Year 2012
VCE
Specialist Mathematics Solutions Trial Examination 2

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SECTION 1

ANSWERS

1	A	B	**C**	D	E
2	A	B	C	D	**E**
3	A	B	C	**D**	E
4	A	B	C	D	**E**
5	**A**	B	C	D	E
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7	A	B	**C**	D	E
8	A	**B**	C	D	E
9	**A**	B	C	D	E
10	A	B	C	**D**	E
11	A	B	C	**D**	E
12	A	**B**	C	D	E
13	A	B	**C**	D	E
14	A	**B**	C	D	E
`15	A	B	C	D	**E**
16	A	B	**C**	D	E
17	A	B	C	D	**E**
18	A	B	C	**D**	E
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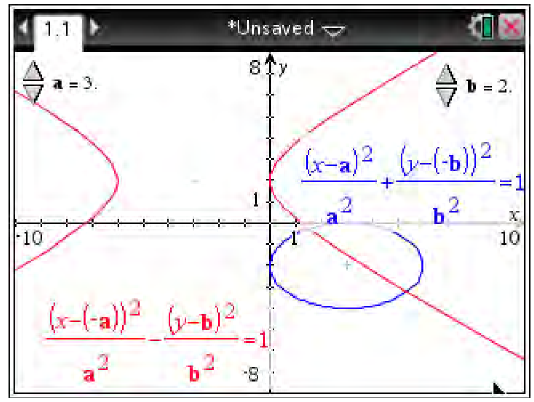
22	**A**	B	C	D	E

SECTION 1

Question 1 **Answer C**

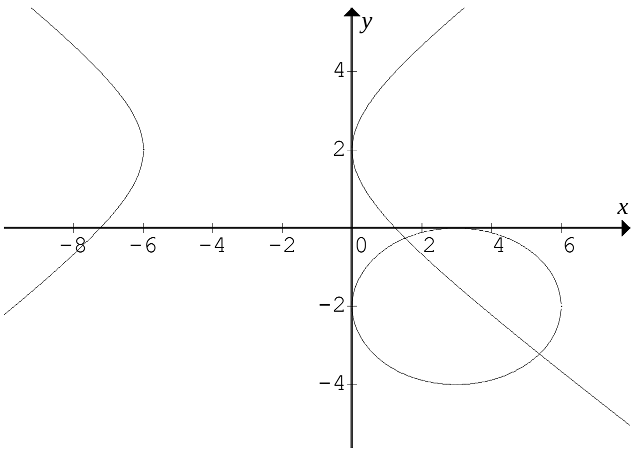


Consider the case when $a=3$ and $b=2$,

the graphs of $\dfrac{(x-3)^2}{9}+\dfrac{(y+2)^2}{4}=1$

and $\dfrac{(x+3)^2}{9}-\dfrac{(y-2)^2}{4}=1$, are shown,

and intersect at two distinct points.



Question 2 **Answer E**

Since $|\underset{\sim}{a}|=\sqrt{2}$ and $|\underset{\sim}{b}|=\sqrt{2}$ the vectors have the same length

$\underset{\sim}{a}.\underset{\sim}{b}=1 \quad \cos(\theta)=\dfrac{\underset{\sim}{a}.\underset{\sim}{b}}{|\underset{\sim}{a}||\underset{\sim}{b}|}=\dfrac{1}{2} \quad \Rightarrow \theta=60^0$

$\underset{\sim}{a}+\underset{\sim}{c}=3\underset{\sim}{i}-3\underset{\sim}{k}$ is parallel to the vector $\underset{\sim}{a}-\underset{\sim}{b}=\underset{\sim}{i}-\underset{\sim}{k}$

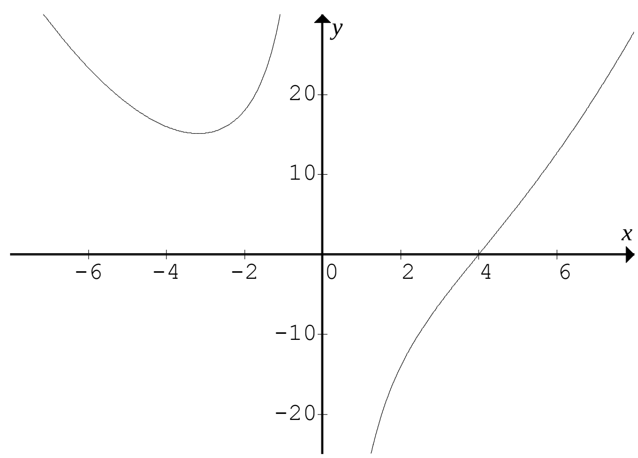
$\underset{\sim}{c}=2\underset{\sim}{a}-3\underset{\sim}{b}$, the vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are linearly dependent.

A. B. C. and **D.** are all true, **E.** is false.

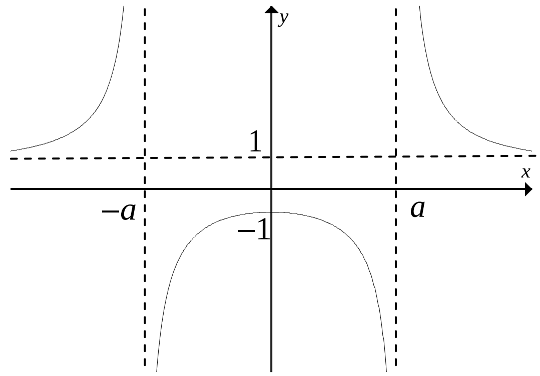
Question 3 **Answer D**

The only possibility is when $n=3$, the graph of

$y=\dfrac{x^3-64}{2x}$ is shown.



Question 4 **Answer E**



$y=\dfrac{x^2+a^2}{x^2-a^2}=1+\dfrac{2a^2}{x^2-a^2}$

the maximal domain is $R\setminus\{\pm a\}$,
the range is $(-\infty,-1]\cup(1,\infty)$.
The graph has vertical asymptotes at $x=\pm a$ and a horizontal asymptote at $y=1$. The graph crosses the *y*-axis at $y=-1$ and this point $(0,-1)$ is a maximum turning point.

Question 5 **Answer A**

The asymptotes intersect at the centre of the hyperbola
$-2x=2x-4k \quad \Rightarrow 4x=4k \quad x=k$ and $y=-2k$, the centre is $(k,-2k)$ and the gradient of the asymptotes is ± 2, the equation of the hyperbola is $\dfrac{(x-k)^2}{k^2}-\dfrac{(y+2k)^2}{4k^2}=1$

Question 6 **Answer B**

The domain and range of $y=\cos^{-1}(x)$ are $|x|\le 1$ and $[0,\pi]$ respectively, the domain of $y=\dfrac{2a}{\pi}\cos^{-1}\left(\dfrac{x}{a}-1\right)$ is $\left|\dfrac{x}{a}-1\right|\le 1$ or $-1\le\dfrac{x}{a}-1\le 1 \Rightarrow 0\le\dfrac{x}{a}\le 2$ or $[0,2a]$
and the range is $[0,\pi]\times\dfrac{2a}{\pi}=[0,2a]$

Question 7 **Answer C**

$y=\cot(px)=\dfrac{\cos(px)}{\sin(px)}$. The graph crosses the *x*-axis, when $y=0$ or $\cos(px)=0$
$\Rightarrow px=(2n+1)\dfrac{\pi}{2}$ or $x=\dfrac{(2n+1)\pi}{2p}$. The graph has vertical asymptotes when
$\sin(px)=0 \Rightarrow px=n\pi$ or $x=\dfrac{n\pi}{p}$.

Question 8 **Answer B**

$P(z)$ is a fourth degree polynomial
$P(z)=(z-ki)(z+ki)(z-2k)(z+k)$ expanding
$P(z)=(z^2+k^2)(z^2-kz-2k^2)$
$P(z)=z^4-kz^3-k^2z^2-k^3z-2k^4$

Question 9 **Answer A**

$\{z: |z-a|^2 - |z-bi|^2 = a^2+b^2\}$ let $z = x+yi$

$|(x-a+yi)|^2 - |x+(y-b)i|^2 = a^2+b^2$

$(x-a)^2 + y^2 - \left(x^2 + \left(y-b^2\right)\right) = a^2+b^2$

$x^2 - 2xa + a^2 + y^2 - \left(x^2+y^2-2by+b^2\right) = a^2+b^2$

$2by - 2xa = 2b^2$

$y = \dfrac{xa}{b} + b$ or $z = x+yi$ $\quad \text{Im}(z) = \dfrac{a}{b}\text{Re}(z) + b$

this represents a straight line in the argand plane.

Question 10 **Answer D**

$u = 4\text{cis}(\theta)$, $v = 2\text{cis}\left(\dfrac{-4\pi}{5}\right)$

$\dfrac{u}{v} = 2\text{cis}\left(\theta + \dfrac{4\pi}{5}\right) = -2i = 2\text{cis}\left(-\dfrac{\pi}{2}\right)$

$\theta + \dfrac{4\pi}{5} = -\dfrac{\pi}{2} \quad \Rightarrow \theta = -\dfrac{4\pi}{5} - \dfrac{\pi}{2} = -\dfrac{13\pi}{10}$

Now $\theta = -\dfrac{13\pi}{10} + 2\pi = \dfrac{7\pi}{10}$

Question 11 **Answer D**

$|\underset{\sim}{b}| = \sqrt{4+t^2+1} = \sqrt{5+t^2} = 3$ if $t = \pm 2$ **A.** is true.

the angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is $\cos(\theta) = \dfrac{\underset{\sim}{a}.\underset{\sim}{b}}{|\underset{\sim}{a}||\underset{\sim}{b}|}$

$\cos(\theta) = \dfrac{-8-t+1}{\sqrt{18}\times\sqrt{5+t^2}} = -\dfrac{1}{\sqrt{2}}$ when $t = 2 \Rightarrow \theta = -135^0$ so **B.** is true.

$\underset{\sim}{a}.\underset{\sim}{b} = -8-t+1 = 0$ when $t = -7$ so **C.** is true.

D. is false, there is no value of t, for which the vector $\underset{\sim}{a}$ is parallel to the vector $\underset{\sim}{b}$.

The scalar resolute of $\underset{\sim}{b}$ in the direction of $\underset{\sim}{a}$ equals

$\underset{\sim}{b}.\underset{\sim}{\hat{a}} = \dfrac{\underset{\sim}{b}.\underset{\sim}{a}}{|\underset{\sim}{a}|} = \dfrac{-8-t+1}{\sqrt{18}} = \dfrac{1}{\sqrt{2}}$ when $t = -10$, so **E.** is true.

Question 12 **Answer B**

$\dot{\underset{\sim}{r}}(t) = 4\cos(2t)\underset{\sim}{i} + 2\sin(2t)\underset{\sim}{j}$ for $t \ge 0$

$\underset{\sim}{r}(t) = \int 4\cos(2t)\,dt\,\underset{\sim}{i} + \int 2\sin(2t)\,dt\,\underset{\sim}{j} = 2\sin(2t)\underset{\sim}{i} - \cos(2t)\underset{\sim}{j} + \underset{\sim}{c}$

Now $\underset{\sim}{r}(0) = \underset{\sim}{0} \Rightarrow -\underset{\sim}{j} + \underset{\sim}{c} = \underset{\sim}{0} \Rightarrow \underset{\sim}{c} = \underset{\sim}{j}$

$\underset{\sim}{r}(t) = 2\sin(2t)\underset{\sim}{i} + (1 - \cos(2t))\underset{\sim}{j}$

$x = 2\sin(2t)$ and $y = 1 - \cos(2t)$

$\sin^2(2t) + \cos^2(2t) = 1$

$$\frac{x^2}{4} + (y-1)^2 = 1$$

an ellipse with centre at $(0,1)$.

Question 13 **Answer C**

let $u = 9 - 4x^2$ $\quad \frac{du}{dx} = -8x$

terminals when $x = \frac{3}{2}$ $u = 0$ and when $x = 0$ $u = 9$

$x^2 = \frac{1}{4}(9 - u) \quad dx = -\frac{1}{8x}du$

$$\int_0^{\frac{3}{2}} \frac{x^3}{\sqrt{9-4x^2}}dx = \int_0^{\frac{3}{2}} \frac{x\,x^2}{\sqrt{9-4x^2}}dx = \int_9^0 -\frac{1}{32}\left(\frac{9-u}{\sqrt{u}}\right)du$$ swap terminals

$$= \frac{1}{32}\int_0^9 \frac{9-u}{\sqrt{u}}du$$

Question 14 **Answer B**

$v(x) = e^{2x} - e^{-2x}$

$\frac{dv}{dx} = 2\left(e^{2x} + e^{-2x}\right)$

$a = v\frac{dv}{dx} = 2\left(e^{2x} - e^{-2x}\right)\left(e^{2x} + e^{-2x}\right)$

$a(x) = 2\left(e^{4x} - e^{-4x}\right)$

alternatively

$v^2(x) = \left(e^{2x} - e^{-2x}\right)^2 = e^{4x} - 2 + e^{-4x}$

$\frac{1}{2}v^2 = \frac{1}{2}e^{4x} - 1 + \frac{1}{2}e^{-4x}$

$a(x) = \frac{d}{dx}\left(\frac{1}{2}v^2\right) = 2\left(e^{4x} - e^{-4x}\right)$

Question 15 **Answer E**

$$\underset{\sim}{r}(t) = 25t\cos(40^0)\underset{\sim}{i} + \left(25t\sin(40^0) - \frac{1}{2}gt^2\right)\underset{\sim}{k}$$

$$\underset{\sim}{r}(t) = Vt\cos(\alpha)\underset{\sim}{i} + \left(Vt\sin(\alpha) - \frac{1}{2}gt^2\right)\underset{\sim}{k} \text{ so that } V = 25 \quad \alpha = 40^0$$

$$\text{time of flight } T = \frac{2V\sin(\alpha)}{g} = \frac{2\times 25\sin(40^0)}{9.8} = 3.28 \text{ seconds}$$

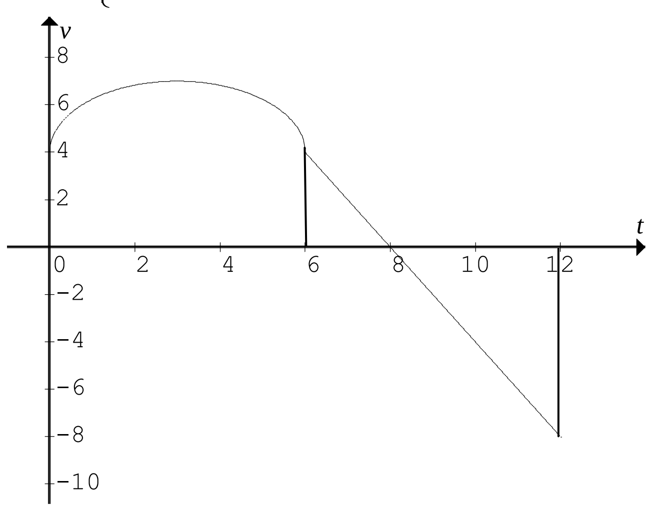
$$\text{maximum height } H = \frac{V^2\sin^2(\alpha)}{2g} = \frac{25^2\sin^2(40^0)}{2\times 9.8} = 13.175 \text{ metres}$$

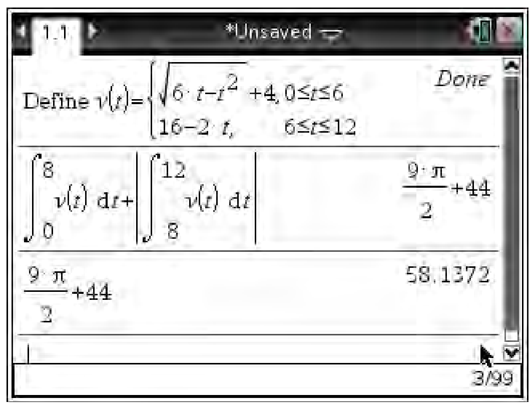
$$\text{the range } R = \frac{V^2\sin(2\alpha)}{g} = \frac{25^2\sin(80^0)}{9.8} = 62.807 \text{ metres}$$

The golf ball travels in a parabolic path, all of **i. ii. iii. iv** and **v.** are correct.

Question 16 **Answer C**

$$v(t) = \begin{cases} \sqrt{6t - t^2} + 4 & \text{for } 0 \le t \le 6 \\ 16 - 2t & \text{for } 6 \le t \le 12 \end{cases}$$

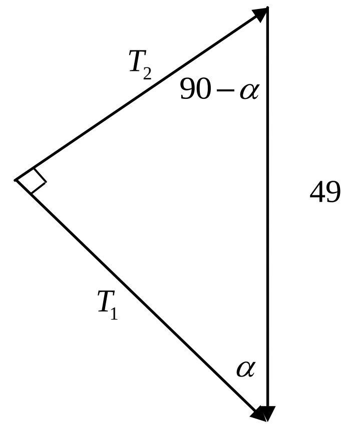




The distance travelled is half the area of a circle of radius 3, plus the area of a rectangle, plus the area of a triangle above the *t*-axis, plus the area of a triangle below the *t*-axis.

$$\frac{9\pi}{2} + 6\times 4 + \frac{1}{2}\times 2\times 4 + \frac{1}{2}\times 4\times 8 \approx 58 \text{ metres.}$$

Question 17 **Answer E**



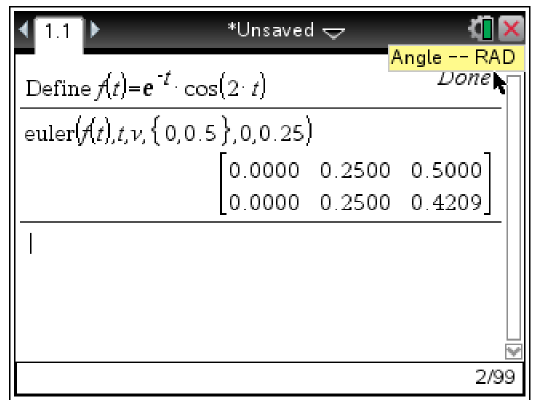
The forces are in newtons, the weight force is

$mg = 5\times 9.8 = 49$ newtons.

By Lami's theorem, $\dfrac{T_1}{\sin(90-\alpha)} = \dfrac{T_2}{\sin(\alpha)} = \dfrac{49}{\sin(90^0)}$

$T_1 = 49\cos(\alpha)$ and $T_2 = 49\sin(\alpha)$

Question 18 **Answer D**



$\dfrac{dv}{dt} = f(t) = e^{-t}\cos(2t)$

$v(0) = 0 \quad h = 0.25 \quad t_0 = 0 \quad t_1 = 0.25$

$v_1 = v_0 + hf(t_0) = 0 + 0.25\times e^0\cos(0) = 0.25$

$v_2 = v_1 + hf(t_1) = 0.25 + 0.25\times e^{-0.25}\cos(0.5) = 0.42$

Question 19 **Answer C**

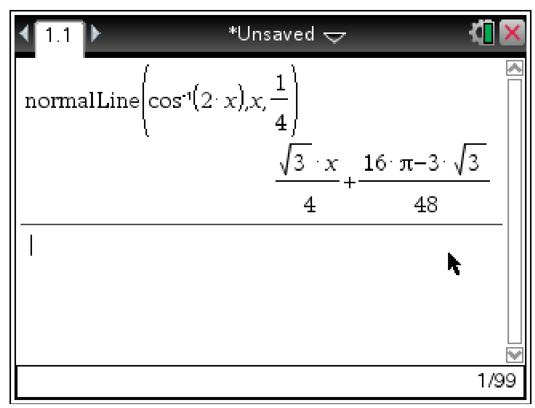
the differential equation is $a = \dfrac{dv}{dt} = 2\cos(2t)$

Question 20 **Answer D**

$F = 10$ newtons $\quad m = 5$ kg $\quad a = \dfrac{F}{m} \quad a = 2\,\text{ms}^{-2} \quad u = 1\ \text{ms}^{-1} \quad s = 20$

$v^2 = u^2 + 2as \ \Rightarrow v^2 = 1 + 2\times 2\times 20 = 81$ so $v = 9$ so $p = mv = 5\times 9 = 45\,\text{kgms}^{-1}$

Question 21 **Answer A**



$y = \cos^{-1}(2x)$ when $x = \dfrac{1}{4} \quad y = \cos^{-1}\left(\dfrac{1}{2}\right) = \dfrac{\pi}{3}$

$\dfrac{dy}{dx} = \dfrac{-2}{\sqrt{1-4x^2}}$ when $x = \dfrac{1}{4} \quad m_T = -\dfrac{4}{\sqrt{3}}$

$\Rightarrow m_N = \dfrac{\sqrt{3}}{4}$ equation of the normal is

$y - \dfrac{\pi}{3} = \dfrac{\sqrt{3}}{4}\left(x - \dfrac{1}{4}\right)$ or $y = \dfrac{\sqrt{3}\,x}{4} - \dfrac{\sqrt{3}}{16} + \dfrac{\pi}{3}$

Question 22 **Answer A**

by Newton's law of cooling, the temperature of surroundings is 200°C, and the initial temperature is 3°C, $\dfrac{dT}{dt} = -k(T-200) \quad T(0) = 3$

END OF SECTION 1 SUGGESTED ANSWERS

SECTION 2

Question 1

a. $\overrightarrow{OA} = -\underset{\sim}{i} + 3\underset{\sim}{j} - 2\underset{\sim}{k}$ $\quad \left|\overrightarrow{OA}\right| = \sqrt{(-1)^2 + 3^2 + (-2)^2} = \sqrt{14}$

$\overrightarrow{OB} = 2\underset{\sim}{i} + \underset{\sim}{j} - 3\underset{\sim}{k}$ $\quad \left|\overrightarrow{OB}\right| = \sqrt{2^2 + 1^2 + (-3)^2} = \sqrt{14}$ M1

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 3\underset{\sim}{i} - 2\underset{\sim}{j} - \underset{\sim}{k}$ $\quad \left|\overrightarrow{AB}\right| = \sqrt{3^2 + (-2)^2 + (-1)^2} = \sqrt{14}$

since $\left|\overrightarrow{OA}\right| = \left|\overrightarrow{OB}\right| = \left|\overrightarrow{AB}\right| = \sqrt{14}$ therefore *OAB* is an equilateral triangle. A1

b. $\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$

$\overrightarrow{OM} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$

$\overrightarrow{OM} = \overrightarrow{OA} + \frac{1}{2}\left(\overrightarrow{OB} - \overrightarrow{OA}\right)$

M1

$\overrightarrow{OM} = \frac{1}{2}\left(\overrightarrow{OA} + \overrightarrow{OB}\right)$

$\overrightarrow{OM} = \frac{1}{2}\left(\underset{\sim}{i} + 4\underset{\sim}{j} - 5\underset{\sim}{k}\right)$

$\overrightarrow{OG} = \frac{2}{3}\overrightarrow{OM} = \frac{1}{3}\left(\underset{\sim}{i} + 4\underset{\sim}{j} - 5\underset{\sim}{k}\right)$ A1

c. $\overrightarrow{OP} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$\overrightarrow{GP} = \overrightarrow{OP} - \overrightarrow{OG} = \left(x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}\right) - \frac{1}{3}\left(\underset{\sim}{i} + 4\underset{\sim}{j} - 5\underset{\sim}{k}\right)$

$\overrightarrow{GP} = \frac{1}{3}\left((3x-1)\underset{\sim}{i} + (3y-4)\underset{\sim}{j} + (3z+5)\underset{\sim}{k}\right)$ M1

Since $\overrightarrow{OG}$ is perpendicular to $\overrightarrow{GP}$

$\overrightarrow{OG}.\overrightarrow{GP} = 0 \Rightarrow \frac{1}{9}\left(1(3x-1) + 4(3y-4) - 5(3z+5)\right) = 0$ M1

$(1) \quad 3x + 12y - 15z = 42$

d. $\left|\overrightarrow{OP}\right| = \sqrt{x^2+y^2+z^2} = 3\sqrt{5}$

$(2)\ \ x^2+y^2+z^2 = 45$ A1

$\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA} = (x+1)\underset{\sim}{i} + (y-3)\underset{\sim}{j} + (z+2)\underset{\sim}{k}$

$\left|\overrightarrow{AP}\right| = \sqrt{(x+1)^2+(y-3)^2+(z+2)^2} = 3\sqrt{5}$ expanding M1

$x^2+2x+1+y^2-6y+9+z^2+4z+4 = 45$ from (2)

$(3)\quad 2x-6y+4z+14=0$

similarly

$\overrightarrow{BP} = \overrightarrow{OP} - \overrightarrow{OB} = (x-2)\underset{\sim}{i} + (y-1)\underset{\sim}{j} + (z+3)\underset{\sim}{k}$

$\left|\overrightarrow{BP}\right| = \sqrt{(x-2)^2+(y-1)^2+(z+3)^2} = 3\sqrt{5}$ expanding M1

$x^2-4x+4+y^2-2y+1+z^2+6z+9 = 45$ from (2)

$(4)\quad -4x-2y+6z+14=0$

e. Solving (1) (2) (3) and (4)

gives $x=4$ $y=5$ and $z=2$

or $x=-\frac{10}{3}$ $y=-\frac{7}{3}$ and $z=-\frac{16}{3}$ A1

both set of answers are acceptable, see **f.**

$eq1:=3\cdot x+12\cdot y-15\cdot z=42$	$3\cdot x+12\cdot y-15\cdot z=42$
$eq2:=x^2+y^2+z^2=45$	$x^2+y^2+z^2=45$
$eq3:=2\cdot x-6\cdot y+4\cdot z+14=0$	$2\cdot x-6\cdot y+4\cdot z+14=0$
$eq4:=-4\cdot x-2\cdot y+6\cdot z+14=0$	$-4\cdot x-2\cdot y+6\cdot z+14=0$
solve$\left(\begin{cases} eq1 \\ eq2 \\ eq3 \\ eq4 \end{cases},\{x,y,z\}\right)$	$x=\frac{-10}{3}$ and $y=\frac{-7}{3}$ and $z=\frac{-16}{3}$ or $x=4$ and $y=5$ and $z=2$

f. $\overrightarrow{GP} = \frac{11}{3}\left(\underset{\sim}{i}+\underset{\sim}{j}+\underset{\sim}{k}\right)$ if $x=4$ $y=5$ and $z=2$ M1

$\overrightarrow{GP} = -\frac{11}{3}\left(\underset{\sim}{i}+\underset{\sim}{j}+\underset{\sim}{k}\right)$ if $x=-\frac{10}{3}$ $y=-\frac{7}{3}$ and $z=-\frac{16}{3}$

The height of the pyramid in both cases is $\left|\overrightarrow{GP}\right| = \frac{11\sqrt{3}}{3}$ A1

Question 2

a. $\underset{\sim}{r}(0)=\left(6\sin(0)-3\sin(0)\right)\underset{\sim}{i}+\left(6\cos(0)+3\cos(0)\right)\underset{\sim}{j}$

$\underset{\sim}{r}(0)=0\underset{\sim}{i}+9\underset{\sim}{j}$

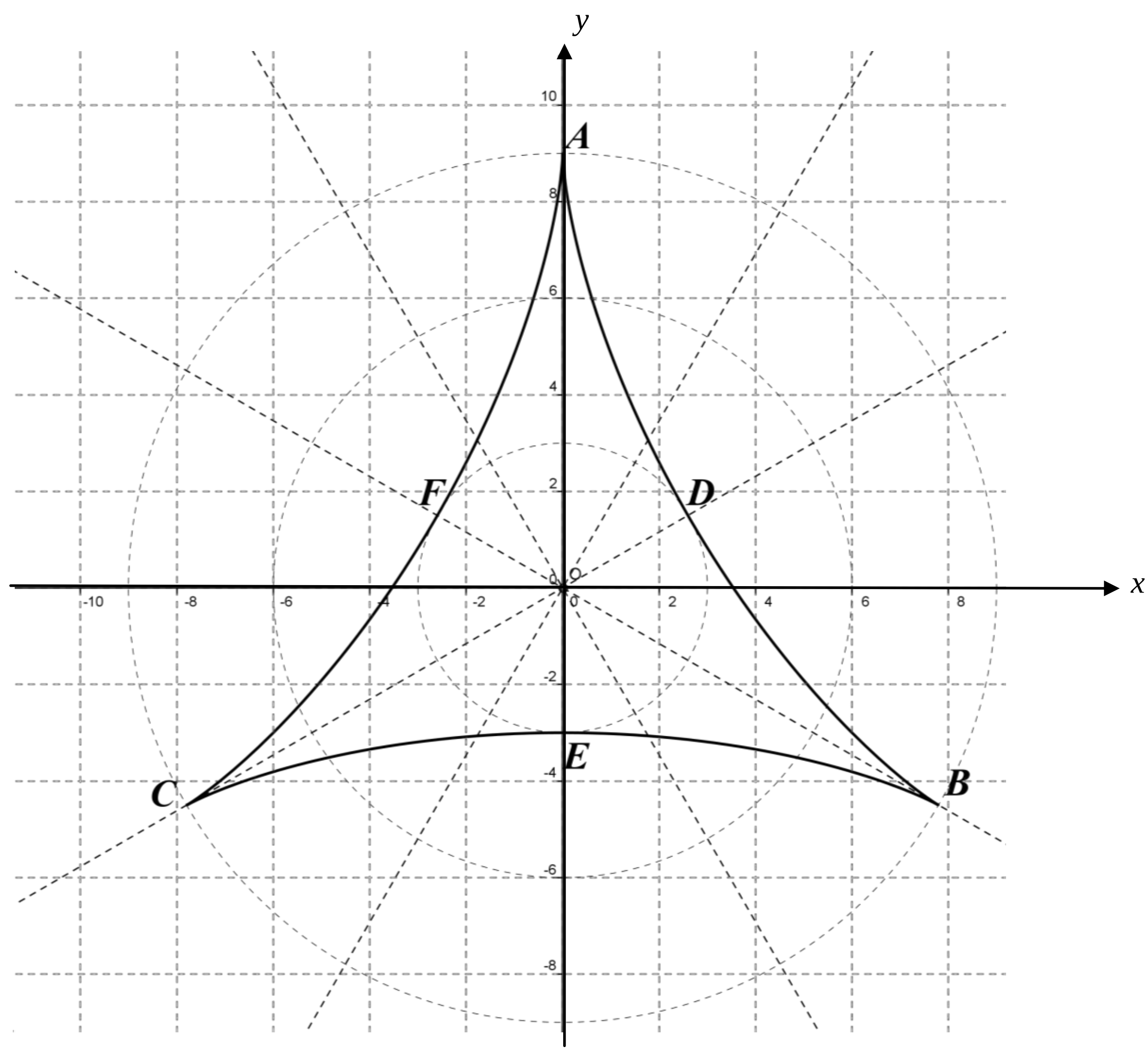
at $(0,9)$ A1

b. $\underset{\sim}{r}(\pi)=\left(6\sin(2\pi)-3\sin(4\pi)\right)\underset{\sim}{i}+\left(6\cos(2\pi)+3\cos(4\pi)\right)\underset{\sim}{j}$

$\underset{\sim}{r}(\pi)=\underset{\sim}{r}(0)=0\underset{\sim}{i}+9\underset{\sim}{j}$ at $(0,9)$ again

after π seconds. A1

c. correct shape and scale factors A2



d. $\dot{\underset{\sim}{r}}(t) = (12\cos(2t) - 12\cos(4t))\underset{\sim}{i} - (12\sin(2t) + 12\sin(4t))\underset{\sim}{j}$

$|\dot{r}(t)| = \sqrt{(12(\cos(2t)-\cos(4t)))^2 + (-12(\sin(2t)+\sin(4t)))^2}$ M1

$|\dot{r}(t)| = \sqrt{144(\cos^2(2t) - 2\cos(2t)\cos(4t) + \cos^2(4t) + \sin^2(2t) + 2\sin(2t)\sin(4t) + \sin^2(4t))}$

$|\dot{r}(t)| = \sqrt{144(2 - 2(\cos(2t)\cos(4t) - \sin(2t)\sin(4t)))}$

$|\dot{r}(t)| = 12\sqrt{2(1-\cos(6t))}$ M1

$|\dot{r}(t)| = 12\sqrt{4\sin^2(3t)}$

$|\dot{r}(t)| = 24|\sin(3t)|$ so that $a = 3$ A1

e. at rest $|\dot{r}(t)| = 0 \Rightarrow \sin(3t) = 0$

$3t = 0, \pi, 2\pi$

$t = 0, \frac{\pi}{3}, \frac{2\pi}{3}$ A1

$\underset{\sim}{r}(0) = 0\underset{\sim}{i} + 9\underset{\sim}{j}$ at $(0,9)$ point A

$\underset{\sim}{r}\left(\frac{\pi}{3}\right) = \frac{9}{2}\left(\sqrt{3}\underset{\sim}{i} - \underset{\sim}{j}\right)$ at $\left(\frac{9\sqrt{3}}{2}, -\frac{9}{2}\right)$ point B

$\underset{\sim}{r}\left(\frac{2\pi}{3}\right) = -\frac{9}{2}\left(\sqrt{3}\underset{\sim}{i} + \underset{\sim}{j}\right)$ at $\left(-\frac{9\sqrt{3}}{2}, -\frac{9}{2}\right)$ point C

correct points and labelled on the diagram in **c.** A1

f. maximum speed $|\dot{r}(t)| = 24\,\text{ms}^{-1}$ and occurs when $|\sin(3t)| = 1$ A1

$3t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

$t = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$ A1

$\underset{\sim}{r}\left(\frac{\pi}{6}\right) = \frac{3}{2}\left(\sqrt{3}\underset{\sim}{i} + \underset{\sim}{j}\right)$ at $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$ point D

$\underset{\sim}{r}\left(\frac{\pi}{2}\right) = -3\underset{\sim}{j}$ at $(0,-3)$ point E

$\underset{\sim}{r}\left(\frac{5\pi}{6}\right) = \frac{3}{2}\left(-\sqrt{3}\underset{\sim}{i} + \underset{\sim}{j}\right)$ at $\left(-\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$ point F

correct points and labelled on the diagram in **c.** A1

g. $\int_0^{\pi} 24|\sin(3t)|\,dt = 3\int_0^{\frac{\pi}{3}} 24\sin(3t)\,dt = 48$ metres A1

Define $r(t)=[6\cdot\sin(2\cdot t)-3\cdot\sin(4\cdot t) \quad 6\cdot\cos(2\cdot t)+3\cdot\cos(4\cdot t)]$	*Done*
$r(0)$	$[0 \quad 9]$
$r(\pi)$	$[0 \quad 9]$
Define $v(t)=\frac{d}{dt}(r(t))$	*Done*
tCollect(norm($v(t)$))	$12\cdot\sqrt{-2\cdot(\cos(6\cdot t)-1)}$
$r\left(\frac{\pi}{3}\right)$	$\left[\frac{9\cdot\sqrt{3}}{2} \quad \frac{-9}{2}\right]$
$r\left(\frac{2\cdot\pi}{3}\right)$	$\left[\frac{-9\cdot\sqrt{3}}{2} \quad \frac{-9}{2}\right]$
$r\left(\frac{\pi}{6}\right)$	$\left[\frac{3\cdot\sqrt{3}}{2} \quad \frac{3}{2}\right]$
$r\left(\frac{\pi}{2}\right)$	$[0 \quad -3]$
$r\left(\frac{5\cdot\pi}{6}\right)$	$\left[\frac{-3\cdot\sqrt{3}}{2} \quad \frac{3}{2}\right]$

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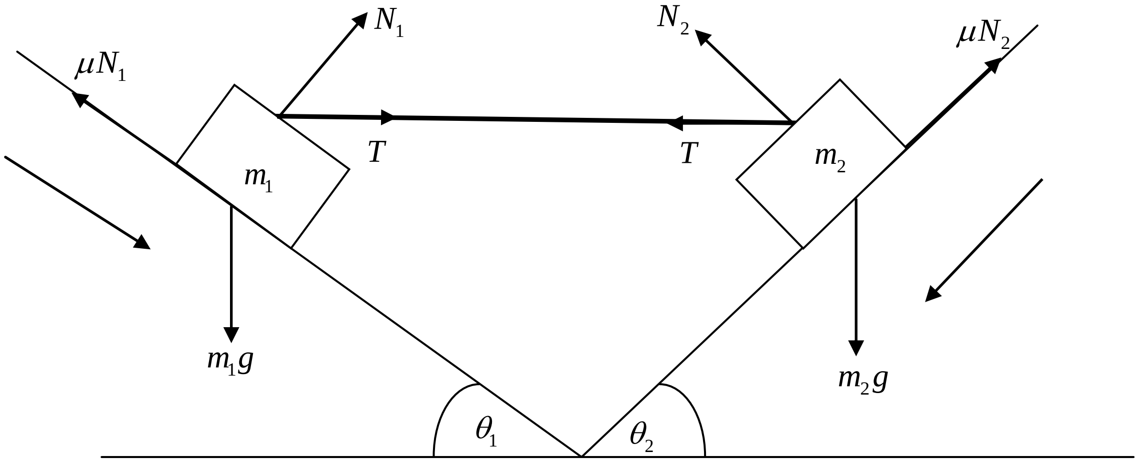
Question 3

a.



The forces all given in newtons, acting on the mass m_1 are the weight force $m_1 g$, the normal reaction N_1, the tension in the rod *T*, and the frictional force μN_1.
The forces acting on the mass m_2 are the weight force $m_2 g$, the normal reaction N_2, the tension in the rod *T*, and the frictional force μN_2. A1

b. For the particle of mass m_1 on the slope inclined at θ_1 to the horizontal,

since m_1 is on the point of moving down the plane,

resolving perpendicular to the slope

$(1)\ N_1 + T\sin(\theta_1) - m_1 g\cos(\theta_1) = 0$ A1

resolving downwards parallel to the slope

$(2)\ m_1 g\sin(\theta_1) + T\cos(\theta_1) - \mu N_1 = 0$ A1

For the particle of mass m_2 on the slope inclined at θ_2 to the horizontal,

since m_2 is on the point of moving down the plane,

resolving perpendicular to the slope

$(3)\ N_2 + T\sin(\theta_2) - m_2 g\cos(\theta_2) = 0$

resolving downwards parallel to the slope

$(4)\ m_2 g\sin(\theta_2) + T\cos(\theta_2) - \mu N_2 = 0$

eliminating the normal reaction, M1

from (1) $N_1 = m_1 g\cos(\theta_1) - T\sin(\theta_1)$ substitute into (2)

(2) $m_1 g\sin(\theta_1) + T\cos(\theta_1) - \mu\left(m_1 g\cos(\theta_1) - T\sin(\theta_1)\right) = 0$

$\Rightarrow \quad T\left(\cos(\theta_1) + \mu\sin(\theta_1)\right) = m_1 g\left(\mu\cos(\theta_1) - \sin(\theta_1)\right)$

$\Rightarrow \quad T = \dfrac{m_1 g\left(\mu\cos(\theta_1) - \sin(\theta_1)\right)}{\cos(\theta_1) + \mu\sin(\theta_1)}$ M1

similarly from (3) $N_2 = m_2 g\cos(\theta_2) - T\sin(\theta_2)$ substitute into (4)

(4) $m_2 g\sin(\theta_2) + T\cos(\theta_2) - \mu\left(m_2 g\cos(\theta_2) - T\sin(\theta_2)\right) = 0$

$\Rightarrow \quad T\left(\cos(\theta_2) + \mu\sin(\theta_2)\right) = m_2 g\left(\mu\cos(\theta_2) - \sin(\theta_2)\right)$

$\Rightarrow \quad T = \dfrac{m_2 g\left(\mu\cos(\theta_2) - \sin(\theta_2)\right)}{\cos(\theta_2) + \mu\sin(\theta_2)}$

now eliminating *T*, gives

$$\frac{m_1 g\left(\mu\cos(\theta_1) - \sin(\theta_1)\right)}{\cos(\theta_1) + \mu\sin(\theta_1)} = \frac{m_2 g\left(\mu\cos(\theta_2) - \sin(\theta_2)\right)}{\cos(\theta_2) + \mu\sin(\theta_2)}$$

$$\frac{m_1}{m_2} = \frac{\left(\mu\cos(\theta_2) - \sin(\theta_2)\right)}{\left(\mu\cos(\theta_1) - \sin(\theta_1)\right)}\frac{\left(\cos(\theta_1) + \mu\sin(\theta_1)\right)}{\left(\cos(\theta_2) + \mu\sin(\theta_2)\right)}$$

now divide both numerator and denominator by $-\cos(\theta_1)\cos(\theta_2)$ M1

$$\frac{m_1}{m_2} = \frac{\left(\tan(\theta_2) - \mu\right)}{\left(\tan(\theta_1) - \mu\right)}\frac{\left(1 + \mu\tan(\theta_1)\right)}{\left(1 + \mu\tan(\theta_2)\right)}$$

c. resolving around m_1 perpendicular to the slope

$(1)\ N_1 - m_1 g\cos(\theta_1) = 0$

resolving downwards parallel to the slope

$(2)\ m_1 g\sin(\theta_1) - \mu N_1 = 0$

$m_1 g\sin(\theta_1) = \mu m_1 g\cos(\theta_1)$

$\Rightarrow\ \mu = \tan(\theta_1)$ M1

resolving around m_2 perpendicular to the slope

$(3)\ N_2 - m_2 g\cos(\theta_2) = 0$

resolving downwards parallel to the slope

$(4)\ m_2 g\sin(\theta_2) - \mu N_2 = m_2 a$

from (3) $N_2 = m_2 g\cos(\theta_2)$ substitute into (4)

$m_2 a = m_2 g\sin(\theta_2) - \mu m_2 g\cos(\theta_2)$

$a = g\left(\sin(\theta_2) - \mu\cos(\theta_2)\right)$ A1

$u = 0 \quad s = D \quad t = T \quad \mu = \tan(\theta_1)$ and $\theta_2 = 2\theta_1$

using $s = ut + \frac{1}{2}at^2$

$D = \frac{g}{2}\left(\sin(\theta_2) - \mu\cos(\theta_2)\right)T^2$ but $\mu = \tan(\theta_1)$ and $\theta_2 = 2\theta_1$

$\frac{2D}{gT^2} = \sin(2\theta_1) - \tan(\theta_1)\cos(2\theta_1)$ M1

$\frac{2D}{gT^2} = \sin(2\theta_1) - \frac{\sin(\theta_1)\cos(2\theta_1)}{\cos(\theta_1)}$

$\frac{2D}{gT^2} = \frac{\sin(2\theta_1)\cos(\theta_1) - \sin(\theta_1)\cos(2\theta_1)}{\cos(\theta_1)}$

$\frac{2D}{gT^2} = \frac{\sin(2\theta_1 - \theta_1)}{\cos(\theta_1)} = \frac{\sin(\theta_1)}{\cos(\theta_1)} = \tan(\theta_1)$ A1

$\theta_1 = \tan^{-1}\left(\frac{2D}{gT^2}\right)$

Question 4

a. $|z-c|=1$, $c=-2+i$ let $z=x+yi$

$|(x+2)+(y-1)i|=1$

$\sqrt{(x+2)^2+(y-1)^2}=1$

$(x+2)^2+(y-1)^2=1$

a circle with centre $(-2,1)$ and radius 1. A1

b. $\left|z-\frac{1}{5}(-2+11i)\right|=|z-c|$, $c=-2+i$ let $z=x+yi$

$\left|\left(x+\frac{2}{5}\right)+\left(y-\frac{11}{5}\right)i\right|=|(x+2)+(y-1)i|$

$\sqrt{\left(x+\frac{2}{5}\right)^2+\left(y-\frac{11}{5}\right)^2}=\sqrt{(x+2)^2+(y-1)^2}$ M1

$x^2+\frac{4x}{5}+\frac{4}{25}+y^2-\frac{22y}{5}+\frac{121}{25}=x^2+4x+4+y^2-2y+1$

$12y+16x=0$, T is the line $y=-\frac{4x}{3}$

$3y+4x=0$ $z=x+yi$ $\text{Re}(z)=x$ and $\text{Im}(z)=y$

$p=3$ and $q=4$ A1

c. from **b.** $y=-\frac{4x}{3}$ substitute into $(x+2)^2+(y-1)^2=1$

$(x+2)^2+\left(-\frac{4x}{3}-1\right)^2=1$

$x^2+4x+4+\frac{16x^2}{9}+\frac{8x}{3}+1=1$ M1

$25x^2+60x+36=0$

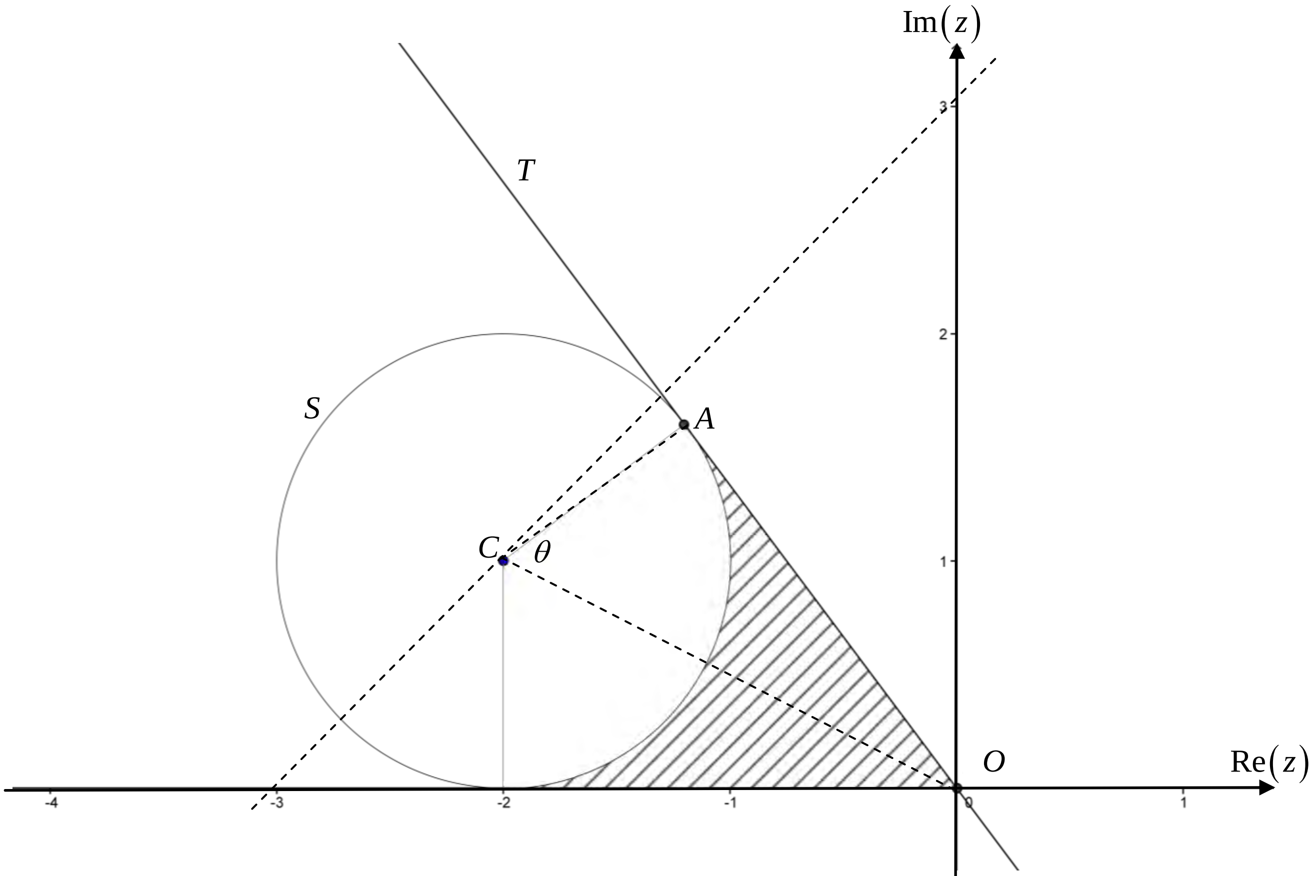
$(5x+6)^2=0$

$x=-\frac{6}{5}\Rightarrow y=\frac{8}{5}$

$A\left(-\frac{6}{5},\frac{8}{5}\right)$ A1

d. S is the circle with centre $C(-2,1)$ and radius 1, and T is the line $y=-\frac{4x}{3}$

intersecting at the point $A\left(-\frac{6}{5},\frac{8}{5}\right)$ A2



e. since the gradient of the line T is $m = -\frac{4}{3}$, α is the angle T makes with the positive real axis. $\operatorname{Arg}(z) = \alpha = \pi - \tan^{-1}\left(\frac{4}{3}\right)$ A1

f. the maximum value of $|z| \in S$, is the furthest point on S, from the origin, since $OC = \sqrt{5}$ and $CA = 1$ the radius of the circle, then

$z \in S \quad |z|_{\max} = \sqrt{5} + 1$ A1

g. The shaded area is twice the area of the triangle OAC, minus the area of the sector of the circle. Now $\angle OAC = 90^0$, since OA is a tangent to the circle, also

$OA = 2 \quad AC = 1 \quad OC = \sqrt{5}$, let $\theta = \angle ACO$, M1

$\sin(\theta) = \frac{2}{\sqrt{5}} \quad \cos(\theta) = \frac{1}{\sqrt{5}}$ and $\tan(\theta) = 2$

$\text{Area} = 2\left(\frac{1}{2} \times 2 \times 1 - \frac{1}{2} \times 1^2 \times \theta\right) = 2 - \theta$

$\text{Area} = 2 - \tan^{-1}(2)$ or alternatively A1

$\text{Area} = 2 - \sin^{-1}\left(\frac{2}{\sqrt{5}}\right) = 2 - \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$

Question 5

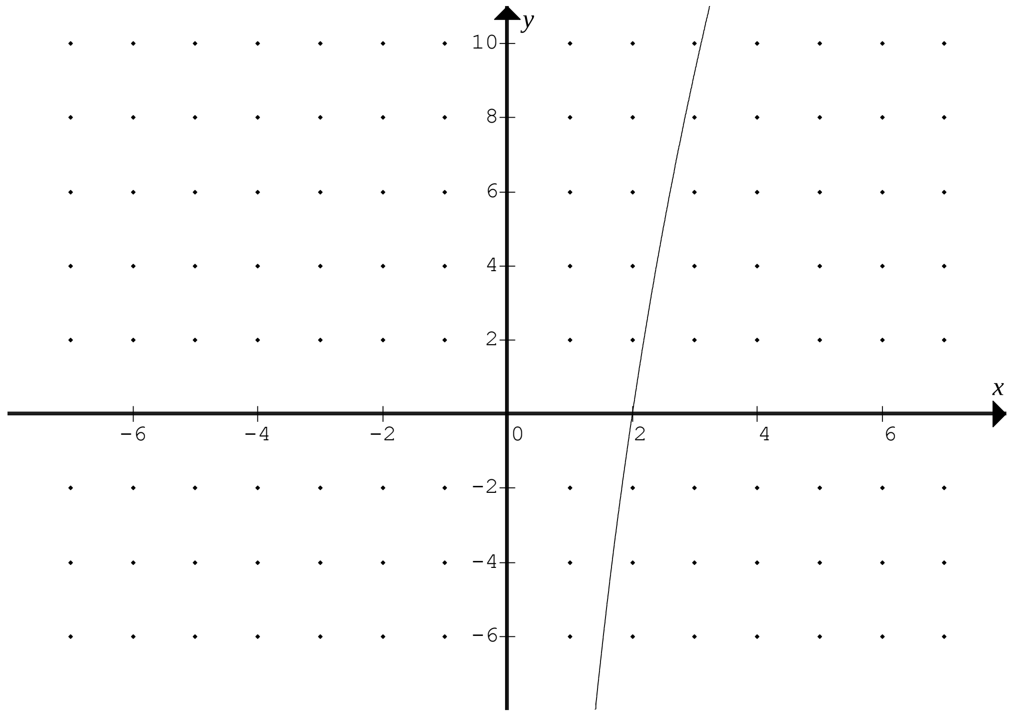
a. $y = \dfrac{16(x-2)}{\sqrt{x}}$

$\dfrac{dy}{dx} = \dfrac{8(x+2)}{\sqrt{x^3}}$ for turning points $\dfrac{dy}{dx} = 0 \Rightarrow x = -2$ A1

but the domain of the function is $x > 0$ so there are no turning points. A1

b. crosses the *x*-axis at $x = 2$, $(2,0)$

and the *y*-axis, $x = 0$ is a vertical asymptote. A1



c. $y = \dfrac{16(x-2)}{\sqrt{x}}$

$16x - 32 = y\sqrt{x}$

$16x - y\sqrt{x} - 32 = 0$ A1

using the quadratic formulae,

$\sqrt{x} = \dfrac{y \pm \sqrt{y^2 - 4\times 16 \times {}^{-}32}}{32}$ since $\sqrt{x} > 0$ must take the positive A1

$\sqrt{x} = \dfrac{y + \sqrt{y^2 + 2048}}{32}$

$x = \dfrac{\left(y + \sqrt{y^2 + 2048}\right)^2}{1024}$

d. solving $y=\dfrac{16(x-2)}{\sqrt{x}}=9$ gives $x=\dfrac{9\sqrt{2129}+1105}{512}$ as the radius

the diameter is $\dfrac{9\sqrt{2129}+1105}{256}$ cm A1

e.i. $V=\pi\int_a^b x^2\,dy$

$$V=\pi\int_0^9 \frac{\left(y+\sqrt{y^2+2048}\right)^4}{1048576}\,dy$$ A1

ii. $V=172.54\ \text{cm}^3$ A1

f. solving $V=\pi\int_0^h \dfrac{\left(y+\sqrt{y^2+2048}\right)^4}{1048576}\,dy=150$

gives $h=8.16\,\text{cm}$ A1

g. $$V=\pi\int_0^h \frac{\left(y+\sqrt{y^2+2048}\right)^4}{1048576}\,dy$$

$$\left.\frac{dV}{dh}\right|_{h=4}=\frac{\pi\left(65\sqrt{129}+2177\right)}{512}$$ A1

given $\dfrac{dV}{dt}=2\,\text{cm}^3/\text{sec}$ and $\dfrac{dh}{dt}=\dfrac{dh}{dV}\dfrac{dV}{dt}$ M1

$$\frac{dh}{dt}=\frac{1024}{\pi\left(65\sqrt{129}+2177\right)}\ \text{cm/s}$$ A1

Define $f1(x)=\dfrac{16\cdot(x-2)}{\sqrt{x}}$	*Done*
$\dfrac{d}{dx}(f1(x))$	$\dfrac{8\cdot(x+2)}{x^{\frac{3}{2}}}$
solve$(f1(x)=9,x)$	$x=\dfrac{9\cdot\sqrt{2129}+1105}{512}$
Define $v(h)=\pi\cdot\displaystyle\int_0^h \dfrac{\left(y+\sqrt{y^2+2048}\right)^4}{1048576}\,dy$	*Done*
$v(9)$	172.537
solve$(v(h)=150,h)$	$h=8.15617$
$\dfrac{d}{dh}(v(h))\vert h=4$	$\dfrac{(65\cdot\sqrt{129}+2177)\cdot\pi}{512}$
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END OF SECTION 2 SUGGESTED ANSWERS