Year 2012

VCE

Specialist Mathematics

Trial Examination 2

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Victorian Certificate of Education **2012**

STUDENT NUMBER

										Letter
Figures										
Words										

SPECIALIST MATHEMATICS

Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer booklet of 35 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Answer sheet for multiple choice questions.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Write your **name** and **student number** on your answer sheet for multiple-choice questions and sign your name in the space provided.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section I

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1 mark, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

The graphs of $\dfrac{(x-a)^2}{a^2}+\dfrac{(y+b)^2}{b^2}=1$ and $\dfrac{(x+a)^2}{a^2}-\dfrac{(y-b)^2}{b^2}=1$, where a and b are positive real constants,

A. do not intersect.

B. touch each other at one point.

C. intersect at two points.

D. intersect at three points.

E. intersect at four points.

Question 2

Let $\underset{\sim}{a}=\underset{\sim}{i}+\underset{\sim}{j}$, $\underset{\sim}{b}=\underset{\sim}{j}+\underset{\sim}{k}$ and $\underset{\sim}{c}=2\underset{\sim}{i}-\underset{\sim}{j}-3\underset{\sim}{k}$. Which of the following is **false?**

A. The vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ have the same length.

B. The angle between the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is 60^0.

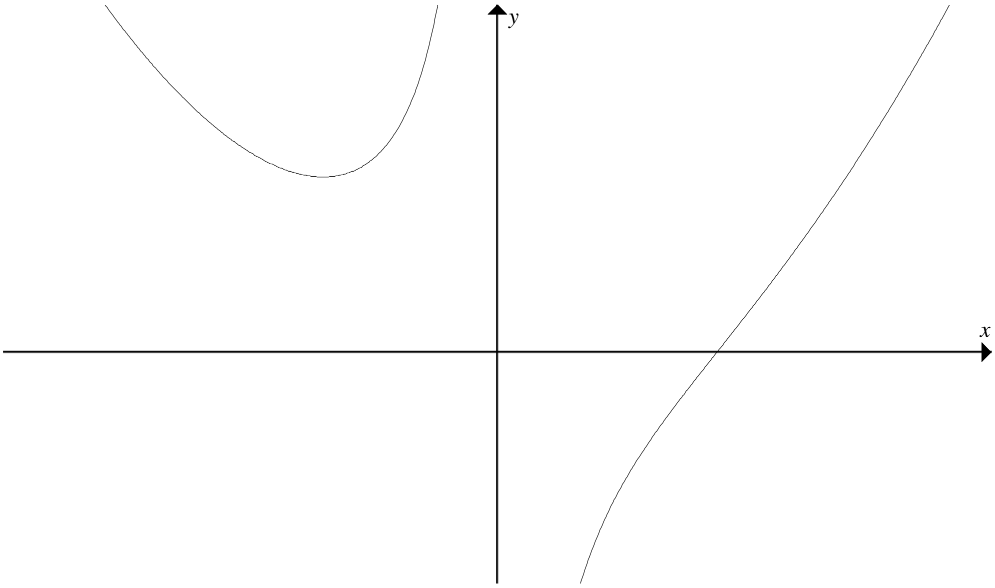
C. The vector $\underset{\sim}{a}+\underset{\sim}{c}$ is parallel to the vector $\underset{\sim}{a}-\underset{\sim}{b}$.

D. $\underset{\sim}{c}=2\underset{\sim}{a}-3\underset{\sim}{b}$.

E. The vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are linearly independent.

Question 3

For the graph of $y=\dfrac{x^n-64}{2x}$ shown below, n is equal to



A. 0

B. 1

C. 2

D. 3

E. 4

Question 4

For the graph of $y=\dfrac{x^2+a^2}{x^2-a^2}$ where a is a non-zero real constant, then

A. the maximal domain is $R\backslash\{\pm a\}$ and the range is $R\backslash\{0\}$.

B. the graph has vertical asymptotes at $x=\pm a$ and a horizontal asymptote at $y=0$.

C. the graph crosses the y-axis at $y=-1$ and this point $(0,-1)$ is a minimum turning point.

D. the graph has vertical asymptotes at $x=\pm a$ and a horizontal asymptote at $y=-1$.

E. the graph does not cross the x-axis and the line $y=1$ is a horizontal asymptote and the range is $(-\infty,-1]\cup(1,\infty)$.

Question 5

The hyperbola that has asymptotes $y=-2x$ and $y=2x-4k$ where k is a positive real number, could be

A. $\dfrac{(x-k)^2}{k^2}-\dfrac{(y+2k)^2}{4k^2}=1$

B. $\dfrac{(x+k)^2}{4k^2}-\dfrac{(y-2k)^2}{k^2}=1$

C. $\dfrac{(x-2k)^2}{4k^2}-\dfrac{(y-k)^2}{k^2}=1$

D. $\dfrac{(x+k)^2}{k^2}-\dfrac{(y+2k)^2}{2k^2}=1$

E. $\dfrac{(y+2k)^2}{2k^2}-\dfrac{(x-k)^2}{k^2}=1$

Question 6

Which of the following, have both the domain and range equal to $[0,2a]$ where $a>0$.

A. $y=\dfrac{a}{\pi}\cos^{-1}\left(\dfrac{x}{a}\right)+a$

B. $y=\dfrac{2a}{\pi}\cos^{-1}\left(\dfrac{x}{a}-1\right)$

C. $y=\dfrac{2a}{\pi}\tan^{-1}\left(\dfrac{x}{a}-1\right)$

D. $y=\dfrac{a}{\pi}\sin^{-1}\left(\dfrac{x}{a}\right)+a$

E. $y=\dfrac{2a}{\pi}\sin^{-1}\left(\dfrac{x}{a}-1\right)$

Question 7

Consider the graph of $y = \cot(px)$ where $p > 0$ and $n \in Z$. Then the graph

A. does not cross the *x*-axis and does not have any asymptotes.

B. has vertical asymptotes $x = \dfrac{(2n+1)\pi}{2p}$ and crosses the *x*-axis at $x = \dfrac{n\pi}{2p}$.

C. has vertical asymptotes $x = \dfrac{n\pi}{p}$ and crosses the *x*-axis at $x = \dfrac{(2n+1)\pi}{2p}$.

D. has vertical asymptotes $x = \dfrac{(2n+1)\pi}{2p}$ and does not cross the *x*-axis.

E. crosses the *x*-axis at $x = \dfrac{n\pi}{p}$ and does not have any asymptotes.

Question 8

The polynomial $P(z)$ has real coefficients. Three of the roots of $P(z) = 0$ are $z = ki$, $z = 2k$ and $z = -k$, where $k \in R \setminus \{0\}$, then $P(z)$ is equal to

A. $z^4 + kz^3 - k^2z^2 + k^3z - 2k^4$

B. $z^4 - kz^3 - k^2z^2 - k^3z - 2k^4$

C. $z^4 - kz^3 - k^2z^2 + k^3z + 2k^4$

D. $z^3 - kz^2 + k^2z - k^3$

E. $z^2 - kz - 2k^2$

Question 9

If *a* and *b* are non-zero real numbers, and $z = x + yi$, then the set of points in the argand plane defined by $\{ z : |z-a|^2 - |z-bi|^2 = a^2 + b^2 \}$ represents

A. a straight line.

B. a parabola.

C. an ellipse.

D. a hyperbola.

E. a circle.

Question 10

If $u = 4\text{cis}(\theta)$, $v = 2\text{cis}\left(\frac{-4\pi}{5}\right)$ and $\frac{u}{v} = -2i$, then θ is equal to

A. $-\frac{\pi}{5}$

B. $-\frac{3\pi}{10}$

C. $\frac{\pi}{5}$

D. $\frac{7\pi}{10}$

E. $\frac{13\pi}{10}$

Question 11

Let $\underset{\sim}{a} = 4\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{b} = -2\underset{\sim}{i} + t\,\underset{\sim}{j} + \underset{\sim}{k}$. Which of the following is **false?**

A. If $t = \pm 2$ then $|\underset{\sim}{b}| = 3$.

B. If $t = 2$ then angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is 135^0.

C. If $t = -7$ then the vector $\underset{\sim}{a}$ is perpendicular to the vector $\underset{\sim}{b}$.

D. If $t = \frac{1}{2}$ then the vector $\underset{\sim}{a}$ is parallel to the vector $\underset{\sim}{b}$.

E. If $t = -10$ then the scalar resolute of $\underset{\sim}{b}$ in the direction of $\underset{\sim}{a}$ is equal $\frac{1}{\sqrt{2}}$.

Question 12

A particle moves so that its velocity vector at time *t*, is given by
$\dot{\underset{\sim}{r}}(t) = 4\cos(2t)\underset{\sim}{i} + 2\sin(2t)\underset{\sim}{j}$ for $t \geq 0$. Given that $\underset{\sim}{r}(0) = \underset{\sim}{0}$, then the particle moves on

A. an ellipse with centre at $(0,0)$.

B. an ellipse with centre at $(0,1)$.

C. a circle with centre at $(0,0)$.

D. a hyperbola with centre at $(0,0)$.

E. a hyperbola with centre at $(0,1)$.

Question 13

Using a suitable substitution, $\int_0^{\frac{3}{2}} \frac{x^3}{\sqrt{9-4x^2}}\,dx$ can be expressed, in terms of *u* as

A. $\frac{1}{2}\int_0^9 \frac{9-u}{\sqrt{u}}\,du$

B. $2\int_0^9 \frac{9-u}{\sqrt{u}}\,du$

C. $\frac{1}{32}\int_0^9 \frac{9-u}{\sqrt{u}}\,du$

D. $\frac{1}{2}\int_0^{\frac{9}{4}} \frac{u^2}{\sqrt{9-4u}}\,du$

E. $2\int_0^{\frac{9}{4}} \frac{u^2}{\sqrt{9-4u}}\,du$

Question 14

A body moves in a straight line such that its velocity v ms^{-1} is given by $v(x)=e^{2x}-e^{-2x}$, where *x* metres is its displacement from the origin. The acceleration of the body in ms^{-2} is given by

A. $4\left(e^{4x}-e^{-4x}\right)$

B. $2\left(e^{4x}-e^{-4x}\right)$

C. $e^{4x}-e^{-4x}$

D. $2\left(e^{2x}+e^{-2x}\right)$

E. $-4x$

Question 15

A golf ball is hit off the ground, its position vector, at a time t seconds after being hit, is given by $\underset{\sim}{r}(t) = 25t\cos\left(40^0\right)\underset{\sim}{i} + \left(25t\sin\left(40^0\right) - \frac{1}{2}gt^2\right)\underset{\sim}{k}$ for $t \ge 0$, where $\underset{\sim}{i}$ is a unit vector in metres horizontally forward, and $\underset{\sim}{k}$ is a unit vector in metres vertically upwards. Students when analysing the motion of the golf ball, stated some propositions

i. The golf ball is hit with an initial velocity of 25 ms^{-1} at an angle of 40^0.

ii. The golf ball hits the ground again after a time of 3.28 seconds.

iii. The golf ball reaches a maximum height of 13.175 metres.

iv. The golf ball first hits the ground at a distance of 62.807 metres from where it was hit.

v. The golf ball travels in a parabolic path.

Then

A. Only **i. ii.** and **v.** are correct.

B. Only **i. iii.** and **v.** are correct.

C. Only **i. iii. iv.** and **v.** are correct.

D. Only **i. ii. iii.** and **iv.** are correct.

E. All of **i. ii. iii. iv** and **v.** are correct.

Question 16

A particle moves so that at a time t seconds, its velocity v ms^{-1} is given by

$$v(t) = \begin{cases} \sqrt{6t - t^2} + 4 & \text{for } 0 \le t \le 6 \\ 16 - 2t & \text{for } 6 \le t \le 12 \end{cases}$$

The distance travelled in metres by the particle over the first 12 seconds, is closest to

A. 76

B. 72

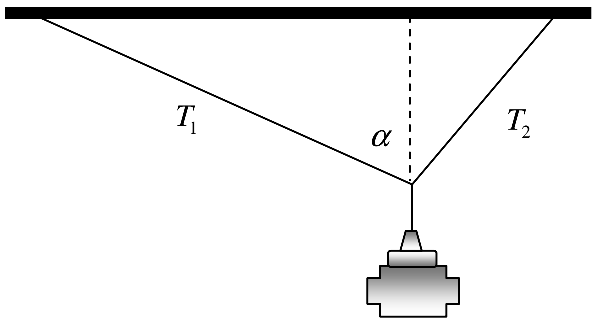
C. 58

D. 40

E. 26

Question 17

An engine weighing 5 kg is suspended by two ropes at right angles to one another, which support tensions of T_1 and T_2 newtons. The rope supporting a tension of T_1 makes an angle of α to the vertical as shown in the diagram below.



Then

A. $T_1 = 5\sin(\alpha)$ and $T_2 = 5\cos(\alpha)$

B. $T_1 = 5\cos(\alpha)$ and $T_2 = 5\sin(\alpha)$

C. $T_1 = 5\tan(\alpha)$ and $T_2 = \dfrac{5}{\tan(\alpha)}$

D. $T_1 = 49\sin(\alpha)$ and $T_2 = 49\cos(\alpha)$

E. $T_1 = 49\cos(\alpha)$ and $T_2 = 49\sin(\alpha)$

Question 18

The acceleration of a body is given by $e^{-t}\cos(2t)$ ms^{-2} at a time *t* seconds and initially the body is at rest. Euler's method is used with a step size of 0.25 in the values of *t*. The velocity of the body in ms^{-1} correct to two decimal places when $t = 0.5$ is equal to

A. 0.25

B. 0.34

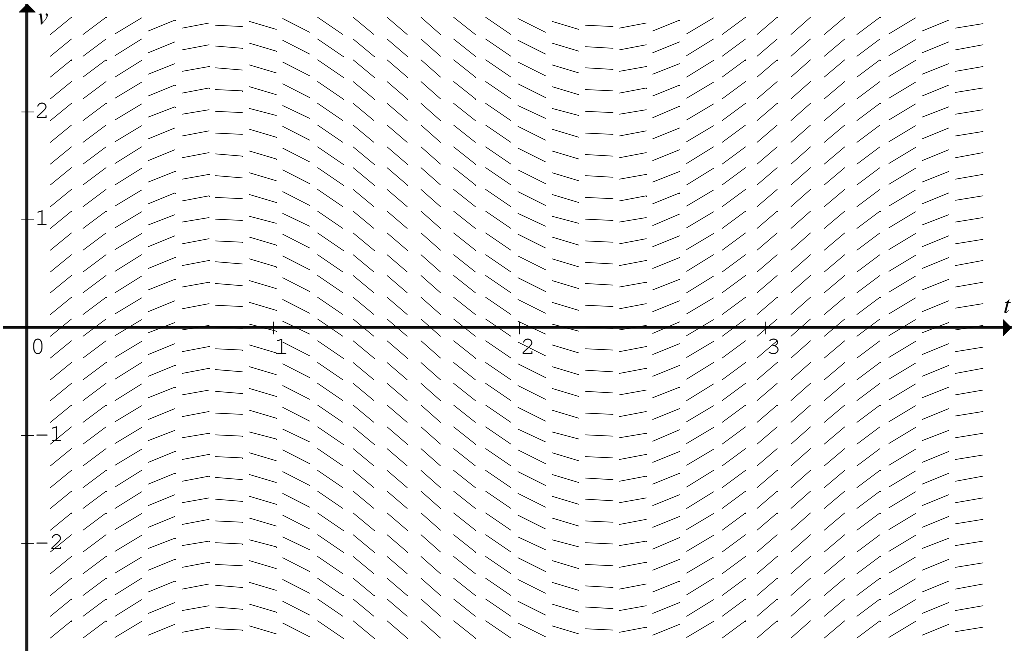
C. 0.39

D. 0.42

E. 0.44

Question 19

The direction field for the velocity v ms^{-1} of a particle moving at a time t seconds, for $t \geq 0$ is shown below.



The acceleration of the particle in ms^{-2} is given by

A. $\sin(2t)$

B. $\cos(2t)$

C. $2\cos(2t)$

D. $-4\cos(2t)$

E. $-4\sin(2t)$

Question 20

A constant force of 10 newtons acts on a mass of 5 kg initially moving at 1 ms^{-1}. After the mass has moved a distance of 20 metres, the magnitude of the momentum in kgms^{-1}, is equal to

A. 9

B. 10

C. 40

D. 45

E. 50

Question 21

The equation of the normal to the curve $y = \cos^{-1}(2x)$ at the point where $x = \frac{1}{4}$ is given by

A. $y = \frac{\sqrt{3}\,x}{4} - \frac{\sqrt{3}}{16} + \frac{\pi}{3}$

B. $y = \frac{\sqrt{3}\,x}{4} - \frac{\sqrt{3}}{16} + \frac{\pi}{6}$

C. $y = -\frac{\sqrt{3}\,x}{4} + \frac{\sqrt{3}}{16} + \frac{\pi}{3}$

D. $y = \frac{-4\sqrt{3}\,x}{3} + \frac{2\sqrt{3} + \pi}{6}$

E. $y = \frac{-4\sqrt{3}\,x}{3} + \frac{\sqrt{3} + \pi}{3}$

Question 22

A chicken is taken from a refrigerator at a temperature of 3°C and placed in a preheated oven at a temperature of 200°C. If k is a positive constant, then the differential equation describing the temperature T°C of the chicken, at a time t after placed in the oven, is given by

A. $\frac{dT}{dt} = -k(T - 200) \quad T(0) = 3$

B. $\frac{dT}{dt} = k(T - 200) \quad T(0) = 3$

C. $\frac{dT}{dt} = -k(T - 197) \quad T(0) = 0$

D. $\frac{dT}{dt} = -k(T - 3) \quad T(0) = 200$

E. $\frac{dT}{dt} = k(T - 3) \quad T(0) = 200$

END OF SECTION 1

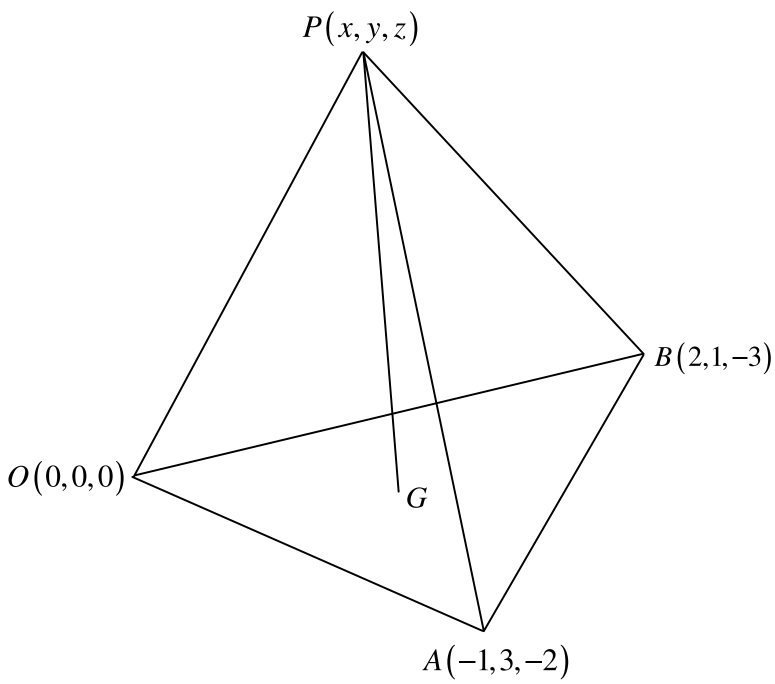
SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
Unless otherwise specified an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

OABP is a pyramid, where *O* is the origin. The coordinates of the points are $A(-1,3,-2)$, $B(2,1,-3)$ and $P(x,y,z)$. The height of the pyramid is the length of *GP*, where *G* is a point on the base of *OAB*, such that *GP* is perpendicular to the base.



a. Show using vectors that *OAB* is an equilateral triangle.

2 marks

b. Let M be the midpoint of AB. The point G is such that $\overrightarrow{OG} = \frac{2}{3}\overrightarrow{OM}$,

find the vector $\overrightarrow{OG}$.

2 marks

c. Find the vector $\overrightarrow{GP}$ and using the fact that $\overrightarrow{GP}$ is perpendicular to $\overrightarrow{OG}$, show that $3x + 12y - 15z = 42$.

2 marks

d. The faces of the pyramid, *OAP*, *ABP* and *OBP* are all similar isoceles triangles, with $OP = AP = BP = 3\sqrt{5}$. Write down another set of three equations involving *x*, *y* and *z*.

3 marks

e. Find the values of x , y and z.

1 mark

f. Determine the height of the pyramid.

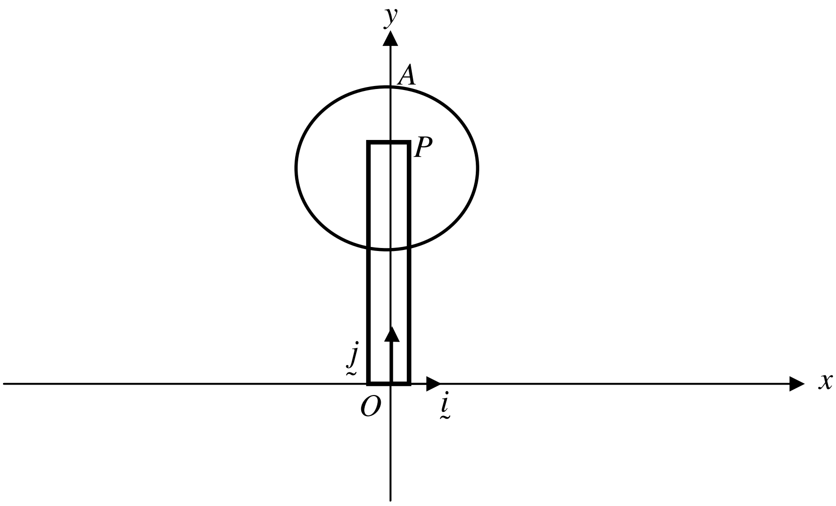
2 marks
Total 12 marks

Question 2

A ride at an amusement park, consists of riders seated on a circular platform. The platform has its centre at a point *P*, and this platform rotates clockwise around the point *P*. At the same time a bar *OP* supporting the platform also rotates clockwise in a plane around the origin *O*. The position vector of Ashley a rider, indicated by the point *A*, on the diagram below at a time *t* seconds after the ride starts is given by

$$\underset{\sim}{r}(t) = (6\sin(2t) - 3\sin(4t))\underset{\sim}{i} + (6\cos(2t) + 3\cos(4t))\underset{\sim}{j} \quad t \geq 0$$

The *x* and *y* axes are displayed, and $\underset{\sim}{i}$ and $\underset{\sim}{j}$ are perpendicular unit vectors in the plane, with units measured in metres.



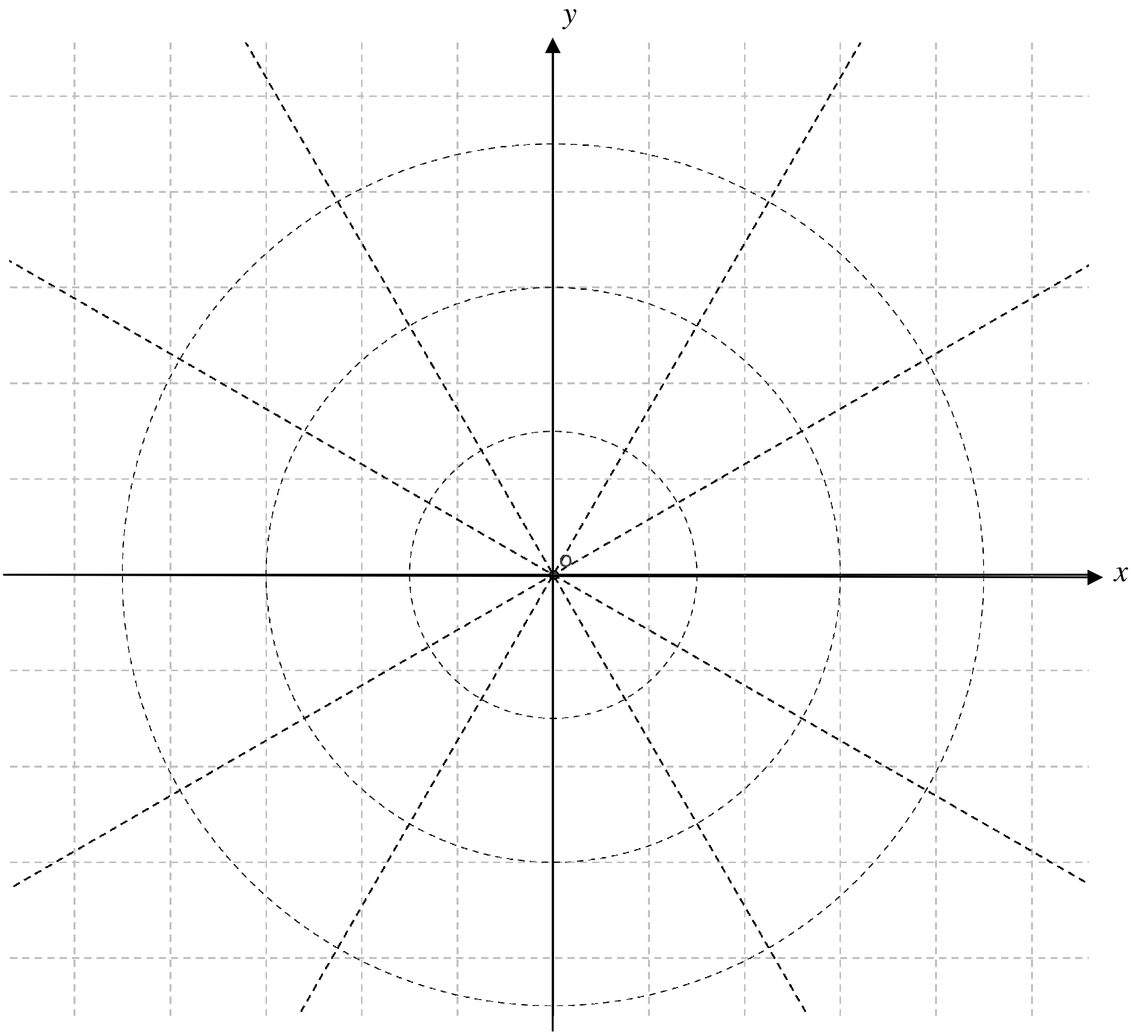
a. Find the coordinates of Ashley initially when the ride starts.

1 mark

b. Find the time in seconds, taken for Ashley to first return to his initial starting point.

1 mark

c. On the diagram below, sketch the graph of the path of Ashley, clearly indicating the scale. Show one cycle, that is from the start of the ride, until he first returns to his initial starting point.



2 marks

d. While on the ride show that Ashley's speed at time t, can be expressed as $8a\left|\sin(at)\right|$ ms^{-1}, where a is a positive integer and find the value a.

3 marks

e. Find the first three times when Ashley is at rest and find his position vector at these times. Indicate these points on the diagram above in **c.**

2 marks

f. Find the maximum speed reached by Ashley on the ride. Find the first three times when Ashley is at the maximum speed and find his position vector at these times. Indicate these points on the diagram above in **c**.

3 marks

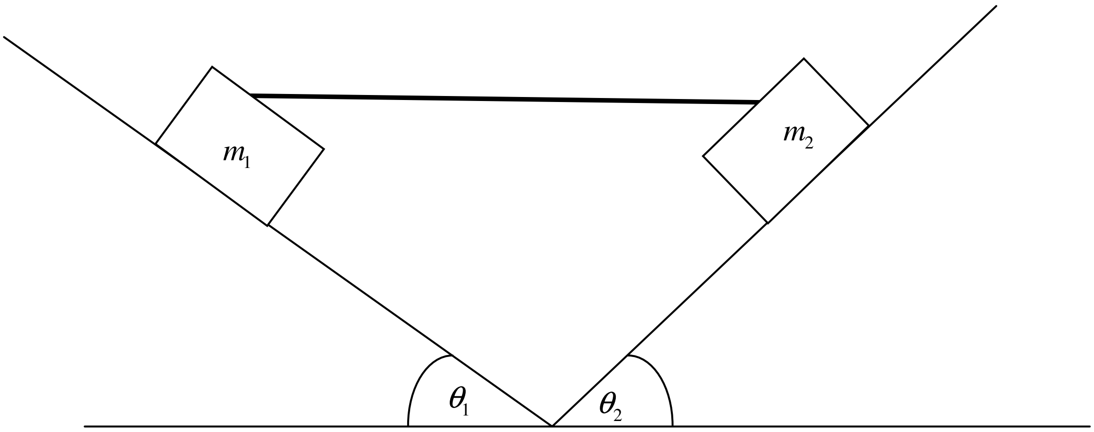
g. The distance travelled between the times of t_0 and t_1 is given by $\int_{t_0}^{t_1} \left|\dot{r}(t)\right| dt$.

Write down a definite integral which gives the distance travelled by Ashley from the start of the ride until he next returns to his initial starting point. Find this distance, giving your answer in metres.

1 mark
Total 13 marks

Question 3

Two particles of masses m_1 and m_2 are connected by a light horizontal rod and rest on two inclined planes. The planes are inclined at angles of θ_1 and θ_2 respectively to the horizontal as shown in the diagram below. The coefficient of friction between both masses and the planes is μ and the rod just prevents both masses from sliding down the plane.



a. On the diagram above, mark in and label all the forces acting on both masses.

1 mark

b. By resolving the forces, show that $\dfrac{m_1}{m_2} = \dfrac{\left(\tan\left(\theta_2\right)-\mu\right)\left(1+\mu\tan\left(\theta_1\right)\right)}{\left(\tan\left(\theta_1\right)-\mu\right)\left(1+\mu\tan\left(\theta_2\right)\right)}$.

5 marks

c. Assume now that $\theta_2 = 2\theta_1$, and the rod is removed. The mass m_1 remains at rest, however the mass m_2 moves from rest down the plane a distance of D metres, in a time of T seconds. Show that $\theta_1 = \tan^{-1}\left(\frac{2D}{gT^2}\right)$.

4 marks
Total 10 marks

Question 4

Given the complex number $c = -2 + i$, and in the complex plane, *S* has the equation $|z - c| = 1$ and *T* has the equation $\left|z - \frac{1}{5}(-2 + 11i)\right| = |z - c|$, where $z \in C$.

a. Find and describe the cartesian equation of *S*.

1 mark

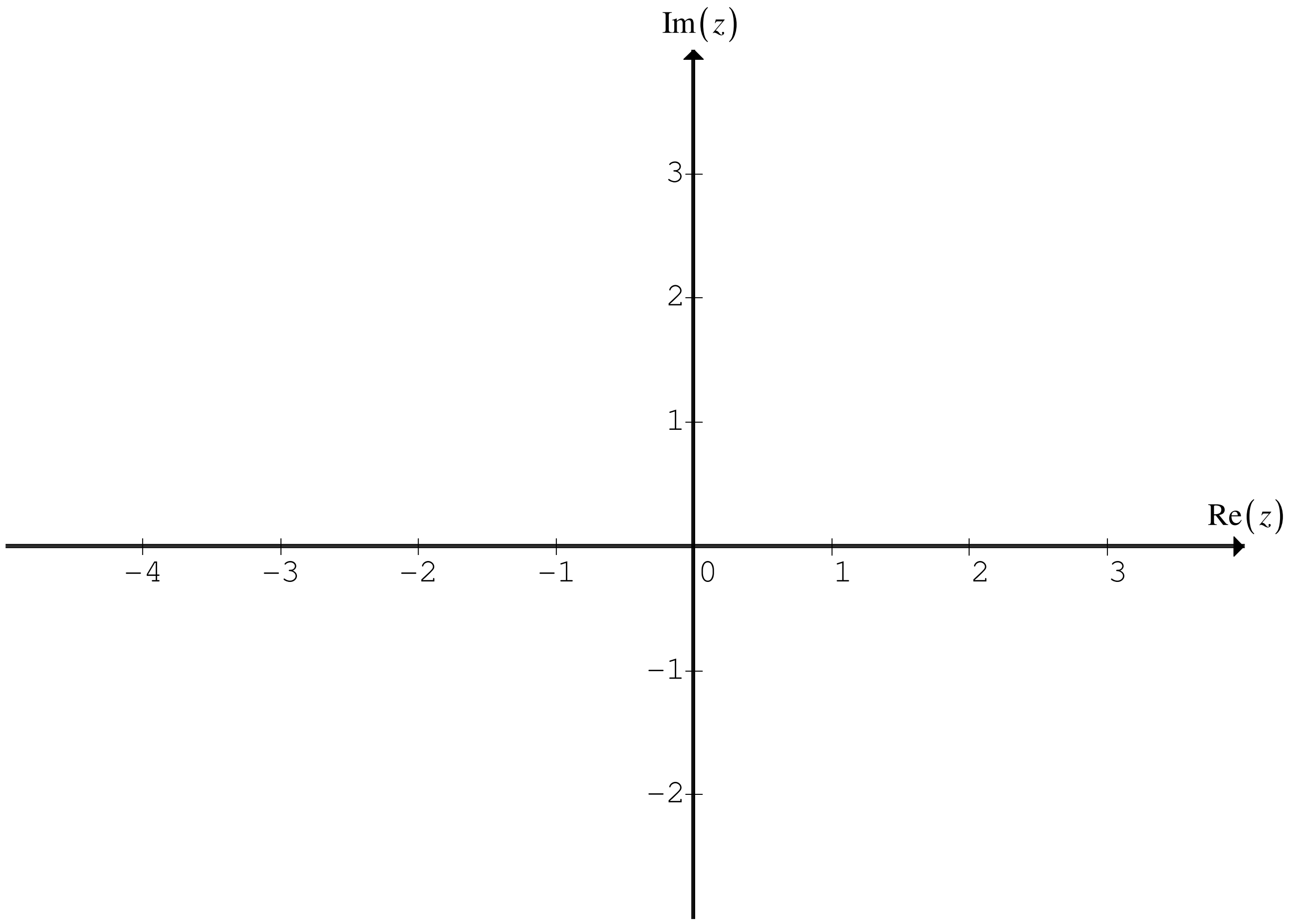
b. The cartesian equation of *T*, can be expressed in the form, $p\,\text{Im}(z) + q\,\text{Re}(z) = 0$ where *p* and *q* are positive integers, with no common factors, find the values of *p* and *q*.

2 marks

c. If $A \in S \cap T$ find the cartesian coordinates of the point A.

2 marks

d. Sketch the graphs of S and T on the argand diagram below.



2 marks

e. The part of T in the second quadrant, can be expressed in the form $\text{Arg}(z) = \alpha$, find the value of α.

1 mark

f. If $z \in S$, find the maximum value of $|z|$.

1 mark

g. On the diagram in part **d.** above, shade the region corresponding to $\{z : |z - c| \geq 1\} \cap \{z : \alpha \leq \text{Arg}(z) \leq \pi\} \cap \{z : \text{Im}(z) - \text{Re}(z) < 3\}$, and find the area of this shaded region.

2 marks

Total 11 marks

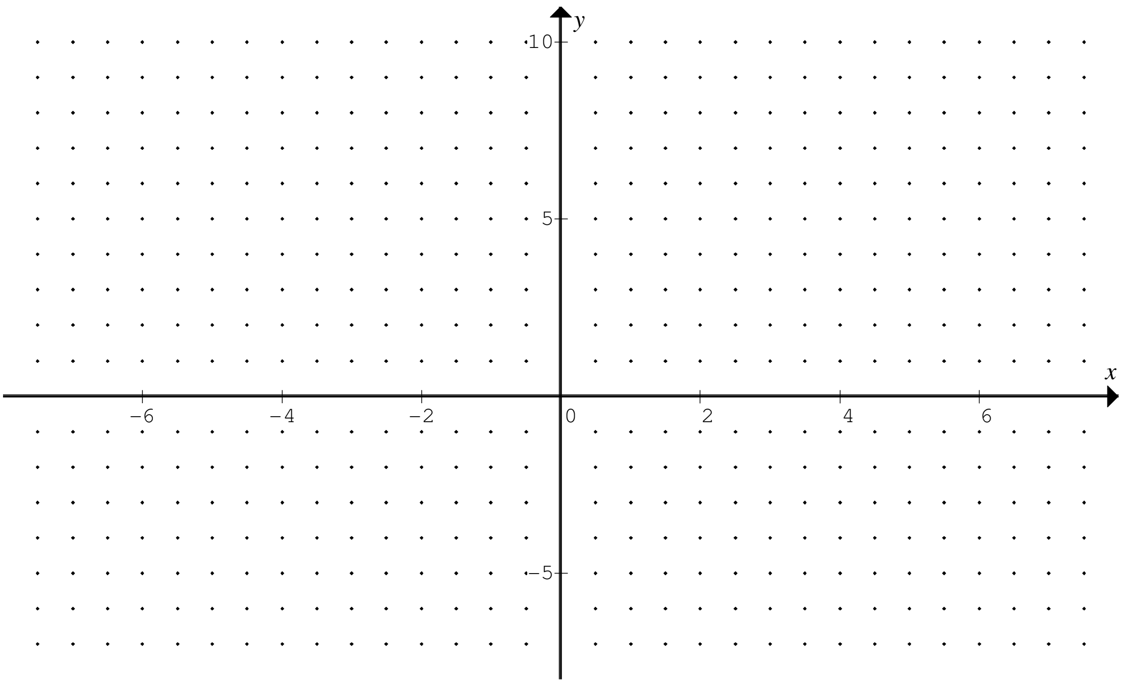
Question 5

Given the function f with the rule $y = \frac{16(x-2)}{\sqrt{x}}$

a. Find $\frac{dy}{dx}$ and hence explain why the function f has no turning points.

2 marks

b. Sketch the graph of the function f, on the set of axes below, clearly showing the x intercept.



1 mark

c. From the rule relating y and x, show that $x = \dfrac{\left(y + \sqrt{y^2 + 2048}\right)^2}{1024}$.

2 marks

When the part of the curve for $y \in [0, 9]$ is rotated about the y-axis, it forms the shape of a wine glass, with the x and y coordinates measured in centimetres.

d. If the base of the wine glass has a diameter of 4 cm, find the diameter in cm of the top of the wine glass.

1 mark

e.i. Write down a definite integral which gives the total volume of the wine glass, in cm^3.

1 mark

ii. Find the total volume of the glass in cm^3, correct to two decimal places.

1 mark

f. The wine glass is used in restaurants, and is filled to a height of h cm, such that the volume of wine in the glass is 150 ml. Find the value of h giving your answer correct to two decimal places.

1 mark

g. Wine is poured into the glass at a rate of $2\ cm^3$ per second. Find the rate in cm/s at which the height of the wine in the glass is rising, when the height is 4 cm from the base of the glass.

3 marks
Total 12 marks

EXTRA WORKING SPACE

END OF QUESTION AND ANSWER BOOKLET

SPECIALIST MATHEMATICS

Written examination 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

<u>Specialist Mathematics Formulas</u>

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of triangle:	$\frac{1}{2}bc\sin(A)$
sine rule:	$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$
cosine rule:	$c^2=a^2+b^2-2ab\cos(C)$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigonometric) functions

$$\cos^2(x)+\sin^2(x)=1$$

$$1+\tan^2(x)=\sec^2(x) \qquad \cot^2(x)+1=\operatorname{cosec}^2(x)$$

$$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y) \qquad \sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$$

$$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y) \qquad \cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$$

$$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)} \qquad \tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$$

$$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$$

$$\sin(2x)=2\sin(x)\cos(x) \qquad \tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1,1]$	$[-1,1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0,\pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex Numbers)

$z = x + yi = r\left(\cos(\theta) + i\sin(\theta)\right) = r\,\text{cis}(\theta)$

$|z| = \sqrt{x^2 + y^2} = r$ $\qquad -\pi < \text{Arg}(z) \le \pi$

$z_1 z_2 = r_1 r_2 \,\text{cis}(\theta_1 + \theta_2)$ $\qquad \dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\,\text{cis}(\theta_1 - \theta_2)$

$z^n = r^n \text{cis}(n\theta)$ (de Moivre's theorem)

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$ $\qquad \underset{\sim}{r_1} \cdot \underset{\sim}{r_2} = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \dfrac{d\underset{\sim}{r}}{dt} = \dfrac{dx}{dt}\underset{\sim}{i} + \dfrac{dy}{dt}\underset{\sim}{j} + \dfrac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

sliding friction: $F \le \mu N$

constant (uniform) acceleration:

$v = u + at \qquad s = ut + \frac{1}{2}at^2 \qquad v^2 = u^2 + 2as \qquad s = \frac{1}{2}(u+v)t$

acceleration: $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\tfrac{1}{2}v^2\right)$

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c\ ,\ n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e(x) + c,$ for $x > 0$
$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$	$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$	$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$	$\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$
$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$
$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$
$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Euler's method If $\frac{dy}{dx} = f(x),\ x_0 = a$ and $y_0 = b,$ then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf\left(x_n\right)$

END OF FORMULA SHEET

ANSWER SHEET

STUDENT NUMBER

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Words										

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