INSIGHT

YEAR 12 *Trial Exam Paper*

2013

SPECIALIST MATHEMATICS

Written examination 2

STUDENT NAME:

Reading time: 15 minutes
Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT have to be cleared.
- Students are NOT permitted to bring sheets of paper or white-out liquid/tape into the examination.

Materials provided

- The question and answer book of 31 pages, a formula sheet, and an answer sheet for the multiple-choice questions.

Instructions

- Write your **name** in the box provided and on the multiple-choice answer sheet.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

SECTION 1

Instructions for Section 1
Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where g = 9.8.

Question 1

For the hyperbola $\frac{x^2}{9}-\frac{(y+1)^2}{16}=1$, the equations of the asymptotes are

A. $y=\pm\frac{4x}{3}-1$

B. $y=\pm\frac{4x}{3}+1$

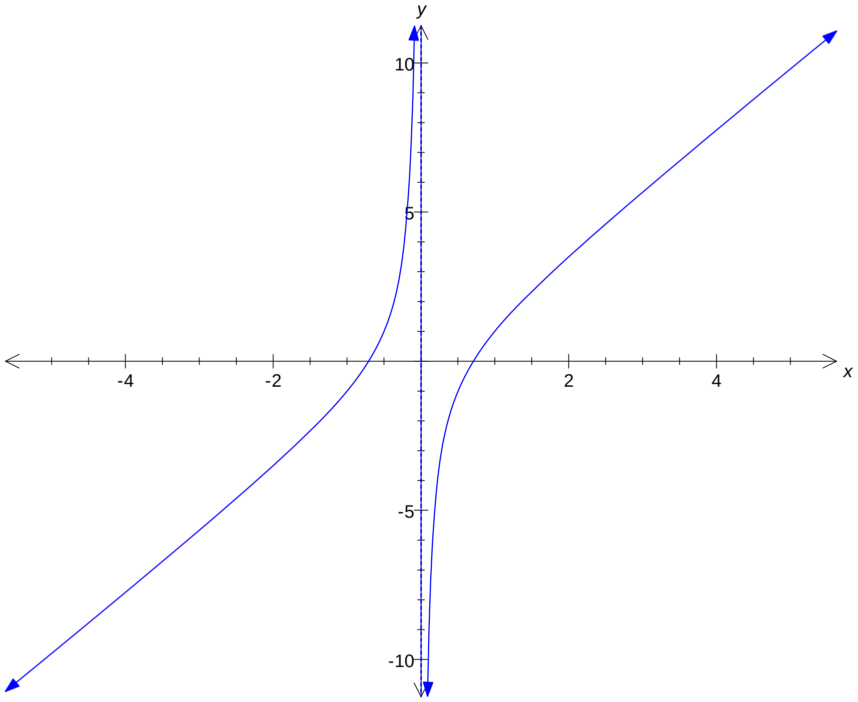
C. $y=\pm\frac{3x}{4}-1$

D. $y=\pm\frac{3}{4}(x-1)$

E. $y=\pm\frac{4}{3}(x-1)$

Question 2

Which expression could define the function shown in the graph?



A. $f(x) = \dfrac{ax^2 + b}{x}$, $a < 0$ and $b > 0$

B. $f(x) = \dfrac{ax^2 + bx}{x}$, $a > 0$ and $b < 0$

C. $f(x) = \dfrac{ax^2 + b}{x}$, $a > 0$ and $b < 0$

D. $f(x) = \dfrac{ax^2 + bx}{x^2}$, $a > 0$ and $b < 0$

E. $f(x) = \dfrac{ax^2 + b}{x^2}$, $a < 0$ and $b > 0$

Question 3

If $\tan(2x) = \dfrac{-3}{4}$ and $\dfrac{\pi}{2} < x < \pi$, then $\text{cosec}(x)$ is

A. $\dfrac{3}{\sqrt{10}}$

B. $\dfrac{\sqrt{10}}{3}$

C. $\dfrac{1}{3}$

D. $\sqrt{10}$

E. $\dfrac{1}{\sqrt{10}}$

Question 4

For $y = a - \pi \sin^{-1}(2x - k),\ a > 0, k > 0,$ the maximal domain and range are

A. domain $\left[\frac{-k}{2}, \frac{k}{2}\right]$, range $\left[\frac{\pi - a\pi}{2}, \frac{\pi + a\pi}{2}\right]$

B. domain $\left[\frac{k-1}{2}, \frac{k+1}{2}\right]$, range $\left[\frac{\pi - a\pi}{2}, \frac{\pi + a\pi}{2}\right]$

C. domain $\left[\frac{k-1}{2}, \frac{k+1}{2}\right]$, range $\left[\frac{2a - \pi^2}{2}, \frac{2a + \pi^2}{2}\right]$

D. domain $\left[\frac{-k}{2}, \frac{k}{2}\right]$, range $\left[\frac{a - 2\pi}{2}, \frac{a + 2\pi}{2}\right]$

E. domain $\left[\frac{k}{2} - 1, \frac{k}{2} + 1\right]$, range $\left[a - \frac{\pi^2}{2}, a + \frac{\pi^2}{2}\right]$

Question 5

If $z = 1 - i$, then $\text{Arg}\left(\frac{i^3}{z}\right)$ is

A. $\frac{\pi}{4}$

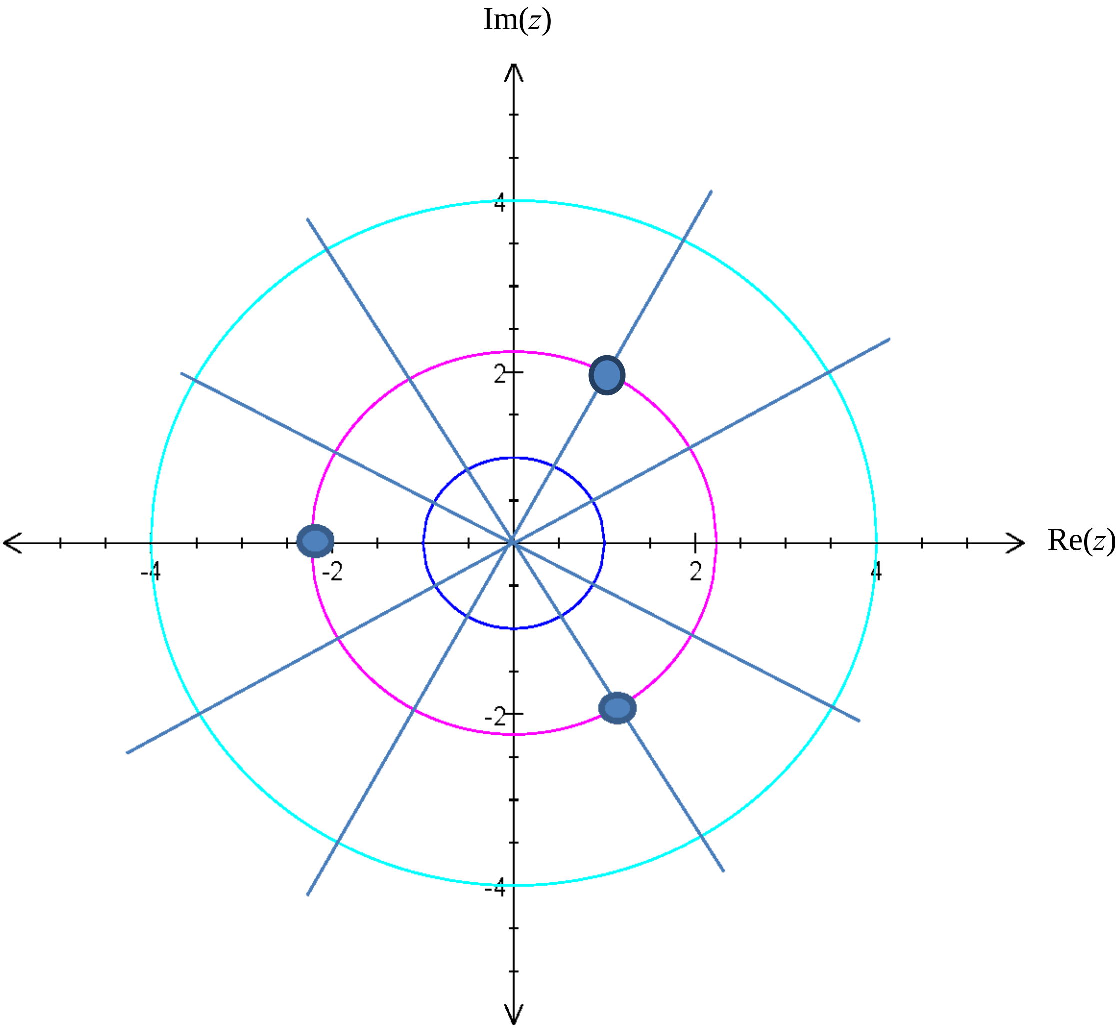
B. $\frac{7\pi}{4}$

C. $\frac{-3\pi}{4}$

D. $\frac{3\pi}{4}$

E. $\frac{-\pi}{4}$

Question 6



The three points marked on the Argand diagram above represent solutions to the cubic equation $z^3 = a + bi$, where

A. $a > 0$ and $b = 0$

B. $a = 0$ and $b < 0$

C. $a = 0$ and $b > 0$

D. $a > 0$ and $b < 0$

E. $a < 0$ and $b = 0$

Question 7

If $P(z) = az^3 + bz^2 + cz + d$ and $P(-ki) = 0$ and $P\left(\frac{m}{n}\right) = 0,$ where $a, b, c, d, k, m, n \in R \setminus \{0\},$

then the possible values of a, b, c and d could be

A. $a = 1,\ b = -m,\ c = -nk^2,\ d = -mk^2$

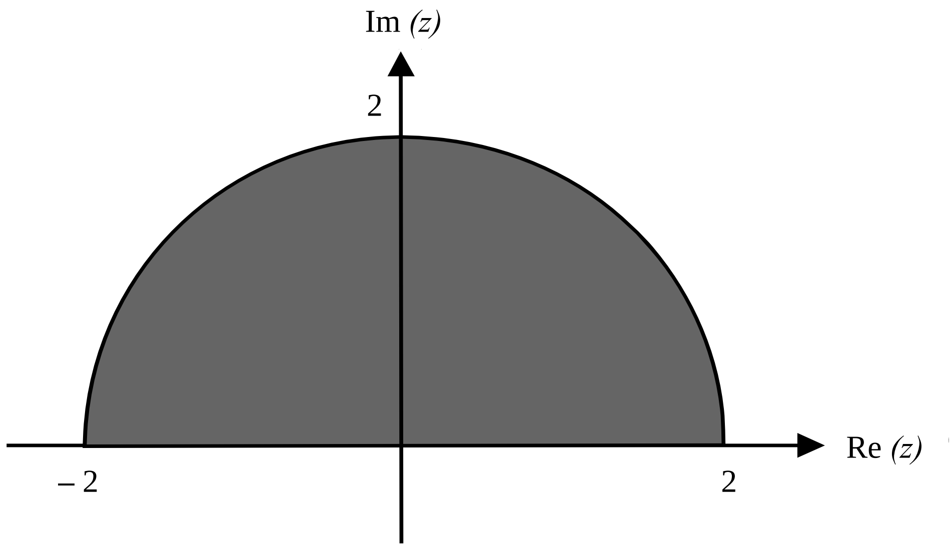
B. $a = 1,\ b = \frac{-m}{n},\ c = -k^2,\ d = \frac{-mk^2}{n}$

C. $a = n,\ b = -m,\ c = -nk^2,\ d = \frac{-mk^2}{n}$

D. $a = 1,\ b = -m,\ c = -k^2,\ d = \frac{-mk^2}{n}$

E. $a = n,\ b = -m,\ c = nk^2,\ d = -mk^2$

Question 8



The shaded region, including the boundaries, shown above on the Argand diagram could be described by

A. $\frac{|z|}{4} \le 2$

B. $|z| \le 2 \cup \text{Im}(z) \ge 0$

C. $|z| \le 4 \cap \text{Im}(z) \ge 0$

D. $|z| \le 2 \cap \text{Re}(z) \ge 0$

E. $|z| \le 2 \cap \text{Im}(z) \ge 0$

Question 9

If $\frac{dx}{dy} = \sqrt{9 - 4x^2}$ then

A. $y = \int \sqrt{9 - 4x^2}\, dx$

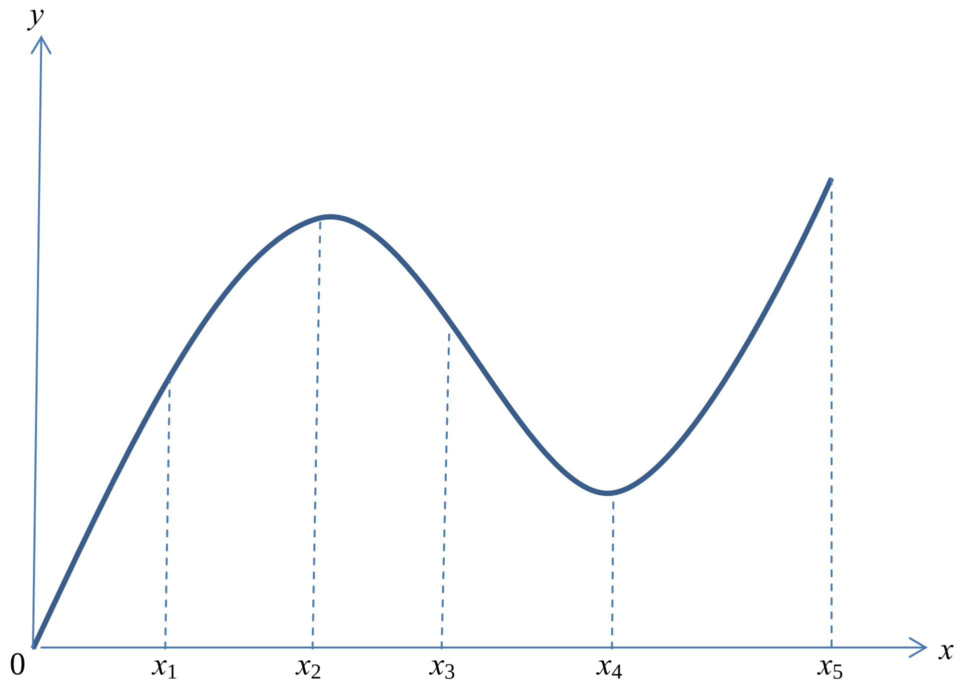
B. $y = \frac{1}{2}\int \frac{1}{\sqrt{\frac{9}{4} - x^2}}\, dx$

C. $y = 2\int \frac{1}{\sqrt{\frac{9}{4} - x^2}}\, dx$

D. $y = \frac{1}{2}\int \frac{1}{\sqrt{\frac{4}{9} - x^2}}\, dx$

E. $y = \frac{2}{3}\int \frac{1}{\sqrt{1 - x^2}}\, dx$

Question 10



Part of the graph of function *f* is shown above for $0 \le x \le x_5$.
For this function, $f'(x_2) = 0$, $f'(x_4) = 0$ and there are points of inflexion at $x = x_1$, $x = x_3$, $x = x_5$.

Then $f''(x) > 0$ for

A. $0 < x < x_1$

B. $0 < x < x_1$ and $x_3 < x < x_5$

C. $x_3 < x < x_5$

D. $x_1 < x < x_3$

E. $x_1 < x < x_2$ and $x_4 < x < x_5$

Question 11

For the curve $x\sin(y) - x^2 = 5$

A. $\dfrac{dy}{dx} = \dfrac{2x - \sin(y)}{x\cos(y)}$

B. $\dfrac{dy}{dx} = x\cos(y) - 2x$

C. $\dfrac{dy}{dx} = x\cos(y) + \sin(y) - 2x$

D. $\dfrac{dy}{dx} = \dfrac{2x - x\sin(y)}{x\cos(y)}$

E. $\dfrac{dy}{dx} = \dfrac{2x - \sin(y)}{\cos(y)}$

Question 12

$\int \frac{3x-2}{(x-1)^2}\,dx$ can be written as

A. $\int \frac{3}{(x-1)^2} - \frac{2}{(x-1)}\,dx$

B. $\int \frac{3}{(x-1)} - \frac{2}{(x-1)^2}\,dx$

C. $\int \frac{3}{(x-1)} + \frac{1}{(x-1)^2}\,dx$

D. $\int \frac{3}{(x-1)} - \frac{1}{(x-1)^2}\,dx$

E. $\int \frac{3}{(x-1)^2} - \frac{1}{(x-1)}\,dx$

Question 13

Initially, a salt solution has a volume of 100 litres. Pure water is poured into the solution at 10 litres per minute and the mixture, after stirring well, is removed at the rate of 7 litres per minute. A differential equation for the mass, x kg, of salt present at time t minutes is given by

A. $\frac{dx}{dt} = 10 - \frac{7x}{100}$

B. $\frac{dx}{dt} = 10 - \frac{7x}{100+t}$

C. $\frac{dx}{dt} = \frac{-7x}{100+t}$

D. $\frac{dx}{dt} = \frac{-7x}{100+3t}$

E. $\frac{dx}{dt} = 10 - \frac{7x}{100+3t}$

Question 14

The position of an object from a fixed point, *O*, is *x* metres at time *t* seconds, where

$$x = \frac{e^{2t}+1}{e^{t}},\ t \geq 0 .$$

The acceleration of the object, in m/s^2, when $t = 2$ seconds is

A. $e^{2}-e^{-2}$

B. $e-e^{-1}$

C. $\dfrac{e^{4}+1}{e^{2}}$

D. $\dfrac{e^{3}-1}{e^{2}}$

E. $\dfrac{e^{4}-e^{2}}{e^{2}}$

Question 15

The velocity of an object *v* metres/second is $v = \sin(x)\cos(x)$, where *x* metres is the distance of the object from a fixed point *O*.
The acceleration of the object is

A. $a = \cos(2x)$

B. $a = -\cos(x)\sin(x)$

C. $a = \dfrac{\sin(2x)}{2}$

D. $a = \dfrac{\sin(4x)}{4}$

E. $a = \cos(2x)\sin 2(x)$

Question 16

If $\underset{\sim}{a} = 5\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$ and $\underset{\sim}{b} = \underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ then the vector resolute of $\underset{\sim}{b}$ in the direction of $\underset{\sim}{a}$ is

A. $\dfrac{3}{25}\underset{\sim}{i} - \dfrac{3}{10}\underset{\sim}{j} + \dfrac{3}{5}\underset{\sim}{k}$

B. $\dfrac{3}{2}\underset{\sim}{i} - \dfrac{3}{10}\underset{\sim}{j} + \dfrac{3}{5}\underset{\sim}{k}$

C. $3\underset{\sim}{i} - 3\underset{\sim}{j} + \underset{\sim}{k}$

D. $\dfrac{3}{2}\underset{\sim}{i} + \dfrac{3}{10}\underset{\sim}{j} + \dfrac{3}{5}\underset{\sim}{k}$

E. $\dfrac{3}{2}\underset{\sim}{i} - \dfrac{3}{10}\underset{\sim}{j} - \dfrac{3}{5}\underset{\sim}{k}$

Question 17

The position vector of a particle at time t is given by $\underset{\sim}{r}(t) = \cos(2t)\underset{\sim}{i} + \sin^2(t)\underset{\sim}{j},\ t \geq 0.$

The equation of the particle's path is

A. $y = \frac{1}{2}(1-x),\ -1 \leq x \leq 1$

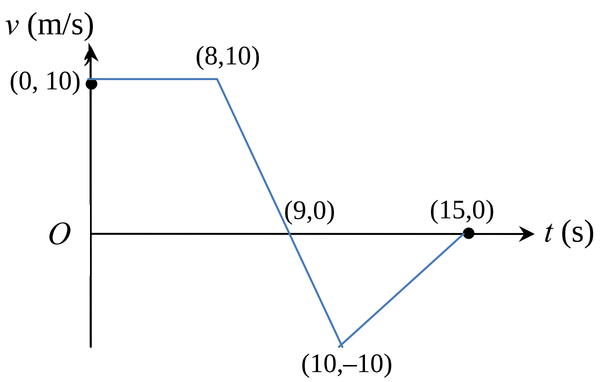
B. $y^2 = \frac{1}{2}(1-x),\ x \geq 0$

C. $y = \frac{1}{2}(1-x)^2,\ x \geq 0$

D. $y^2 = \frac{1}{2}(1-x)^2,\ -1 \leq x \leq 1$

E. $y^2 = \frac{1}{2}(1-x),\ -1 \leq x \leq 1$

Question 18



The velocity–time graph of a particle moving in a straight line starting from a fixed position, O, is shown above. The particle moves with a constant velocity for 8 seconds in a westerly direction. Where is the particle located 7 seconds later?

A. 55 m west of O
B. 115 m west of O
C. 120 m east of O
D. 55 m east of O
E. 100 m east of O

Question 19

An approximate solution to the differential equation $\frac{dy}{dx} = \log_e(2x+1)$ is found using Euler's method, with a step size of 0.1 and with $y_0 = 0$ and $x_0 = 0$.
The value obtained for y_2 would be

A. $\frac{\log_e 0.2}{10}$

B. $\frac{\log_e 1.2}{10}$

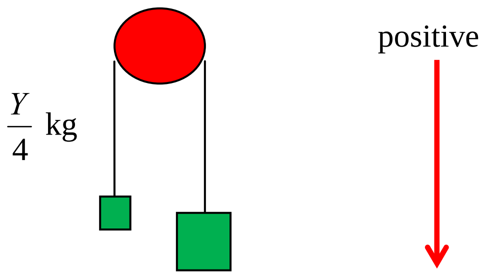
C. $\frac{1+\log_3 1.2}{10}$

D. $\log_e 0.1$

E. $\log_e 1.2$

Question 20

The diagram shows a smooth pulley with two objects attached to either end of an inextensible string. The mass of the smaller object is 40% less than the mass of the larger object.



The acceleration of the larger object is

A. $\frac{7g}{13}$ m/s^2

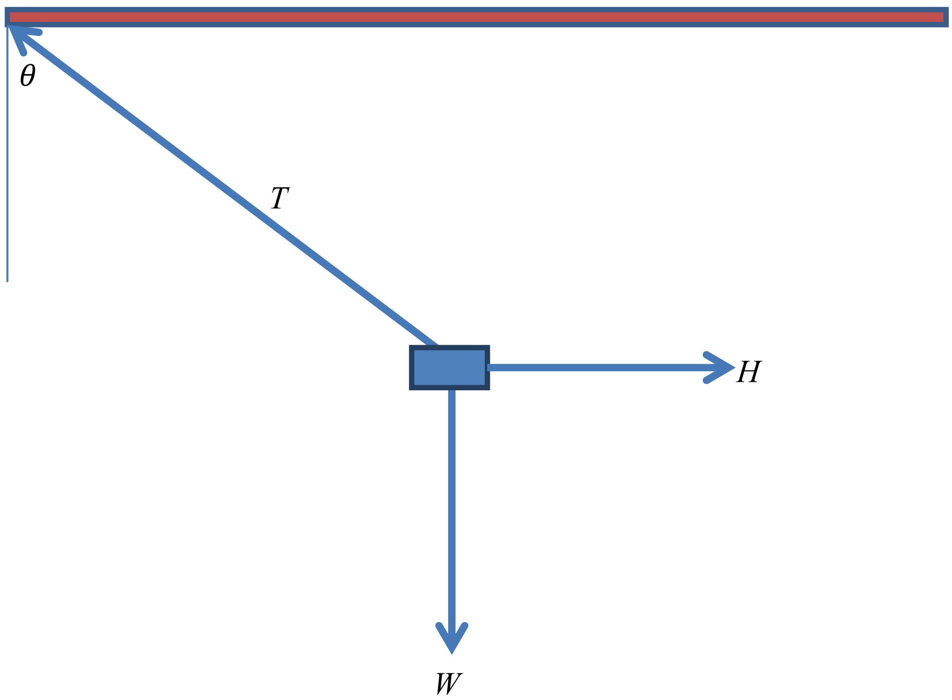
B. $\frac{g}{3}$ m/s^2

C. $\frac{g}{4}$ m/s^2

D. $\frac{1}{4}$ m/s^2

E. $4g$ m/s^2

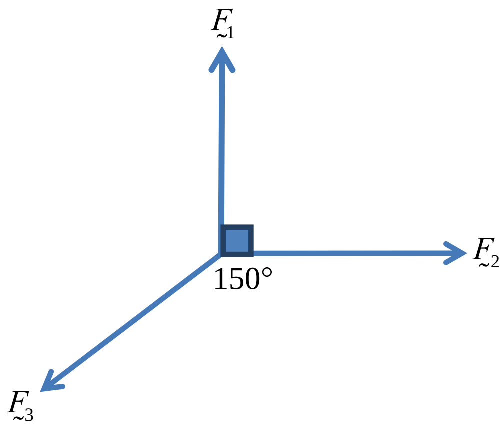
Question 21



An object of mass 10 kg is attached to a beam by a light inelastic string, making an angle of θ with the vertical. The tension force acting on the object is T newtons, and W newtons is the weight force. A horizontal force, H, of 60 newtons keeps the object in equilibrium. The value of θ (in degrees, to 2 decimal places) is

A. 0.55°
B. 31.47°
C. 31.48°
D. 58.52°
E. 30.96°

Question 22



Three co-planar forces, $\underset{\sim}{F}_1$, $\underset{\sim}{F}_2$ and $\underset{\sim}{F}_3$, act on a particle in equilibrium, then

A. $2\sqrt{3}F_1 = 2F_2 = \sqrt{3}F_3$

B. $F_1 = F_2 = F_3$

C. $2F_1 = 2\sqrt{3}F_2 = F_3$

D. $\sqrt{3}F_1 = 2F_2 = F_3$

E. $2\sqrt{3}F_1 = F_2 = 2F_3$

END OF SECTION 1

CONTINUES OVER PAGE

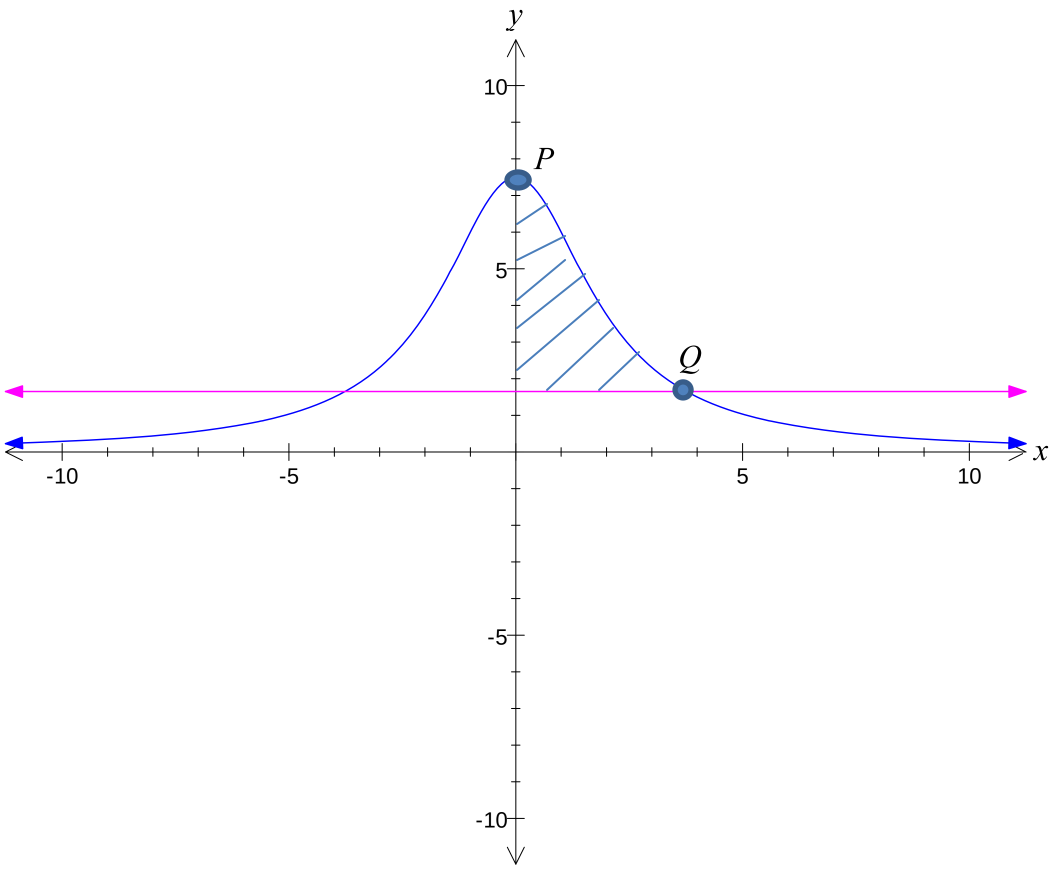
SECTION 2

Instructions for Section 2
Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1 (8 marks)

The graphs of the functions $y = \dfrac{a}{x^2 + b}$, $a > 0$, $b > 0$, and $y = c$, $c > 0$ are shown on the axes below.

P is the y-intercept of $y = \dfrac{a}{x^2 + b}$ and Q is one of the points of intersection of $y = \dfrac{a}{x^2 + b}$ and $y = c$.



SECTION 2 – Question 1 – continued

a. Find the coordinates of P and Q in terms of the constants a, b and c.

3 marks

b. **i.** Write a definite integral for A, the shaded area enclosed by the graphs and the y-axis, using the constants a, b, and c.

1 mark

ii. Write a definite integral for V, the volume of the shaded area rotated around the y-axis, using the constants a, b and c.

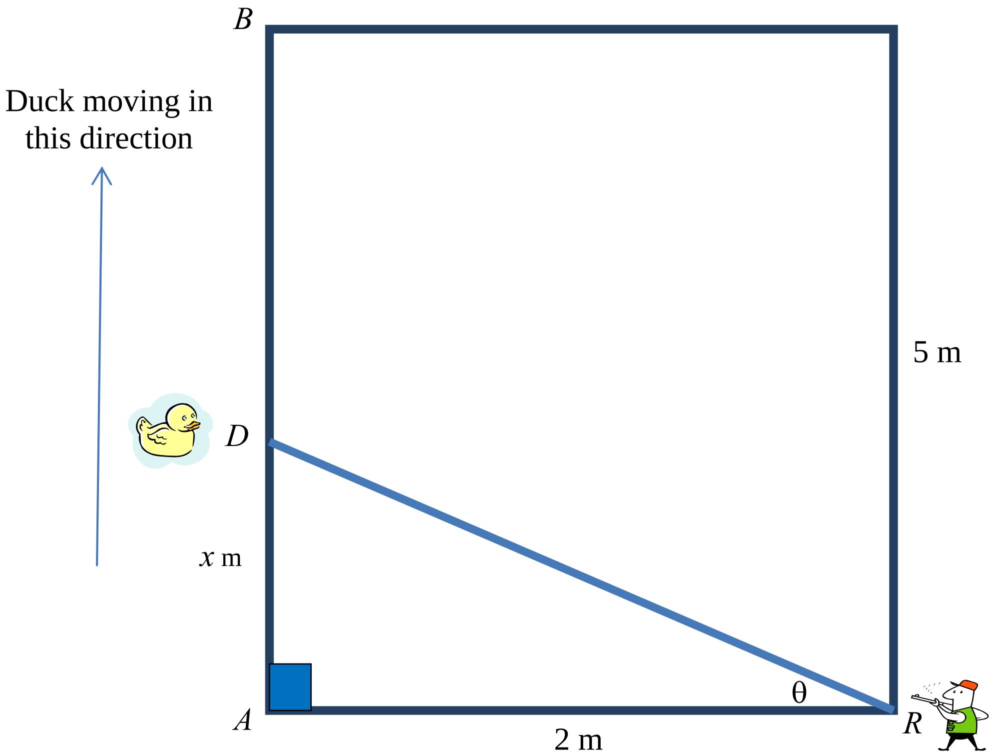
2 marks

c. If $a = 30$, $b = 4$ and $c = 1.6$, evaluate A and V, to 1 decimal place.

2 marks

Question 2 (14 marks)

The rectangle in the diagram represents an aerial view of a shooting gallery at an amusement park. A rifle is fixed at one side of the gallery at point *R* and a duck, *D*, moves along the line *AB* (5 m in length). The duck's velocity, in m/s, is given by $\frac{dx}{dt} = \frac{3x}{2}$, where x m is the distance from *A* to the duck. Initially, the duck is at the point where $x = 1$ and is moving towards *B*.



a. Find θ in terms of x.

1 mark

___

___

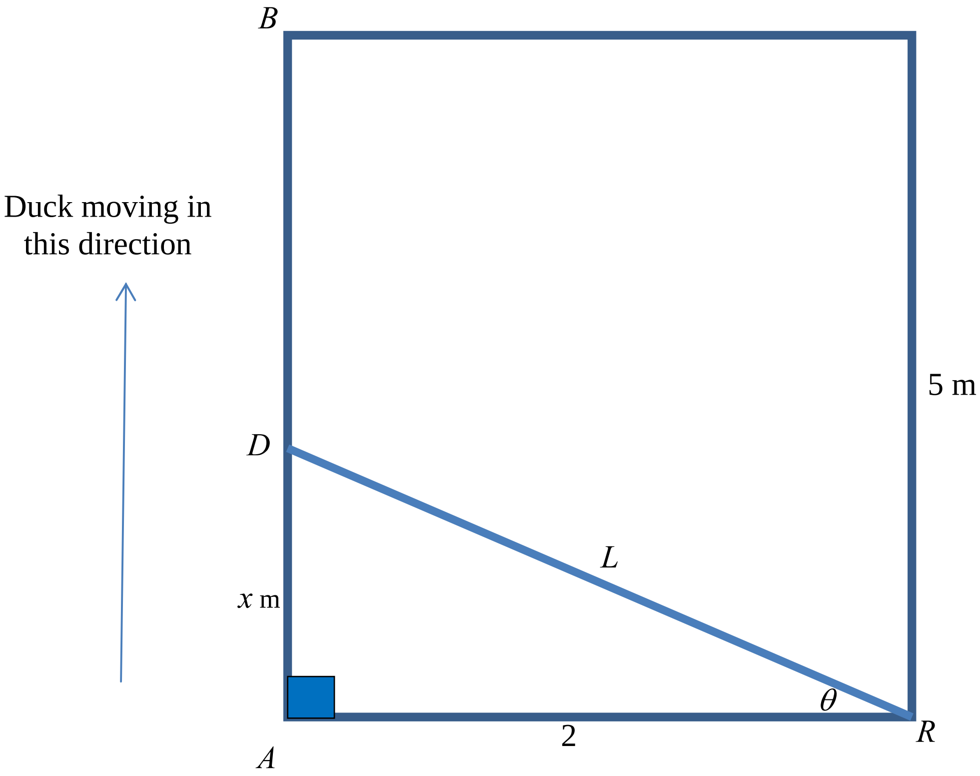
b. Find the value of $\frac{d\theta}{dt}$ when $x = 1$.

2 marks

c. Find a, b and c such that $\frac{d\theta}{dt} = \frac{a\sin(b\theta)}{c}$.

3 marks

d. Let L be the direct distance from the rifle to the duck.



i. Find L in terms of θ only.

1 mark

ii. Find $\frac{dL}{dt}$ in terms of θ.

2 marks

iii. Find $\frac{dL}{dt}$ in terms of L and, hence, find L in terms of t.

5 marks

Question 3 (12 marks)

In a chemical reaction, a compound C (kg) is formed by combining one part of chemical L and two parts of chemical M. The rate at which C is formed is proportional to the product of the amounts of chemicals L and M that are still present in the mixture at any given time *t* minutes. Initially, there are 5 kg of L, 8 kg of M and no amount of C present, but after some time, *x* kg of compound C has been formed.

a. How much of this *x* kg of C formed is chemical M and how much is chemical L?

1 mark

b. How many kilograms of chemical M and chemical L remain in the mixture?

1 mark

c. Show that $\frac{dx}{dt} = k(12 - x)(15 - x)$, where *k* is a constant.

2 marks

After 1 minute, 2 kg of chemical C has been formed.

d. Using calculus, solve the differential equation, writing t in terms of x.

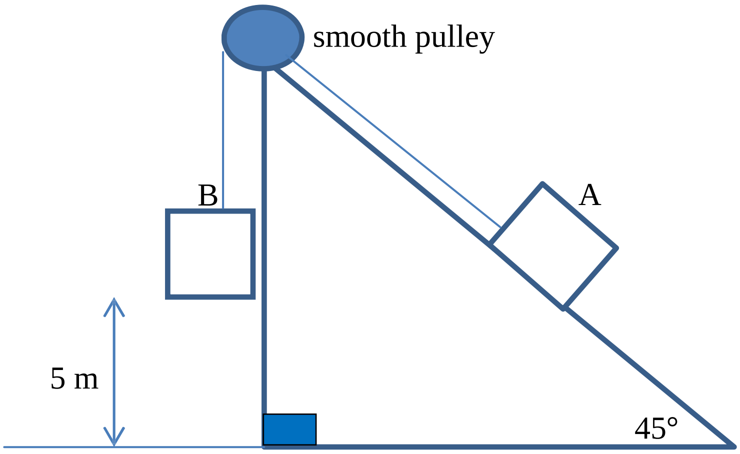
7 marks

e. How long, to the nearest minute, would it take to form 10 kg of compound C?

1 mark

Question 4 (10 marks)

The diagram shows a load, A, of mass M kg, being held stationary on a rough inclined slope by another load, B, also of mass M kg, which is hanging vertically 5 m above the ground. The two loads are connected by a light inelastic rope passing over a smooth pulley.



a. Initially, the load A is on the verge of moving up the slope. Show that μ, the coefficient of friction, is $\sqrt{2}-1$.

3 marks

b. Load A is now reduced by 20% and both loads begin to move from rest. Show that the acceleration of B is $\frac{g}{9}$ m/s^2.

4 marks

SECTION 2 – Question 4 – continued

c. The rope breaks after load B has been moving for 2 seconds. Find the values of *a*, *b* and *c*, if the speed with which B hits the ground after the rope breaks is $\frac{\sqrt{ag+bg^2}}{c}$.

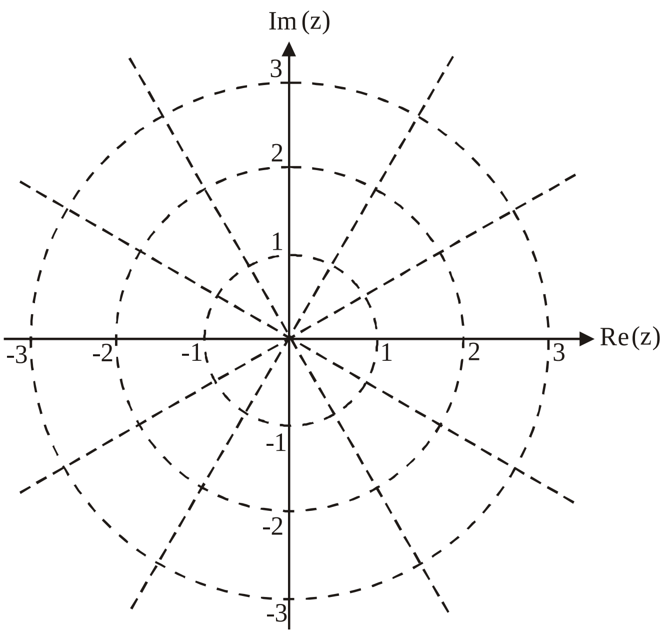
3 marks

Question 5 (14 marks)

a. Let $u = -1 + \sqrt{3}i$.

i. Find and plot z_1 and z_2, where z_1 and z_2 are solutions to $z^2 = \bar{u}$ where Arg $z_1 < 0$ and Arg $z_2 > 0$.

3 marks



ii. The complex relationship given by $|z - z_1| = 2|z - z_2|$ represents a circle. Find the Cartesian equation of the circle and state the exact coordinates of its centre and the value of its radius.

4 marks

b. Let $z = \cos\theta + i\sin\theta$.

i. Show that $z^3 + \dfrac{1}{z^3} = 2\cos(3\theta)$.

2 marks

ii. Show $z + \dfrac{1}{z} = 2\cos(\theta)$.

1 mark

iii. Hence, using parts **i** and **ii** or otherwise, show that
$4\cos^3(\theta) = \cos(3\theta) + 3\cos(\theta)$.

4 marks

END OF QUESTION AND ANSWER BOOK