



Units 3 and 4 Specialist Maths: Exam 1

Practice Exam Solutions

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Question 1

$$f(2+i) = 0 \text{ implies } f(2-i) = 0 \quad \text{property of complex conjugates [1]}$$

$$(z-2-i)(z-2+i) = z^2 - 4z + 5$$

$$\frac{z^4 - 2z^3 - 2z^2 + 15}{z^2 - 4z + 5} = z^2 + 2z + 2 \quad \text{long division [1]}$$

$$\begin{aligned} z^2 + 2z + 2 &= (z+1)^2 + 1 \\ &= (z+1-i)(z+1+i) \end{aligned} \quad \text{factorizing quotient over the complex numbers [1]}$$

$$\therefore f(z) = (z-2-i)(z-2+i)(z+1-i)(z+1+i) \quad [1]$$

Question 2

$$a = e^{-v^2}$$

$$v \frac{dv}{dx} = e^{-v^2} \quad \text{identifying correct differential [1]}$$

$$\frac{dx}{dv} = ve^{v^2} \quad \text{finding an expression for } \frac{dx}{dv} [1]$$

$$x = \int ve^{v^2} dv = \frac{1}{2}e^{v^2} + c \quad \text{integration w.r.t. } v \text{ (incl. constant) [1]}$$

$$e = \frac{1}{2}e^0 + c \Rightarrow c = e - \frac{1}{2} \quad \text{evaluating at given point to find } c$$

$$\therefore x = \frac{1}{2}e^{v^2} + e - \frac{1}{2} \quad \text{correct equation relating } x \text{ and } v [1]$$

Question 3a

$$f(x) = \frac{2}{4+x^2}$$

$$\int_{-\sqrt{3}}^{\sqrt{3}} \frac{2}{4+x^2} = \left[\tan^{-1} \frac{x}{2} \right]_{-\sqrt{3}}^{\sqrt{3}} \quad \text{correctly identifying the antiderivative [1]}$$

$$= \tan^{-1} \frac{\sqrt{3}}{2} - \tan^{-1} \frac{-\sqrt{3}}{2} = 2 \tan^{-1} \frac{\sqrt{3}}{2} \quad \text{correctly evaluating the antiderivative [1]}$$

Question 3b

$$\int f(x) = \int \frac{x^2}{\sqrt{2x-1}} dx$$

$$\text{Let } u = \sqrt{2x-1} \quad \text{appropriate substitution [1]}$$

$$\text{Then } \frac{u^2+1}{2} = x, \text{ hence } \frac{dx}{du} = u \quad \text{finding } x(u) \text{ and } \frac{dx}{du}$$

$$\int f(x) = \int \frac{(u^2+1)^2}{4} du \quad \text{expressing antiderivative in terms of } u [1]$$

$$\begin{aligned} \int \frac{(u^2+1)^2}{4} du &= \frac{1}{4} \int u^4 + 2u^2 + 1 du \\ &= \frac{1}{4} \left(\frac{u^5}{5} + \frac{2u^3}{3} + u \right) \end{aligned} \quad \text{correctly antidifferentiating [1]}$$

$$\int f(x) dx = \frac{\sqrt{2x-1}}{4} \left(\frac{(2x-1)^2}{5} + \frac{4x-2}{3} + 1 \right) \quad \text{substituting } u = u(x)$$

$$\int f(x) = \sqrt{2x-1} \left(\frac{3x^2+2x+2}{15} \right) \quad \text{simplifying [1]}$$

Question 4

$$\sin(3\theta) - \cos(3\theta) = \sin(2\theta)\cos(\theta) + \sin(\theta)\cos(2\theta) - (\cos(2\theta)\cos(\theta) - \sin(2\theta)\sin(\theta))$$

using angle sum identities [1]

$$= 2\sin\theta\cos^2\theta + \sin\theta(\cos^2\theta - \sin^2\theta) - (\cos^2\theta - \sin^2\theta)\cos\theta + 2\sin^2\theta\cos\theta$$

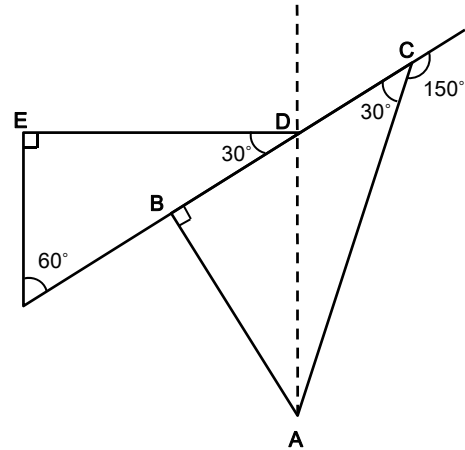
using double angle formulae [1]

$$= 3\sin\theta\cos^2\theta + 3\cos\theta\sin^2\theta - \sin^3\theta - \cos^3\theta$$

collecting like terms [1]

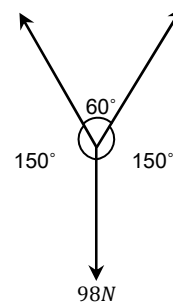
Question 5a

We can simplify the diagram and fill in obvious angles as shown. There are many ways to go about solving this problem. Suppose we label points A, B, C and D as shown. If the dotted line is the perpendicular bisector of $\angle BAC$, then $\angle BAD = \angle CAD = \alpha$. Then $\angle ADC = 150 - \alpha$ and $\angle BDA = 90 - \alpha$. As $\angle BDA + \angle ADC = 180$, $\alpha = 30$. Hence, $\angle ADE = 90$, and the bisector is parallel to the wall, as required. [3]

**Question 5b**

As the bisector is perpendicular to the wall, it is parallel to the direction of mg . We can make a freeform force diagram and solve using the sine rule [1], which gives

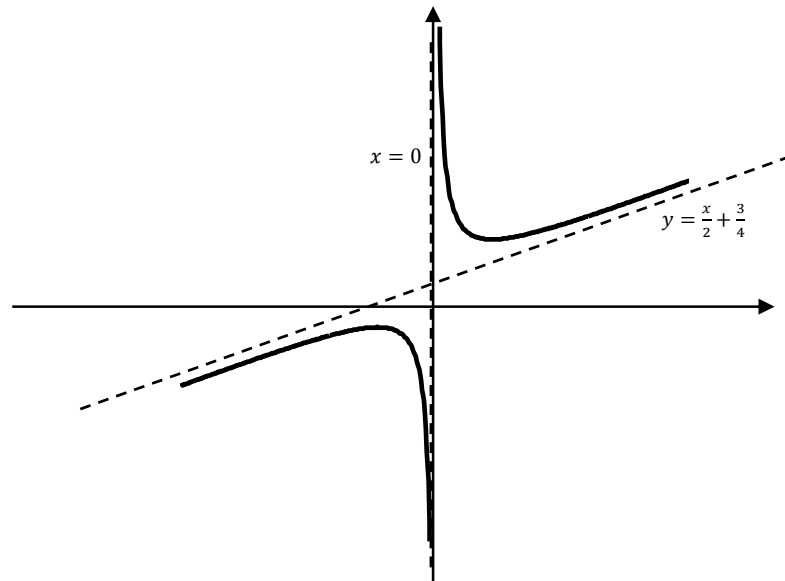
$$F_{AB} = F_{AC} = \frac{49}{\sin(60)} = 49 \times \frac{2}{\sqrt{3}} = \frac{98}{\sqrt{3}} \quad [1]$$



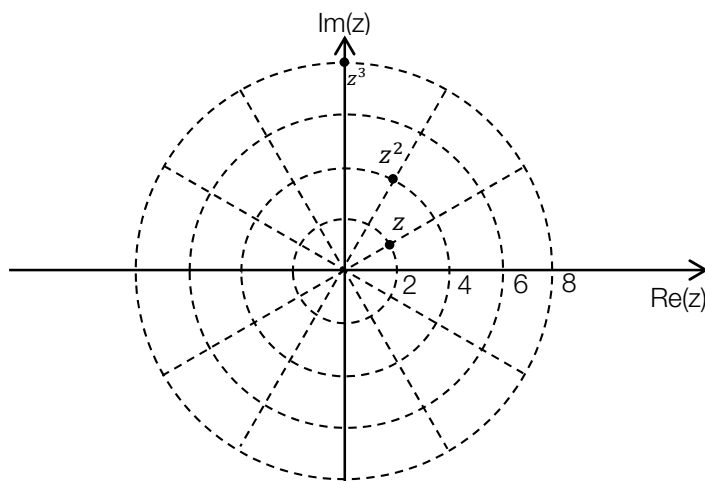
Question 6

correct shape (incl. absence of intercepts) [2]

correct asymptotes [1]



Question 7



Question 8

$$xy \log_e(xy) = 1$$

using log rules to simplify the expression [1]

$$\log_e(xy)^{xy} = 1$$

$$y \log_e x^x + x \log_e y^y = 1$$

$$\frac{dy}{dx} x \log_e x + y(1 + \log_e x) + x \frac{dy}{dx} (1 + \log_e y) + y \log_e y = 0$$

implicit differentiation [1]

$$\frac{dy}{dx} = \frac{-y - y \log_e x - y \log_e y}{x + x \log_e y + x \log_e x} = -\frac{y(1 + \log_e(xy))}{x(1 + \log_e(xy))} = -\frac{y}{x}$$

correct, simplified expression [2]

Question 9a

$$\dot{\mathbf{r}}(t) = \frac{d}{dt} \mathbf{r}(t) = -3 \sin t \mathbf{i} + 3 \cos t \mathbf{j} + \mathbf{k} \quad [1]$$

$$\ddot{\mathbf{r}}(t) = \frac{d}{dt} \dot{\mathbf{r}}(t) = -3 \cos t \mathbf{i} - 3 \sin t \mathbf{j} \quad [1]$$

Question 9b

$$\mathbf{r}(t) \cdot \ddot{\mathbf{r}}(t) = 9 \sin t \cos t - 9 \sin t \cos t = 0 \quad [1]$$

This suggests that the acceleration of the particle is always perpendicular to its velocity [1].

Question 10

To prove the claim, it is sufficient to show that $\overrightarrow{PQ} = \overrightarrow{SR}$ and that $\overrightarrow{SP} = \overrightarrow{RQ}$. [1]

First observe that $\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d} = \mathbf{0}$ [1]

Then: $\overrightarrow{PQ} = \frac{1}{2}(\mathbf{a} + \mathbf{b})$ and $\overrightarrow{SR} = -\frac{1}{2}(\mathbf{c} + \mathbf{d})$

So, $\overrightarrow{PQ} - \overrightarrow{SR} = \frac{1}{2}(\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d}) = \mathbf{0}$

Therefore, $\overrightarrow{PQ} = \overrightarrow{SR}$ [1]

Similarly,

$\overrightarrow{SP} = \frac{1}{2}(\mathbf{d} + \mathbf{a})$ and $\overrightarrow{RQ} = -\frac{1}{2}(\mathbf{c} + \mathbf{b})$

So

$\overrightarrow{SP} - \overrightarrow{RQ} = \frac{1}{2}(\mathbf{a} + \mathbf{d} + \mathbf{c} + \mathbf{b}) = \mathbf{0}$

Therefore, $\overrightarrow{SP} = \overrightarrow{RQ}$, which completes the proof. [1]

