - 36 = 0$$

$$4(x-1)^2 + 9(y+2)^2 = 36$$

$$\frac{(x-1)^2}{9} + \frac{(y+2)^2}{4} = 1$$

Question 4 **Answer D**

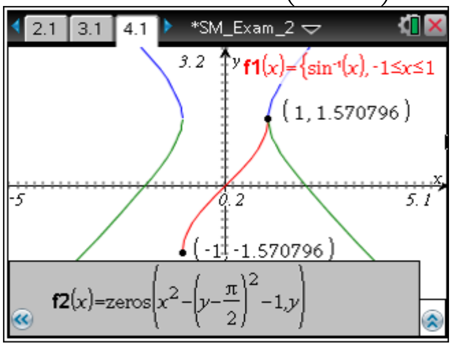
$x = t^2$ and $y = t(t^2 + 1)$, $t \ge 0$

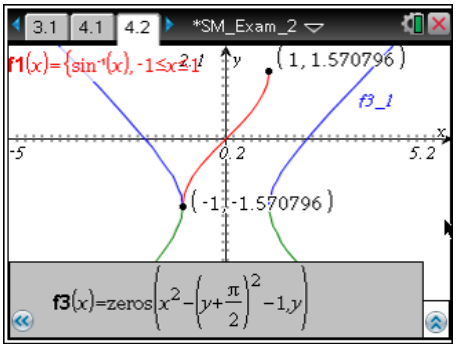
$$y = x^{\frac{1}{2}}(x+1)$$

$$y = x^{\frac{3}{2}} + x^{\frac{1}{2}}$$

Question 5 **Answer E**

The graph of g has endpoints at $\left(1, \frac{\pi}{2}\right), \left(-1, -\frac{\pi}{2}\right)$. The hyperbola $x^2 - y^2 = 1$ has vertices at $(1, 0)$ and $(-1, 0)$ and needs to be translated $\frac{\pi}{2}$ units up or down to intersect with the graph of g. Hence $x^2 - \left(y \pm \frac{\pi}{2}\right)^2 = 1$.





Question 6 **Answer A**

The point of intersection of $\operatorname{Re}(z) = -2$ and $\operatorname{Im}(z) = i$ is $z = -2 + i$.

Question 7 **Answer C**

Using the conjugate root theorem, the roots include $z = 1,\ z = 2\sqrt{2}$ and $z = 1 \pm i\sqrt{3}$.

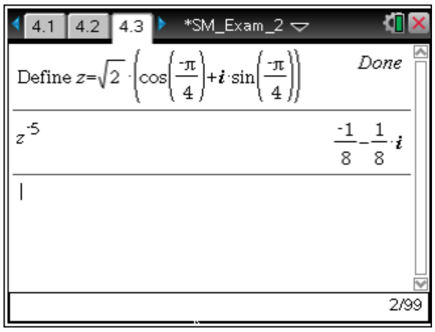
Therefore the degree of the polynomial must be at least 4.

Question 8 **Answer A**

$$z = \sqrt{2}\operatorname{cis}\left(-\frac{\pi}{4}\right)$$

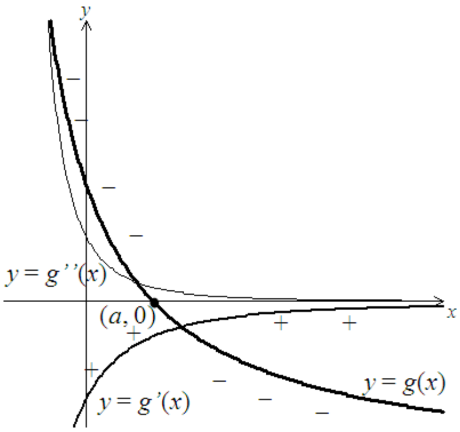
$$z^{-5} = \left(\sqrt{2}\right)^{-5} \operatorname{cis}\left(\frac{5\pi}{4}\right)$$

$$z^{-5} = -\frac{1}{8}(1 + i)$$



Question 9 **Answer B**

Consider the sketch graphs of $y = g'(x)$ and $y = g''(x)$.

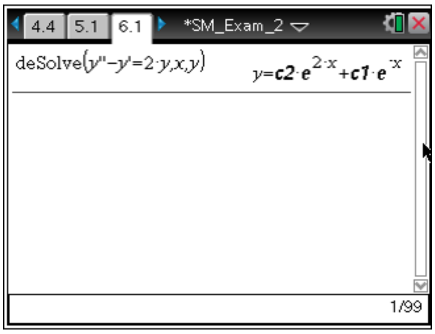


$$g(a) = 0,\ g'(a) < 0,\ g''(a) > 0$$

Therefore, $g''(a) > g(a) > g'(a)$

Question 10 **Answer D**

The solution to the differential equation is of the form $y = Ae^{-x} + Be^{2x}$. Therefore, $a = 1,\ b = 2$.



Question 11 **Answer B**

$$\text{Area} = \int_0^{\frac{\pi}{2}} \left(\sin(x)\cos^3(x)\right)dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\sin(x)\left(1-\sin^2(x)\right)\cos(x)\right)dx$$

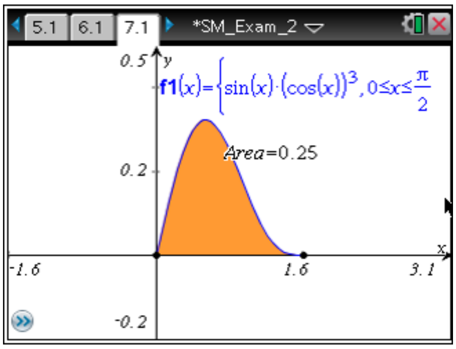
Let $u = \sin(x)$ therefore $\dfrac{du}{dx} = \cos(x)$

Terminals: $x = 0,\ u = \sin(0) = 0$ and

$x = \dfrac{\pi}{2},\ u = \sin\left(\dfrac{\pi}{2}\right) = 1$

Substituting,

$$\text{Area} = \int_0^1 u\left(1-u^2\right)\frac{du}{dx}dx$$



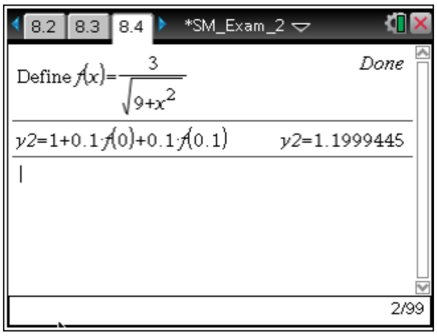
Question 12 **Answer D**

$h = 0.1$, therefore $n = 2$ for $x = 0.2$

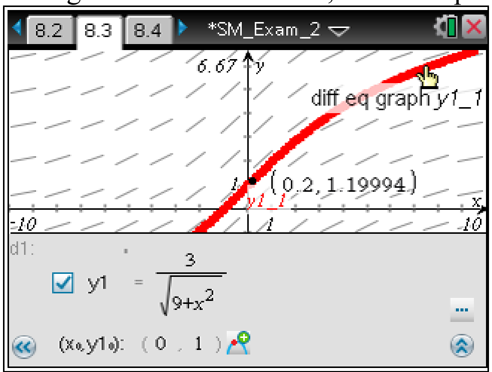
$y_1 = 1 + 0.1f(0)$ and

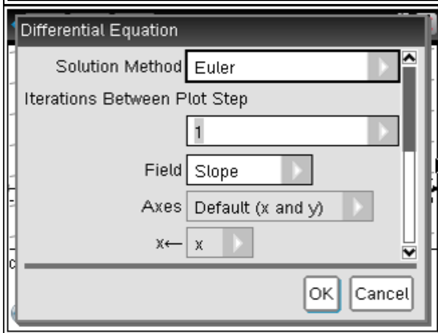
$y_2 = \left(1 + 0.1f(0)\right) + 0.1f(0.1)$

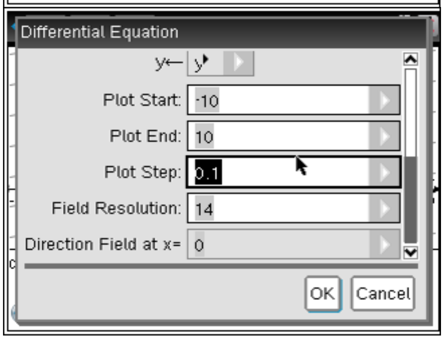
$y_2 \approx 1.1999$



Alternatively, use the *Differential Equation Graph* functionality of your CAS, ensuring that the settings use Euler's method, with a step size of 0.1.







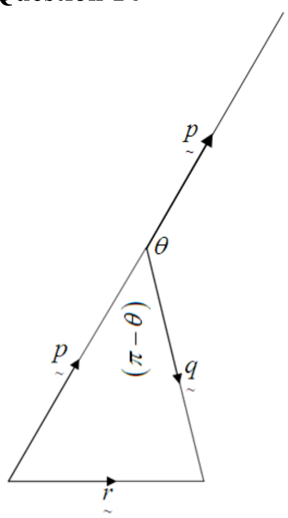
Question 13 **Answer C**

The direction field could be a family of parabolas of the form $y = ax^2 + c$, where $a > 0,\ c \in R$.
From the options available, the only differential equation that could have a solution of this form is

$$\frac{dy}{dx} = x$$

$$y = \int x\, dx = \frac{1}{2}x^2 + c.$$

Question 14 **Answer E**



$\underset{\sim}{p} \cdot \underset{\sim}{q} = |\underset{\sim}{p}| \times |\underset{\sim}{q}| \cos(\theta)$ ... equation (1)

Using the cosine rule

$$|\underset{\sim}{r}|^2 = |\underset{\sim}{p}|^2 + |\underset{\sim}{q}|^2 - 2|\underset{\sim}{p}| \times |\underset{\sim}{q}| \cos(\pi - \theta)$$

$$\cos(\pi - \theta) = \frac{|\underset{\sim}{p}|^2 + |\underset{\sim}{q}|^2 - |\underset{\sim}{r}|^2}{2|\underset{\sim}{p}| \times |\underset{\sim}{q}|}$$

However, $\cos(\pi - \theta) = -\cos(\theta)$

$\cos(\theta) = -\left(\dfrac{|\underset{\sim}{p}|^2 + |\underset{\sim}{q}|^2 - |\underset{\sim}{r}|^2}{2|\underset{\sim}{p}| \times |\underset{\sim}{q}|}\right)$ ...equation (2)

Substituting equation (2) in equation (1)

$$\underset{\sim}{p} \cdot \underset{\sim}{q} = |\underset{\sim}{p}| \times |\underset{\sim}{q}| \times -\left(\frac{|\underset{\sim}{p}|^2 + |\underset{\sim}{q}|^2 - |\underset{\sim}{r}|^2}{2|\underset{\sim}{p}| \times |\underset{\sim}{q}|}\right)$$

$$\underset{\sim}{p} \cdot \underset{\sim}{q} = -\frac{1}{2}\left(|\underset{\sim}{p}|^2 + |\underset{\sim}{q}|^2 - |\underset{\sim}{r}|^2\right)$$

Question 15 **Answer D**

Since the vectors are perpendicular,

$$\left(a\underset{\sim}{i} + 6\underset{\sim}{j} + 2\underset{\sim}{k}\right) \cdot \left(2\underset{\sim}{i} - 3\underset{\sim}{j} + 6\underset{\sim}{k}\right) = 0$$

$$2a - 18 + 12 = 0$$

$$a = 3$$

For the unit vector to $3\underset{\sim}{i} + 6\underset{\sim}{j} + 2\underset{\sim}{k}$,

$$m = \frac{1}{\sqrt{3^2 + 6^2 + 2^2}} = \pm\frac{1}{7}$$

Question 16 **Answer B**

$\underset{\sim}{a} = 2\underset{\sim}{i} + 4\underset{\sim}{k}$ and $\underset{\sim}{b} = 3\underset{\sim}{j} - 5\underset{\sim}{k}$.

Consider option **A**. If $\underset{\sim}{c} = 2\underset{\sim}{i} - 3\underset{\sim}{j} - \underset{\sim}{k}$ is linearly dependent with $\underset{\sim}{a}$ and $\underset{\sim}{b}$, then $\underset{\sim}{c} = \underset{\sim}{a} - \underset{\sim}{b}$. But $\underset{\sim}{c} = \underset{\sim}{a} - \underset{\sim}{b} = 2\underset{\sim}{i} - 3\underset{\sim}{j} + 9\underset{\sim}{k}$. Hence, not **A**.

Consider option **B**. If $\underset{\sim}{c} = -4\underset{\sim}{i} - 9\underset{\sim}{j} + 7\underset{\sim}{k}$ is linearly dependent with $\underset{\sim}{a}$ and $\underset{\sim}{b}$, then

$\underset{\sim}{c} = -2\underset{\sim}{a} - 3\underset{\sim}{b}$.

$\underset{\sim}{c} = -4\underset{\sim}{i} - 9\underset{\sim}{j} + (-8 + 15)\underset{\sim}{k}$

$\underset{\sim}{c} = -4\underset{\sim}{i} - 9\underset{\sim}{j} + 7\underset{\sim}{k}$, as required. Hence **B**.

Alternatively,

$\underset{\sim}{c}$ can be expressed as:

$\underset{\sim}{c} = \alpha\underset{\sim}{a} + \beta\underset{\sim}{b} \quad (1)$

$\therefore \underset{\sim}{c} = 2\alpha\underset{\sim}{i} + 3\beta\underset{\sim}{j} + \left(4\alpha - 5\beta\right)\underset{\sim}{k}$

For option A, using the above equation (1)

$2\alpha = 2 \to \alpha = 1;\ 3\beta = -3 \to \beta = -1$

However, substituting theses values into the $\underset{\sim}{k}$ term above
gives the wrong answer
because $4\alpha - 5\beta = 4 + 5 = 9$.

For option B, using the above equation (1)

$2\alpha = -4 \to \alpha = -2;\ 3\beta = -9 \to \beta = -3$

For the $\underset{\sim}{k}$ term, $4\alpha - 5\beta = -8 + 15 = 7$.

This is consistent with the $\underset{\sim}{k}$ term given in option B.

Therefore option B is correct.

Question 17 **Answer C**

$$\frac{dv}{dt} = -3v$$

$$t = \int \frac{1}{-3v}\,dv$$

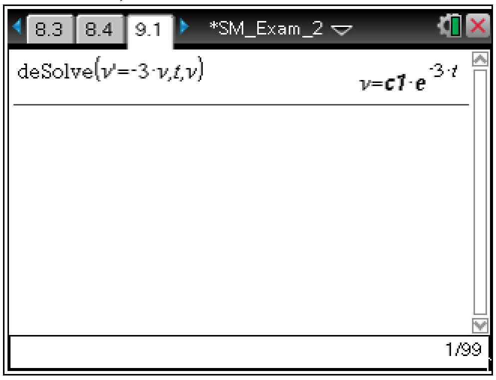
$$t = -\frac{1}{3}\log_e\left(|v|\right) + C$$

Given that $v(0) = 1$,

$$0 = -\frac{1}{3}\log_e(1) + C$$

$$C = 0$$

Therefore, $v = e^{-3t}$.



Question 18 **Answer B**

Let m be the mass of the body and g be acceleration due to gravity.

$$N = mg\cos\left(60^\circ\right)$$

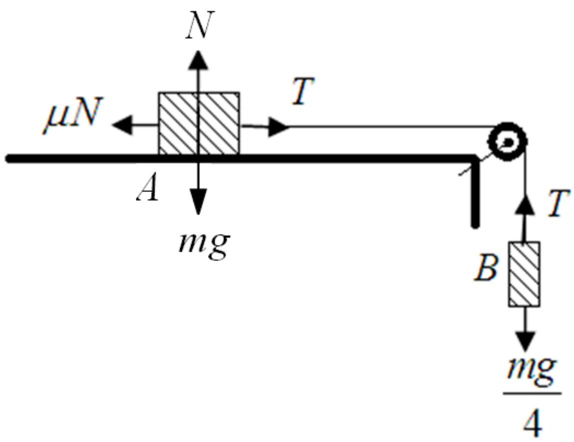
When the body is on the point of sliding,

$$\mu N = mg\sin\left(60^\circ\right)$$

$$\mu\left(mg\cos\left(60^\circ\right)\right) = mg\sin\left(60^\circ\right)$$

$$\mu = \tan\left(60^\circ\right) = \sqrt{3}$$

Question 19 **Answer B**



Consider the resultant force at B.

$$R = ma$$

$$\frac{mg}{4} - T = \frac{m}{4} \times \frac{g}{5}$$

$$T = \frac{mg}{4} - \frac{mg}{20}$$

$$T = \frac{mg}{5}$$

Question 20 **Answer C**

Using calculus or kinematics formulas, $v = \sqrt{492}$.

$$p = \frac{\sqrt{492}}{5} = 4.44 \text{ ms}^{-1}$$

Using kinematics formula

$$v^2 = u^2 + 2as$$

$$v^2 = 10^2 + 2 \times -9.8 \times -20$$

$$v = \sqrt{492}$$

$$p = mv$$

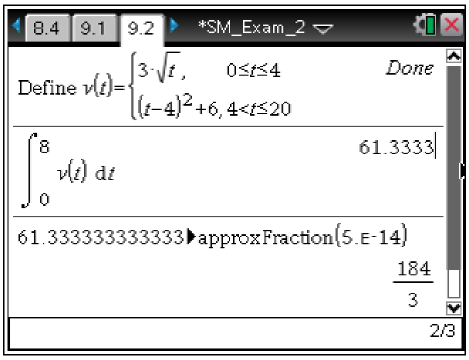
$$p = \frac{\sqrt{492}}{5}$$

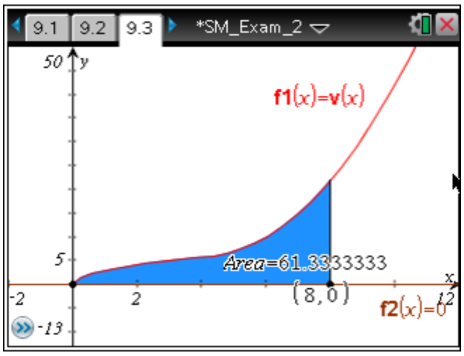
$p = 4.44 \text{ kgms}^{-1}$ (correct to 2 decimal places).

Question 21 **Answer E**

$$v(t) = \begin{cases} 3\sqrt{t} & 0 \le t \le 4 \\ (t-4)^2 + 6 & 4 < t \le 20 \end{cases}$$

$$x = \int_0^8 v(t)\,dt = \frac{184}{3} \text{ m}$$





Question 22
Answer B
Consider the resultant force on the vehicle:

$R = ma$

$-4800 = 1200a$

$a = -4 \text{ ms}^{-2}$

To find the braking time, use calculus or kinematics formulas.

$v = u + at$

$0 = 25 - 4t$

$t = \frac{25}{4} = 6.25 \text{ s}$

To find the braking distance, use calculus or kinematics formulas.

$x = ut + \frac{1}{2}at^2$

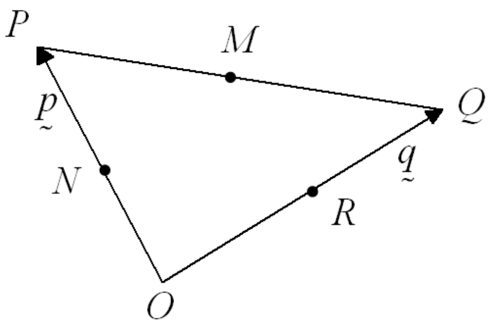
$x = 25 \times 6.25 - \frac{1}{2} \times 4 \times (6.25)^2$

$x = \frac{625}{8} = 78.125 \text{ m}$

END OF SECTION 1 SOLUTIONS

SECTION 2 SOLUTIONS

Question 1



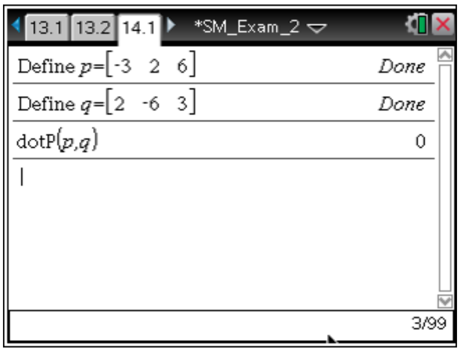
a.

$\left|\underset{\sim}{p}\right| = \left|\underset{\sim}{q}\right| = \sqrt{9+4+36}$

$\left|\underset{\sim}{p}\right| = \left|\underset{\sim}{q}\right| = 7$, therefore V$OPQ$ is isosceles. **1A**

$\underset{\sim}{p} \cdot \underset{\sim}{q} = (-3\times 2)+(2\times -6)+(6\times 3)=0$, therefore $\angle POQ = 90^{o}$ **1A**

VOPQ is an isosceles right-angles triangle.



b. i.

$\overrightarrow{OM} = \overrightarrow{OP} + \frac{1}{2}\overrightarrow{PQ}$

$\overrightarrow{OM} = \underset{\sim}{p} + \frac{1}{2}\left(\underset{\sim}{q} - \underset{\sim}{p}\right)$

$\overrightarrow{OM} = \frac{1}{2}\left(\underset{\sim}{p} + \underset{\sim}{q}\right)$ (This result effectively proves that $\overrightarrow{NM} = \overrightarrow{OR} = \overrightarrow{RQ}$)

$\overrightarrow{OM} = -\frac{1}{2}\underset{\sim}{i} - 2\underset{\sim}{j} + \frac{9}{2}\underset{\sim}{k}$ **1A**

b. ii.

From the previous result,

$\overrightarrow{NM} = \overrightarrow{OR} = \frac{1}{2}\underset{\sim}{q}$, therefore

$\overrightarrow{MN} = -\overrightarrow{OR} = -\frac{1}{2}\underset{\sim}{q}$

$\overrightarrow{MN} = -\underset{\sim}{i} + 3\underset{\sim}{j} - \frac{3}{2}\underset{\sim}{k}$ **1A**

Alternatively,

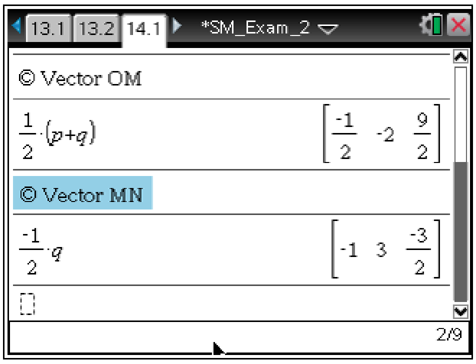
$$\overrightarrow{MN} = -\overrightarrow{OM} + \overrightarrow{ON}$$

From part **b.i.**

$$\overrightarrow{MN} = -\frac{1}{2}\left(\underset{\sim}{p} + \underset{\sim}{q}\right) + \frac{1}{2}\underset{\sim}{p}$$

$$\overrightarrow{MN} = -\frac{1}{2}\underset{\sim}{q}$$

$$\overrightarrow{MN} = -\underset{\sim}{i} + 3\underset{\sim}{j} - \frac{3}{2}\underset{\sim}{k}$$



c.

Required to show that $\overrightarrow{OM} \perp \overrightarrow{PQ}$

$$\overrightarrow{OM} = \overrightarrow{OP} + \frac{1}{2}\overrightarrow{PQ}$$

$$= \underset{\sim}{p} + \frac{1}{2}\left(\underset{\sim}{q} - \underset{\sim}{p}\right)$$

$$= \frac{1}{2}\left(\underset{\sim}{q} + \underset{\sim}{p}\right)$$

Therefore, **1M**

$$\overrightarrow{OM} \cdot \overrightarrow{PQ} = \frac{1}{2}\left(\underset{\sim}{q} + \underset{\sim}{p}\right) \cdot \left(\underset{\sim}{q} - \underset{\sim}{p}\right)$$

$$= \frac{1}{2}\left(\underset{\sim}{q} \cdot \underset{\sim}{q} - \underset{\sim}{p} \cdot \underset{\sim}{p}\right)$$

$$= \frac{1}{2}\left(\left|\underset{\sim}{q}\right|^2 - \left|\underset{\sim}{p}\right|^2\right)$$ **1A**

$$= 0 \quad \text{(because } \left|\underset{\sim}{q}\right| = \left|\underset{\sim}{p}\right| \text{: isosceles triangle)}$$

Therefore the line *OM* is perpendicular to the hypotenuse *PQ*, as required.

d.

$\underset{\sim}{s} \cdot \underset{\sim}{p} = 0$ because $\underset{\sim}{s}$ is perpendicular to $\underset{\sim}{p}$. Therefore

$-3a + 2b + 6c = 0$ … equation 1 **1M**

$\underset{\sim}{s} \cdot \underset{\sim}{q} = 0$ because $\underset{\sim}{s}$ is perpendicular to $\underset{\sim}{p}$. Therefore

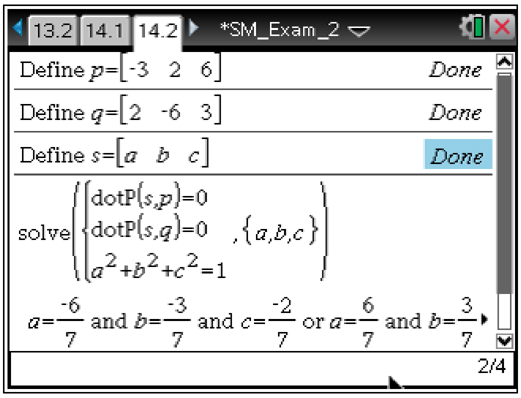
$2a - 6b + 3c = 0$ … equation 2 **1M**

$\underset{\sim}{s}$ is a unit vector, therefore

$a^2 + b^2 + c^2 = 1$ … equation 3 **1A**

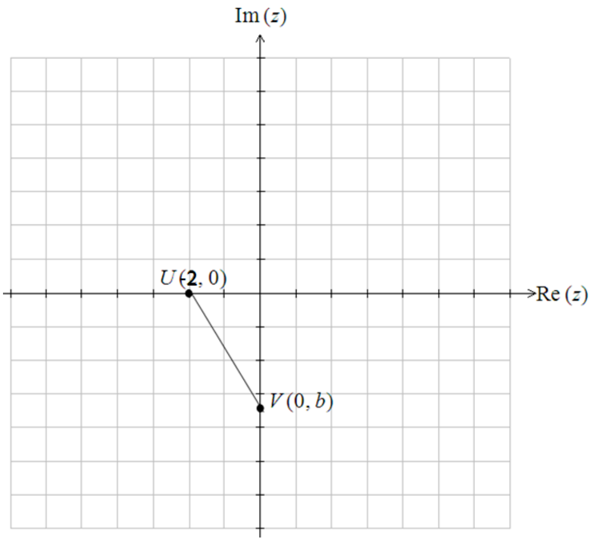
Solve equations 1, 2 and 3 simultaneously

$a=\frac{6}{7}$, $b=\frac{3}{7}$ and $c=\frac{2}{7}$ or $a=-\frac{6}{7}$, $b=-\frac{3}{7}$ and $c=-\frac{2}{7}$ **1A**



Question 2

a. i.



1A

a. ii.

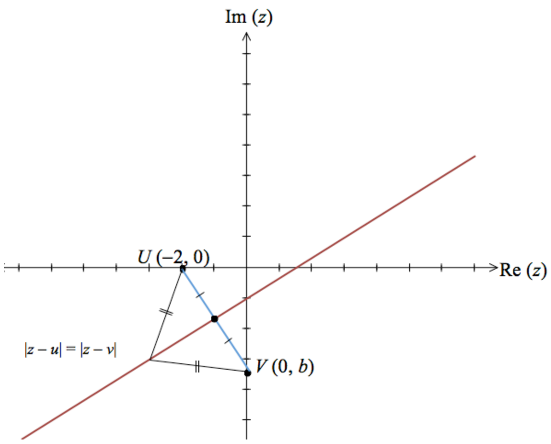
Let the equation of the line segment be of the form $y = mx + c$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{b}{2}$$

$$c = b$$

Equation is $y = \frac{b}{2}x + b$, Domain is $[-2, 0]$ **1A**

b. i.

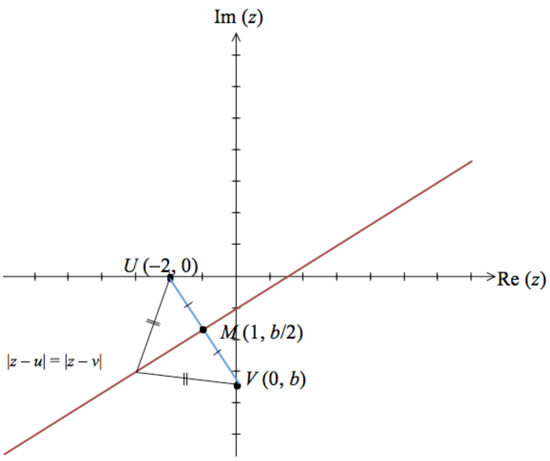


1A

b. ii.
By definition, P is equidistant from U and V. Every point in P will therefore be perpendicular to UV, as illustrated in the diagram below.

Alternatively, from part **a.ii.** above, $m_{uv} = \frac{b}{2}$ and from part **b.iii.** below, $m_P = -\frac{2}{b}$.

$m_P \times m_{uv} = -1$, therefore P is perpendicular to UV. **1A**



b. iii.

$$|z-u| = |z-v|$$

$$|x+iy+2| = |x+i(y-b)|$$

$$\sqrt{(x+2)^2+y^2} = \sqrt{x^2+i(y-b)^2} \quad \textbf{1M}$$

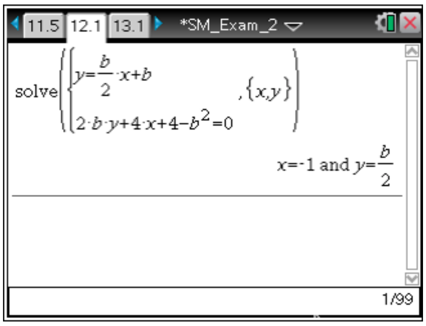
$$x^2+4x+4+y^2 = x^2+y^2-2by+b^2$$

$2by+4x+4-b^2=0$, as required **1A**

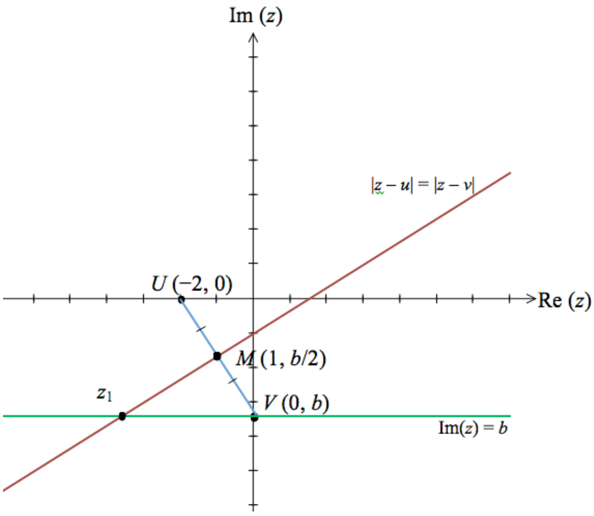
c. Solve simultaneously for x and y,

$y = \frac{b}{2}x+b$ and $2by+4x+4-b^2=0$ **1M**

$M\left(-1, \frac{b}{2}\right)$ **1A**



d. i.



1A

d. ii.

The cartesian equation of P is $2by+4x+4-b^2=0$ (equation 1)

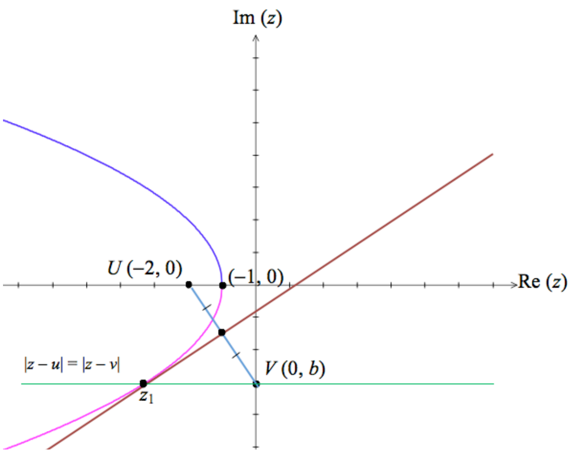
At the point of intersection with $y=b$ (equation 2), substitute equation 2 in equation 1 **1M**

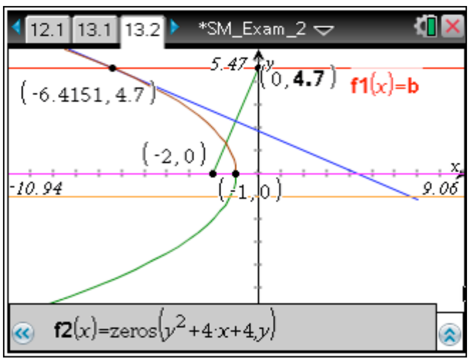
$2y^2+4x+4-y^2=0$ **1A**

$y^2+4x+4=0$, as required.

d. iii.

The equation of the curve can be graphed on a CAS, as shown below.

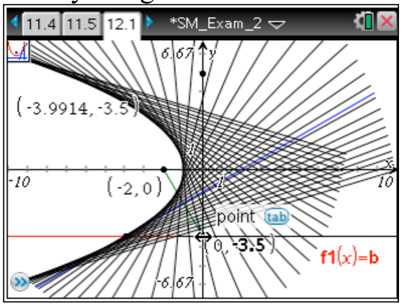




Correct shape **1A**

Correct vertex and correctly labelled **1A**

Alternatively, the path of can be explored using a dynamic geometry application on a CAS device, as illustrated in the screen dump below. This clearly shows that the vertex of the curve is at $(-1,0)$, and that *P* is always tangential to the curve.



Question 3

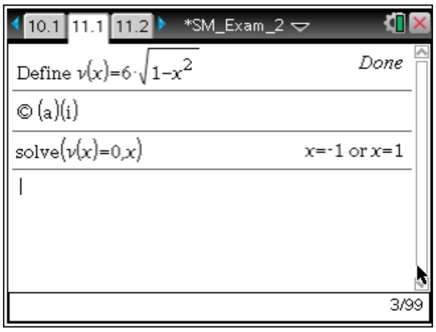
a. i. Consider the velocity of the particle as a function of its displacement.

$$v(x)=\frac{dx}{dt}$$

$$0=6\sqrt{1-x^2}$$

$$1-x^2=0$$

$x=1$ or $x=-1$ **1A**

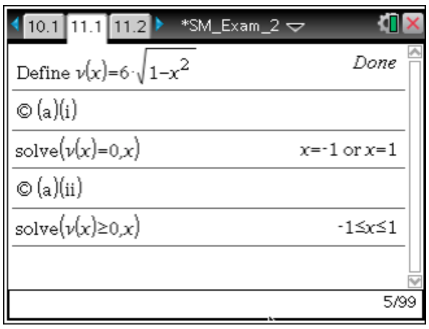


a. ii.

$$1-x^2\geq 0$$

$$-1\leq x\leq 1$$

Domain is $\{x : x\in[-1, 1]\}$ **1A**



b.i.

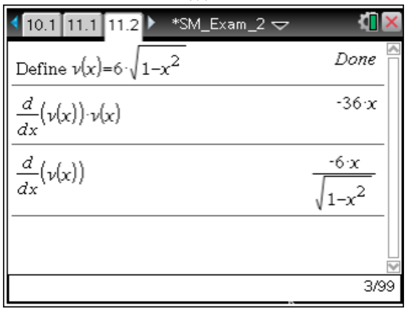
$a=\frac{dv}{dt}=\frac{dv}{dx}\times\frac{dx}{dt}=v\frac{dv}{dx}$, as required **1A**

b.ii.

$$v=\frac{dx}{dt}=6\sqrt{1-x^2}$$

$a = v\dfrac{dv}{dx} = 6\sqrt{1-x^2} \times \dfrac{-6x}{\sqrt{1-x^2}} = -36x$ **1A**

Therefore, $a = \dfrac{d^2x}{dt^2} + 36x = 0$, as required.



c. i.

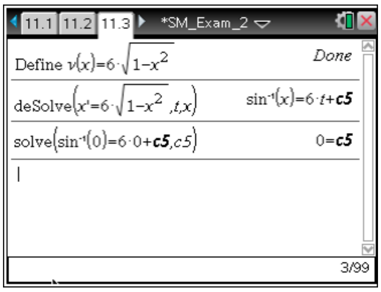
$\dfrac{dx}{dt} = 6\sqrt{1-x^2}$

$t = \dfrac{1}{6}\int \dfrac{dx}{\sqrt{1-x^2}}$ **1M**

$t = \dfrac{1}{6}\sin^{-1}(x) + c$

When $t = 0$, $x = 0$, therefore $c = 0$. **1A**

$t = \dfrac{1}{6}\sin^{-1}(x)$, as required.

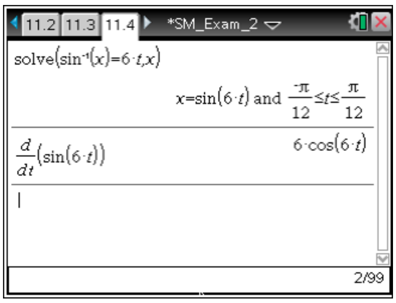


c. ii.

$t = \dfrac{1}{6}\sin^{-1}(x)$

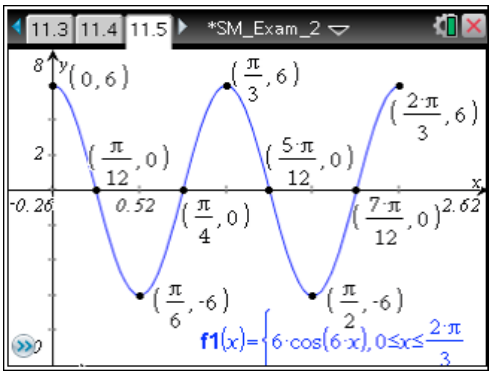
$x = \sin(6t)$ **1M**

$v(t) = \dfrac{dx}{dt} = 6\cos(6t)$ **1A**



c. iii.

Period $= \frac{2\pi}{6} = \frac{\pi}{3}$, amplitude = 6.

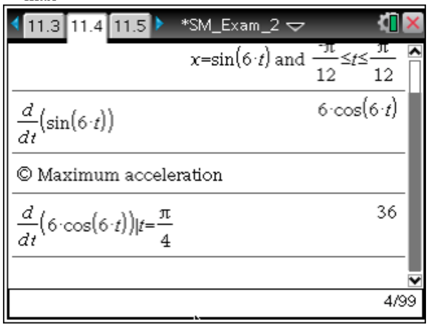


Correct shape, amplitude and period **1A**
Correct x-intercepts and coordinates of turning points **1A**

d. The acceleration is a maximum where $\frac{dv}{dt}$ is greatest, that is, where the gradient of the velocity-time graph is steepest. By inspection, this occurs when $t = \frac{\pi}{4}$.

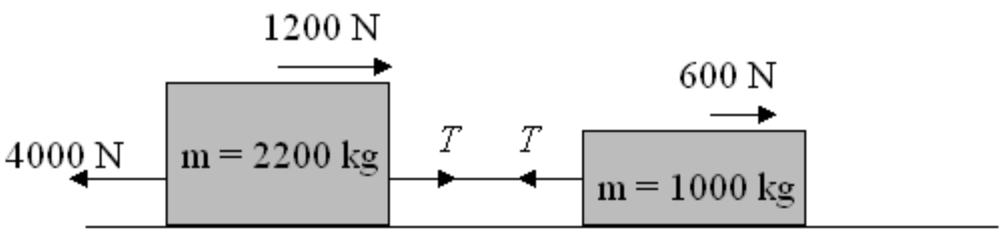
Maximum acceleration is given by $\frac{d(6\cos(6t))}{dt}$ at $t = \frac{\pi}{4}$. **1M**

$a_{\max} = 36$ **1A**



Question 4

a.



1A

b.

Consider the resultant force on the vehicles

$\underset{\sim}{R} = m\underset{\sim}{a}$ **1M**

$(4000 - 600 - 1200) = (2200 + 1000)a$

$a = \frac{2200}{3200}$

$a = \frac{11}{16}\text{ms}^{-2} = 0.6875\text{ms}^{-2}$ **1A**

c.
For the car,

$T - 600 = 1000a$, with $a = \frac{11}{16}\text{ms}^{-2}$ **1M**

$T = 1000 \times \frac{11}{16} + 600$

$T = 1287.5$ N **1A**

Alternatively, for the truck,

$4000 - 1200 - T = 2200a$, with $a = \frac{11}{16}\text{ms}^{-2}$

$T = 4000 - 1200 - 2200 \times \frac{11}{16}$

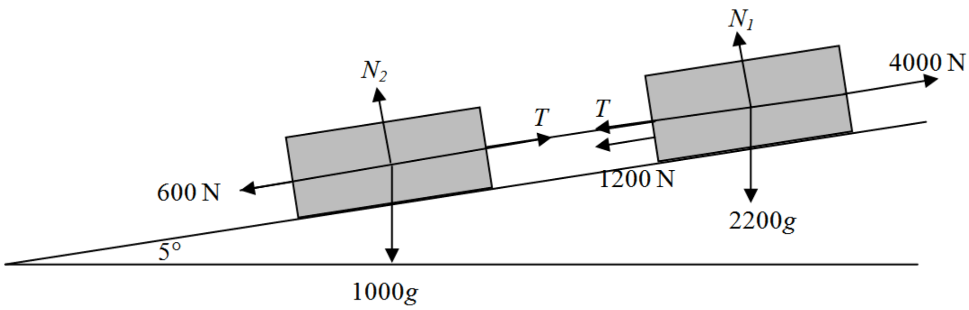
$T = 1287.5$ N

d.
The acceleration of the car and truck is constant, therefore
$v = u + at$

$22 = 0 + \frac{11}{16}t$

$t = \frac{22 \times 16}{11} = 32\text{ s}$ **1A**

e.



The component of the weight of the vehicles parallel to the inclined road is $(2200+1000)g\sin(5^{\circ})$
Resolving forces parallel to the plane,
$\underset{\sim}{R} = m\underset{\sim}{a}$

$4000 - 1200 - 600 - (2200+1000)g\sin(5^{\circ}) = (2200+1000)a$ **1M**

$a = \frac{4000 - 1200 - 600 - (2200+1000)g\sin(5^{\circ})}{(2200+1000)}$

$a \approx -0.167\text{ms}^{-2}$ (i.e. the speed of the vehicles is decreasing by 0.17 ms^{-1} every second) **1A**

f.

$a = \dfrac{F_T - kv}{m}$, where $F_T = 4000\,\text{N}$, $m = 3200\,\text{kg}$ and $k = 250$.

$\dfrac{dv}{dt} = \dfrac{4000 - 250v}{3200}$

Solving the differential equation ,

$v = Ae^{-\frac{5}{64}t} + 16$ **1A**

Starting from rest, $v = 0$ at $t = 0$

$0 = Ae^{-\frac{5}{64}\times 0} + 16$

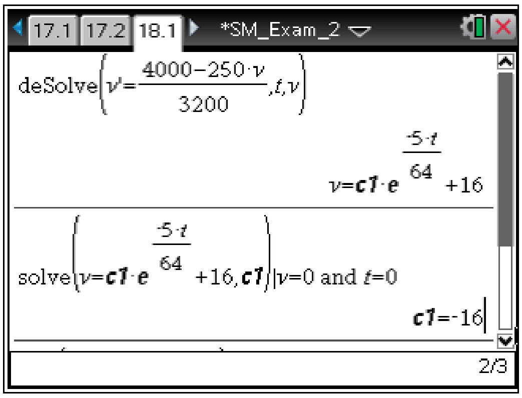
$A = -16$

Therefore,

$v = 16\left(1 - e^{-\frac{5}{64}t}\right)$

As $t \to \infty$, $e^{-\frac{5}{64}t} \to 0$ and $v \to 16$

The limiting (terminal) velocity is 16 ms^{-1} **1A**



Question 5

a.

To find the maximal domain, $x^6 - 64 \geq 0$, but the range $0 \leq y < 6$.

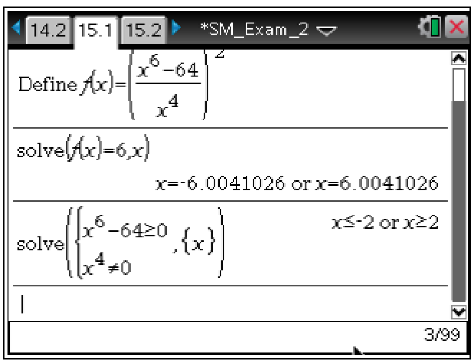
($x^4 \neq 0$, but this condition is made irrelevant by the maximal domain excluding the possibility of $x = 0$.)

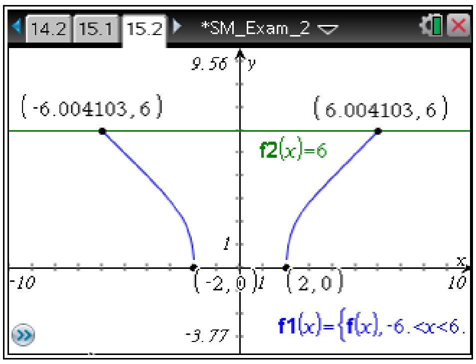
Solving the inequation $x^6 - 64 \geq 0$ for *x*, and

solving the equation $f(x) = 6$ for *x*, gives the results **1M**

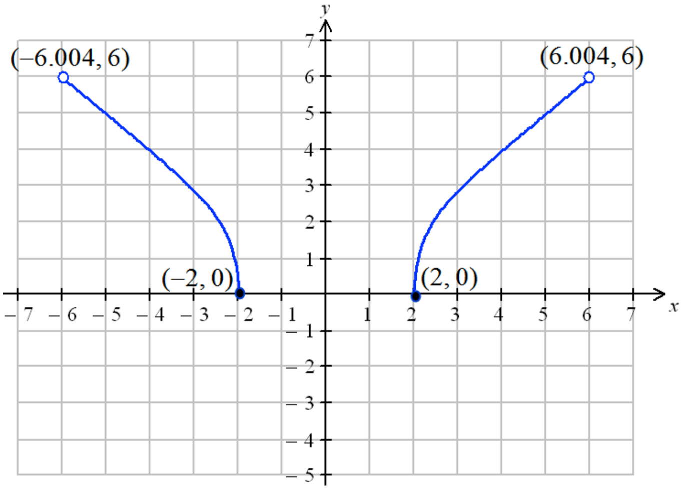
$x \leq -2$ or $x \geq -2$ and $x \approx -6.004$ or $x \approx 6.004$

$D = \{x : -6.004 < x \leq -2\} \cup \{x : 2 \leq x < 6.004\}$, as shown on the graph **1A**





b.



Correct shape and domain **1A**

Endpoints correctly labelled and shown as open or closed **1A**

c.i.

$$\delta V \approx \pi y^2 \delta x$$

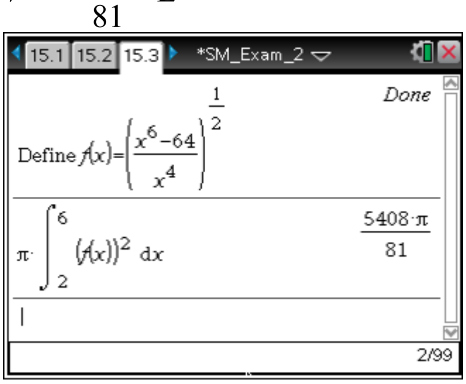
$$V = \pi \int_2^5 \left(f(x)^2 \right) dx$$

$$V = \pi \int_2^6 \left(\frac{x^6 - 64}{x^4} \right) dx$$ **1A**

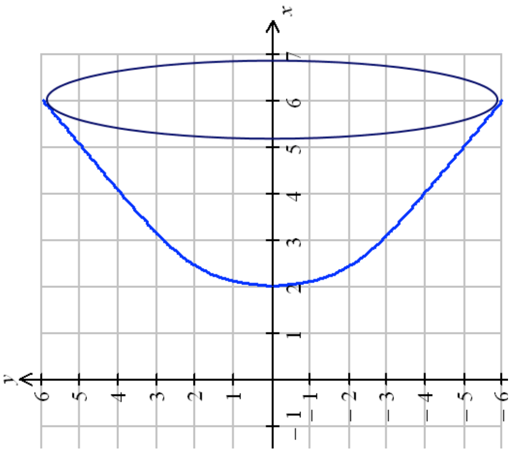
c.ii.

$$V = \pi \int_2^6 \left(\frac{x^6 - 64}{x^4} \right) dx$$

$$V = \frac{5408\pi}{81} \text{ L}$$ **1A**



d.



d.i.

$$\frac{dT}{dx} = k(80 - T),\ k > 0$$

$$x = \frac{1}{k}\int \frac{dT}{80 - T}$$

$$x = -\frac{1}{k}\log_e(|80 - T|) + C$$ **1M**

When $x = 0,\ T = 15$, therefore $C = \frac{1}{k}\log_e(65)$

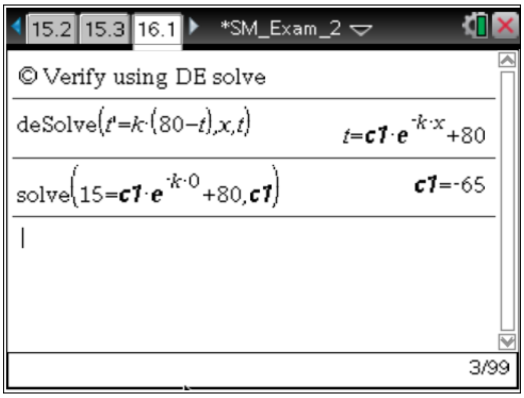
Substituting for C and simplifying the logarithms,

$$x = -\frac{1}{k}\log_e\left(\left|\frac{80 - T}{65}\right|\right)$$

$$\frac{80 - T}{65} = e^{-kx}$$ **1A**

$T = 80 - 65e^{-kx}$, as required

(This meets the initial condition that $T(0) = 80 - 65e^{-k\times 0} = 15$)



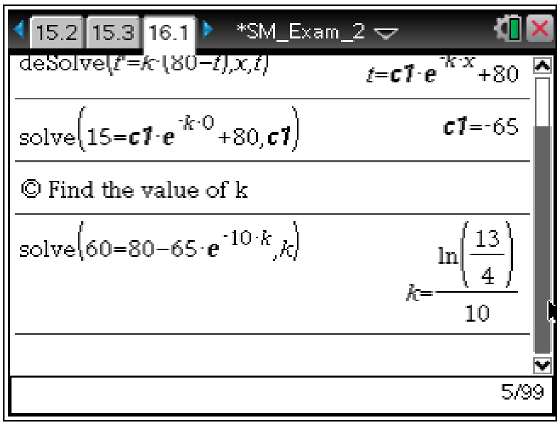
d.ii.

$T = 80 - 65e^{-kx}$

When $T = 60,\ x = 10$

Solve for k, $60 = 80 - 65e^{-10k}$

$$k = \frac{1}{10}\log_e\left(\frac{13}{4}\right)$$ **1A**



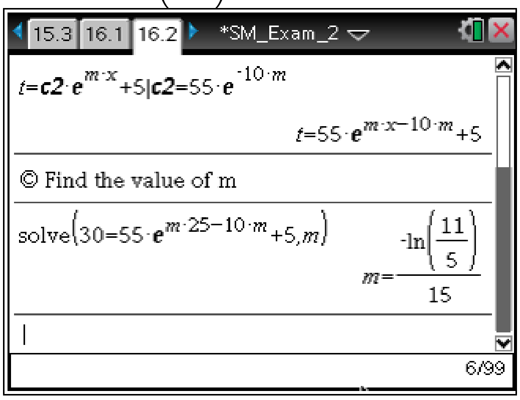
e.i.

$T = 55e^{-m(x-10)} + 5$

When $T = 30$, $x = 10 + 15 = 25$

Solve for m, $30 = 55e^{-m(25-10)} + 5$

$m = \frac{1}{15}\log_e\left(\frac{11}{5}\right) = \frac{\log_e(2.2)}{15}$, as required **1A**

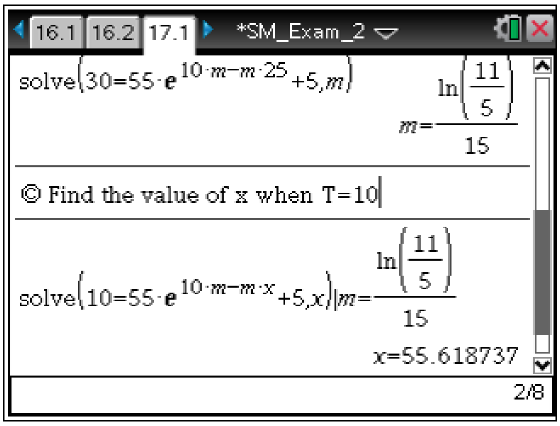


e.ii.

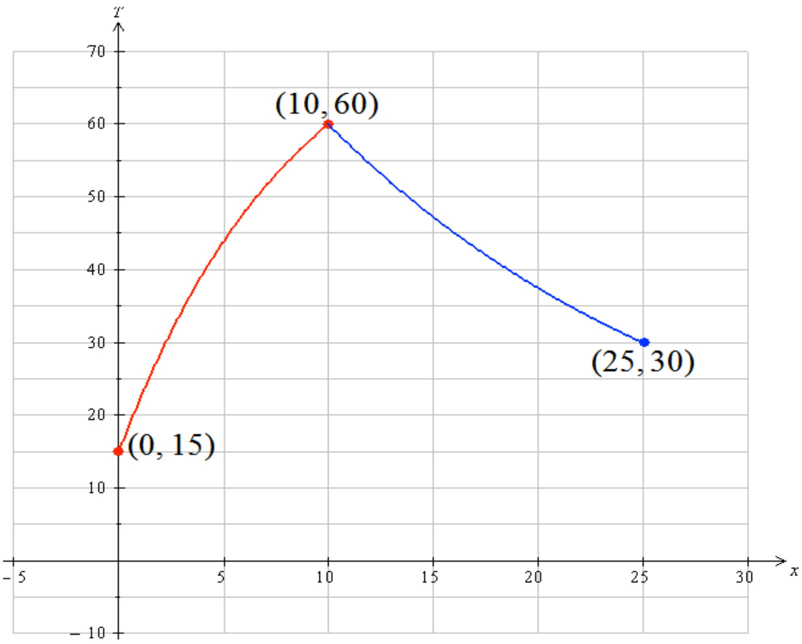
Solve for x, $10 = 55e^{-\frac{\log_e(2.2)}{15}(x-10)} + 5$

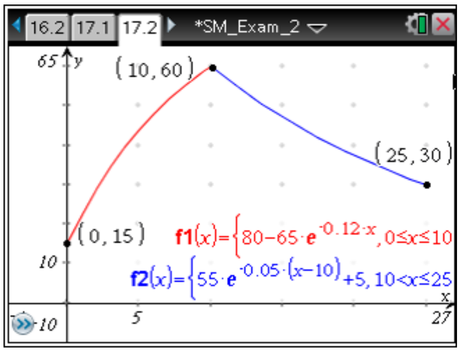
$x \approx 56$

The time that the solution needs to be in the cooling room is (56 – 10) = 46 minutes **1A**



f.





Correct shape (growth followed by decay)	**1A**
Correct domains, with endpoints correctly labelled	**1A**

END OF SOLUTIONS