2012 Specialist Mathematics GA 2: Written examination 1

GENERAL COMMENTS

In 2012, 3800 students sat for the 2012 Specialist Maths examination 1. Students were required to answer ten short-answer questions worth a total of 40 marks. Students were not permitted to bring calculators or notes into the examination.

The common errors highlighted in this report should be brought to the attention of students so that they can develop strategies to avoid them. In particular, students are reminded to read the questions carefully. Responses to several questions in the 2012 examination indicated that students had not done so.

Another concern was the quality of students' handwriting and the manner in which they set out their mathematics. Students are reminded that if an assessor cannot read their writing or is not certain as to what it is conveying, that assessor cannot award marks. Furthermore, students are expected to set out their work properly. If an assessor is unable to follow a student's working (or reasoning), full marks will not be awarded. Equals signs should be placed between quantities that are equal – the working should not appear to be a number of disjointed statements. If there are inconsistencies in student working, full marks will not be awarded. For example, if an equals sign is placed between quantities that are not equal.

Areas for improvement include the following.

- reading the question carefully – When this is not done students may not answer the question properly, may proceed further than required or may not give the answer in the specified form. The latter was common and particularly evident in Questions 4a., 4c., 7, 8, 9a. and 9d. Students are reminded that good examination technique includes re-reading the question after it has been answered to ensure that they have answered what was required and that they have given their answer in the correct form
- algebraic skills – Difficulty with algebra was evident in several questions. The inability to simplify expressions often prevented students from completing the question. Incorrect attempts to factorise, expand and simplify were common. Poor use of brackets was also common
- showing a given result – This was required in Questions 9c. In such questions, the onus is on students to include sufficient relevant working to demonstrate that they know how to derive the result. Students are reminded that they can use a given value in the remaining part(s) of the question whether they were able to derive it or not
- recognising the need to use the chain rule when differentiating implicitly (Question 6)
- recognising the need to use the product rule when differentiating implicitly (Question 6)
- recognising the need to use the chain rule when differentiating (Questions 5 and 9)
- recognising the need to use the product rule when differentiating (Question 8)
- recognising the method of integration required (Questions 1 and 7)
- changing the terminals when integrating using substitution (Question 7)
- knowing the exact values for circular functions (Questions 2, 3, 4 and 9)
- consideration of the quadrant for values of circular functions (Questions 2, 3 and 9)
- giving answers in the required form (Questions 3a., 4c., 8, 9b., 9d. and 10b.)

In this examination, students are expected to be able to apply techniques, routines and processes involving rational, real and complex arithmetic, without the use of technology. Students are expected to be able to evaluate arithmetic expressions. Many students found this difficult and missed out on marks as a consequence. Unfortunately, many students made algebraic slips at the end of an answer, which meant the final mark could not be awarded. This was especially unfortunate when they had a correct answer and there was no need for further simplification.

There were several cases where incorrect working fortuitously led to a correct answer. Students are reminded that in such cases, the mark for the final answer will not be awarded if the answer is not supported by relevant and correct working.

SPECIFIC INFORMATION

This report provides sample answers or an indication of what the answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

Question 1

Marks	0	1	2	Average
%	39	16	46	**1.1**

$3\arctan\left(\frac{x}{2}\right)+\frac{1}{2}\log_e(x^2+4)$ (other equivalent answers were accepted)

This question was quite well done. Most students recognised the need to split the fraction into two parts. Those who did sometimes had incorrect constants multiplying the arctan or the log term, or an incorrect constant inside the arctan term. A surprising number of students erroneously attempted to use partial fractions, mainly using a difference of squares and sometimes a perfect square. A few attempted to use a difference of squares using complex numbers. A small number attempted a substitution or divided the fraction, sometimes dividing the numerator into the denominator.

Question 2

Marks	0	1	2	3	Average
%	40	46	3	11	**0.9**

$x=\frac{(2n+1)\pi}{2},\ \frac{\pi}{3}+2n\pi,\ \frac{2\pi}{3}+2n\pi,\quad n\in Z$ (other equivalent answers were accepted)

This question was not well done, with the majority of students 'losing' the cosine term entirely by cancelling it from both sides of the equation. Students should be cautioned regularly about the danger of using cancellation of a variable term as a simplifying technique. Of those students who did include solutions of $\cos(x) = 0$, a large proportion wrote the answer incorrectly as $x=\frac{n\pi}{2}$. Of the students who found general solutions to the sine part, quite a few wrote the general solution incorrectly as $x=\pm\frac{\pi}{3}+2n\pi$. Some students attempted to solve an equation found by squaring both sides of the original equation and then using the relation between $\sec^2(x)$ and $\tan^2(x)$. This approach was rarely successful and, even if progress had been made, they would have needed to realise that squaring can introduce new solutions that are not valid in the original equation. A number of students found one or two solutions, but did not find a general solution. Some wrote generalisations with $n\in R$ and others gave incorrect generalisations.

Question 3a.

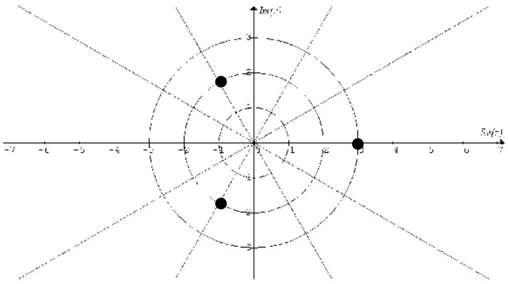
Marks	0	1	2	3	Average
%	28	21	7	44	**1.7**

$-1-i\sqrt{3}$, 3

Most students were able to make a start with this question, finding the complex number in cartesian form and giving the conjugate solution. Typical mistakes included errors with exact values and writing down the real solution as $z = 2$ (to make it lie on the circle of radius 2) or $z = 4$. Some students confused roots and factors. Several students used long division when the factor theorem to find the real root was a much simpler approach.

Question 3b.

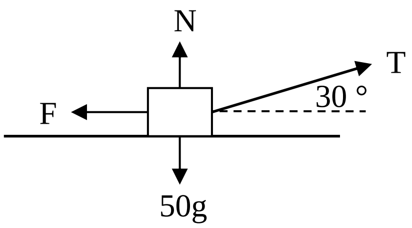
Marks	0	1	Average
%	67	33	**0.4**



This question was not well done, even by several students who found the correct solutions in part a. Many wanted to put $z = 3$ on the circle radius 2; others plotted the complex solution on the $\pm\frac{5\pi}{6}$ rays; a few plotted 3 at $3i$. Quite a few students plotted the complex roots at a distance of $\sqrt{3}$ from the origin.

Question 4a.

Marks	0	1	2	Average
%	9	14	77	**1.7**



Students performed very well on this question, with most of them labelling all of the forces correctly. Typical errors included forgetting N or T, writing the friction force as μ rather than F or μN or simply ‘friction’, and writing g for the weight force rather than mg or W or similar. A small number of students drew the tension force acting in the opposite direction or in both directions.

Question 4b.

Marks	0	1	Average
%	56	44	**0.5**

$100g$

A high proportion of students were able to find that the tension $T = 100g - 2N$, but did not realise that ‘leaving the floor’ meant that the normal reaction force $N = 0$. A few students solved part b. as though it were part c. A surprising number of students wrote $T = 100g = 98$. Some stated that $T < 100g$.

Question 4c.

Marks	0	1	2	3	Average
%	37	4	19	40	**1.6**

$$T = \frac{100g}{5\sqrt{3}+1}$$

This question was quite well done by the majority of students. A number of students made the error $N = 50g$, leading to no further progress. Other errors included incorrect exact values, a weakness with negative signs leading to an incorrect negative in the denominator of the answer, and some poor algebraic skills (often involving a lack of brackets, which led to many different values for the constants in the answer, even when the resolution of forces was correctly done). A small number of students included an acceleration term *a*, which prevented them from finding the correct answer (some used *g* for *a*).

Question 5

Marks	0	1	2	3	Average
%	20	24	12	44	1.8

$a = -4$

This question was reasonably well done by many students, though it was disappointing that some were unable to differentiate an arctan function when the *x* in the argument has a coefficient other than 1. Most students were able to apply the chain rule in order to find the second derivative (even if not the first). Generally, students were able to perform the substitution step quite well (using their first and second derivatives). Many of the students who made algebraic mistakes gave an answer such as $a = -8x$ or similar, notwithstanding the statement in the question that 'a is a real constant'. A surprising number of students interpreted arctan (2*x*) as $\tan^{-1}(2x) = \dfrac{1}{\tan(2x)}$.

Question 6

Marks	0	1	2	3	Average
%	14	30	11	45	1.9

$-\dfrac{4}{13}$

Students who recognised that they needed the product and chain rules in the context of implicit differentiation answered this question relatively well. A few students appeared to forget that the derivative of a constant such as the number 14 is 0. Some of the errors included writing the squared log term as 2ln(*x* – 2) before differentiating and, more commonly, differentiating the log term to obtain 2ln(*x* – 2) or having *x* as the denominator. Several students 'missed' the derivative of the single *y* term. Some students chose to simplify the derivative algebraically before substituting values and many of these students made mistakes with signs or other algebraic errors. Owing to the fact that $\log_e(3-2) = \log_e(1) = 0$, many students obtained the correct answer after they had made one or more differentiation and/or algebraic mistakes.

Question 7

Marks	0	1	2	3	Average
%	30	19	10	41	1.7

$\dfrac{4}{15}$

Most students recognised the need for a substitution. Many of these students used the appropriate substitution $u = 2 - x$, although some used one that did not lead to an answer. There were many students who made sign errors, yet ended up with the correct answer. Some students tried to use a modulus or simply changed the sign from negative to positive when it suited. Equals signs must not be placed between quantities that are not equal. A statement such as $-\dfrac{4}{15} = \dfrac{4}{15}$ is not valid, nor are statements that equate an indefinite integral with a definite integral. Common errors included not changing the terminals and an absence of '*du*' or continuation of '*dx*' throughout, or no indicator at all. Several of the students who used the appropriate substitution but kept the *x* terminals achieved the correct answer by substituting back for *x* before using the terminals. They could not be awarded full marks due to the inconsistency in their working. Many errors involved negatives, often due to not using $\dfrac{du}{dx}$. Some students interpreted this question as a solid of revolution.

Question 8

Marks	0	1	2	3	Average
%	17	24	32	28	**1.7**

$$\frac{4x}{\left(1+x^2\right)^2}$$

This question was answered reasonably well by most students who recognised the need to use $v\frac{dv}{dx}$ or $\frac{d}{dx}\left(\frac{1}{2}v^2\right)$. Many of those who used the former got into significant difficulty with square roots and the associated algebraic simplification. On the other hand, those who used the latter were usually more successful, since squaring first eliminated the square root. There were, however, a small number of students who started from this alternative form and attempted to integrate. Quite a few students found the correct answer, but left it in an unsimplified form, often as two separate fractions. A large proportion of students thought that $a=\frac{dv}{dx}$.

Question 9a.

Marks	0	1	Average
%	27	73	**0.8**

$$\mathbf{v}(t)=\left(\frac{2t}{\sqrt{t^2+2}}-2t\right)\mathbf{i}+\left(\frac{2t}{\sqrt{t^2+2}}+2\right)\mathbf{j}$$

This question was generally well done, although there were some students who had 2 rather than 2*t* in the numerator of the fractions in the two components of the velocity vector. Sign errors were the main source of problems, but some students left out the 2 from the numerators.

Question 9b.

Marks	0	1	2	Average
%	23	39	38	**1.2**

$$\frac{4\sqrt{6}}{3}$$

This question was reasonably well done, with most students realising that Pythagoras's theorem was needed. Those who substituted $t=1$ first tended to have more success, with fewer errors seen. Some students squared the terms inside the square root and often made mistakes with the middle terms. While these generally cancelled, it meant that the final mark could not be awarded. Some students were not able to put the answer into the correct form.

Question 9c.

Marks	0	1	2	Average
%	54	11	35	**0.8**

Students had to show the given result $\frac{dy}{dx}=\frac{1+\sqrt{3}}{1-\sqrt{3}}$.

Those students who realised that the chain rule $\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$ was required handled this question well. As often happens in a 'show that' type of question, some students were unable to do any convincing algebra, yet still managed to obtain the result stated. Many students did not use their previously calculated values of $\frac{dx}{dt}$ and $\frac{dy}{dt}$ for $t=1$, which was previously calculated in part a., but set about repeating the differentiation.

Question 9d.

Marks	0	1	2	Average
%	48	38	13	**0.7**

$\frac{7\pi}{12}$

Most students struggled with this question. Those who used arguments were often successful, but the majority used the dot product without considering other possible methods. Drawing a simple diagram was a useful start. Of those who got a correct expression for the cosine, very few were able to reach the correct answer.

Question 10ai.

Marks	0	1	Average
%	33	67	**0.7**

$x \in [-2, 2]$

Most students handled this question well. The most common error was to obtain $\left[-\frac{1}{2}, \frac{1}{2}\right]$, but $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ was also seen.

Question 10aii.

Marks	0	1	Average
%	67	33	**0.4**

$\left\{x < -\frac{1}{5}\right\} \cup \left\{x > \frac{1}{5}\right\}$ or $R \setminus \left[-\frac{1}{5}, \frac{1}{5}\right]$

Many of the attempts to solve this question were quite disappointing as it drew on assumed knowledge from Mathematical Methods with which students should be familiar. A small percentage of students were able to obtain the correct answer. Many made unfortunate slips with inclusion/exclusion of values at the boundaries. Other errors that were commonly made were $\left\{x < -\frac{1}{25}\right\} \cup \left\{x > \frac{1}{25}\right\}$, $R \setminus \left\{-\frac{1}{5}, \frac{1}{5}\right\}$, $x > \frac{1}{5}$ and $x > \pm\frac{1}{5}$.

Question 10aiii.

Marks	0	1	Average
%	74	26	**0.3**

$x \in \left[-2, -\frac{1}{5}\right) \cup \left(\frac{1}{5}, 2\right]$ or $[-2, 2] \setminus \left[-\frac{1}{5}, \frac{1}{5}\right]$

This question involved finding the intersection of the domains found in the previous two parts. Many students seemed not to realise this, typically giving answers such as $R \setminus \left[-\frac{1}{5}, \frac{1}{5}\right]$. The inclusion/exclusion of values at the boundaries was again an issue.

Question 10b.

Marks	0	1	2	3	Average
%	74	5	8	13	**0.6**

$\frac{5\sqrt{7}}{16}$

Most students were unable to make any meaningful attempt at this question as most failed to recognise that they were dealing with the sine of a compound angle. A high proportion of students were able to obtain $\sin(\theta) = \sin\left(\arcsin\left(\frac{1}{8}\right) + \arcsin\left(\frac{3}{4}\right)\right)$, but could go no further, not realising that a compound angle formula could be used to simplify this expression. Of the few who did recognise the situation, many were unable to find $\cos\alpha$ when given the value of $\sin\alpha$. Most students who were able to find $\cos\alpha$ made reasonably good progress towards the answer, with the most common error being a failure to simplify the surd terms correctly. The most common answer from those who made a promising start but did not use the compound angle formula was $\frac{7}{8}$. Some students failed to simplify their answer, including some who got as close as $\frac{\sqrt{7} + 3\sqrt{63}}{32}$.