TSSM
Creating VCE Success

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2011 Trial Examination

STUDENT NUMBER

									Letter
Figures									
Words									

SPECIALIST MATHEMATICS

Units 3 & 4 – Written examination 1

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, and rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 12 pages.
- Working space is provided throughout the book.

Instructions
- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

On an Argand diagram, O is the origin and point A represents a complex number $a + ib$ in the fourth quadrant. Point B represents the complex number $-\sqrt{3} - 3i$. If the triangle OAB is equilateral, find the values of a and b.

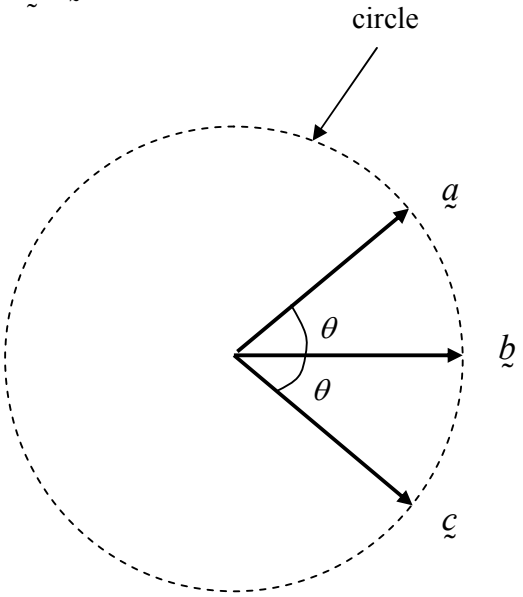
3 marks

TURN OVER

Question 2

The diagram below shows three coplanar vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ with the same magnitude.

$\underset{\sim}{a} = \underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{b} = \underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$.



The angles separating $\underset{\sim}{a}$ and $\underset{\sim}{b}$ and $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are equal. Find vector $\underset{\sim}{c}$.

4 marks

Question 3

Consider the identity: $\sin^{-1} x + \frac{\pi}{6} = \sin^{-1}\left(\frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{1-x^2}\right)$

a. Check that this identity is true for $x = \frac{\sqrt{3}}{2}$

2 marks

b. Prove that the identity is true for all values of x, $0 \le x \le 1$

3 marks

TURN OVER

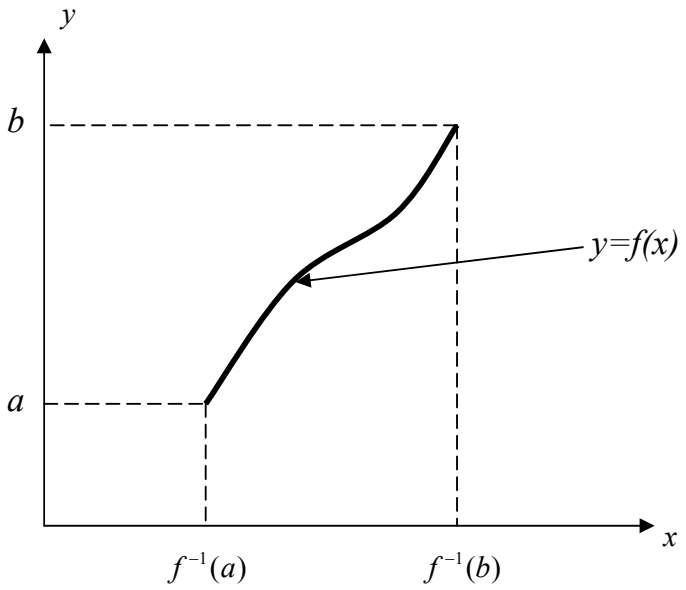
Question 4

Find the equation of the normal to the curve with equation $3x^2 - 2xy + 2y^3 = 7$ at the point where $y = 1$ and $x < 0$.

4 marks

Question 5

The graph of a function $y = f(x)$ is shown below.



Prove that:

$$\int_{f^{-1}(a)}^{f^{-1}(b)} f(x)dx + \int_{a}^{b} f^{-1}(y)dy = b\,f^{-1}(b) - a\,f^{-1}(a)$$

3 marks

TURN OVER

Question 6

Use a geometrical interpretation, or otherwise, to evaluate the integral $\int_{-3}^{-1} \log_e |x| \, dx$

3 marks

Question 7

Two particles A and B are moving with constant velocities $\underset{\sim}{v}_A = -2\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{v}_B$ respectively. Particle A starts at the point $(4, -3)$ and particle B starts at the point $(-1, 1)$. The particles collide after 1 second.

a. Find the velocity $\underset{\sim}{v}_B$ of particle B.

3 marks

b. Find the distance covered by particle B until collision.

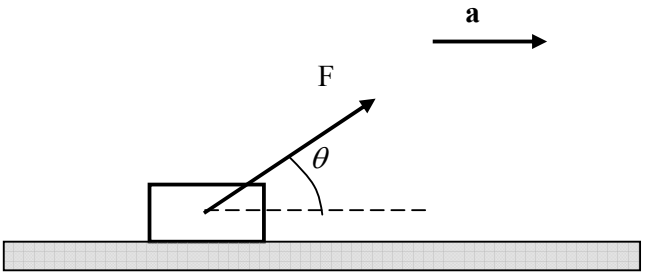
1 mark

TURN OVER

Question 8

A block of mass *m* lies on a rough horizontal table, with coefficient of friction μ.

A force of magnitude *F,* acting on the object at an angle θ to the horizontal, causes the block to accelerate on the table, as seen in the diagram below.



a. Draw in all the forces acting on the block.

1 mark

b. Show that the acceleration of the block is $a = \frac{F}{m}(\cos\theta + \mu\sin\theta) - \mu g$

4 marks

Question 9

The path of a particle is described by the cartesian equation: $\frac{x^2}{4}+\frac{y^2}{9}=1$. Both x and y coordinates are circular functions of time t, with the same period. The particle starts moving anticlockwise from the point $(-2,0)$, with a speed of 6 m/s.

a. Find the parametric equations for x and y.

3 marks

b. Find the speed of the particle after a quarter of a period.

2 marks

TURN OVER

Question 10

A particle moving in a straight line slows down under the influence of an acceleration $a\ ms^{-2}$ such that $a = -kv$, where $v\ ms^{-1}$ is the velocity of the particle at any instant. The initial velocity is u. After 2 seconds the speed of the particle has decreased by a factor of e, where e is Euler's number.

a. Find the constant k and the expression for the velocity v at any time t.

2 marks

b. Find the distance covered by the particle in the first 2 seconds in terms of u, the initial speed.

2 marks

END OF QUESTION AND ANSWER BOOK