Neap:

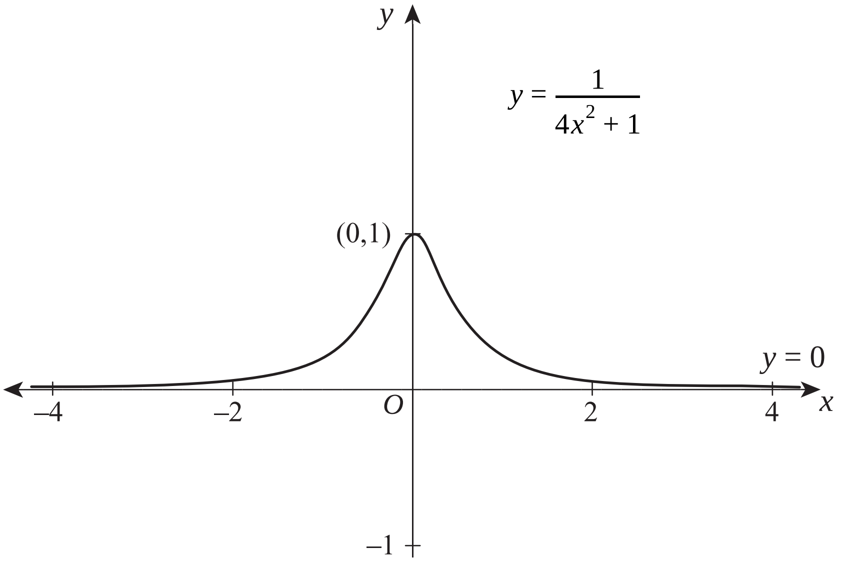
Trial Examination 2013

VCE Specialist Mathematics Units 3 & 4

Written Examination 1

Suggested Solutions

Question 1 (2 marks)

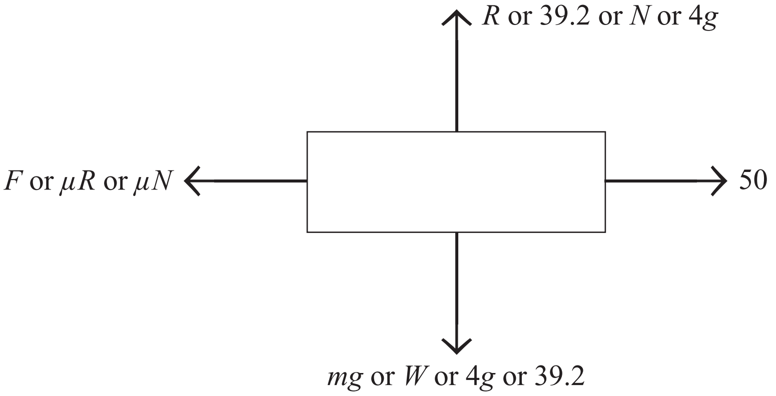


correct shape A1

horizontal asymptote $y = 0$; y-axis intercept $(0, 1)$ A1

Question 2 (4 marks)

a.



A1

b. 39.2 (newtons) or $4g$ (newtons) A1

c. $50 - F = 4 \times 2$ M1

$F = 42$ (newtons) A1

Question 3 (4 marks)

$g'(x) = (2x - x^2)e^{-x}$ M1

$g''(x) = (x^2 - 4x + 2)e^{-x}$ M1 A1

Solving $g''(x) = 0$ for x gives $x = 2 \pm \sqrt{2}$. A1

Question 4 (4 marks)

Let $u = 1 - \cos(x)$ and so $\frac{du}{dx} = \sin(x)$. M1

$\left[\log_e(1 - \cos(x))\right]_{\frac{\pi}{2}}^{k} = \frac{1}{2}$ $\left(\log_e(1 - \cos(x)) > 0 \text{ for } \frac{\pi}{2} < k < \pi\right)$ A1

$1 - \cos(k) = e^{\frac{1}{2}}$ A1

$k = \cos^{-1}(1 - \sqrt{e})$ A1

Question 5 (4 marks)

Use of $\cot^2(x) = \text{cosec}^2(x) - 1$ to obtain $3\text{cosec}^2(x) + 5\text{cosec}(x) - 2 = 0$. M1

Attempting to factorise or using the quadratic formula. M1

$\text{cosec}(x) = \frac{1}{3}$ or $\text{cosec}(x) = -2$ A1

$\sin(x) = -\frac{1}{2} \Rightarrow x = \frac{7\pi}{6}$ A1

Note: Award marks as above if a substitution, e.g. $u = \text{cosec}(x)$, *is used.*

Question 6 (4 marks)

$(x + yi)^2 = 5 - 12i$

$x^2 - y^2 + 2xyi = 5 - 12i$ M1

Equating real and imaginary parts we obtain:

$x^2 - y^2 = 5$ (1)

$xy = -6$ (2) (or equivalent)

A1

Note: Award A1 for two correct equations.

Solving (1) and (2) for x and y we obtain $x = \pm 3, y = \mp 2$. M1

So $z = 3 - 2i$ and $z = -3 + 2i$. A1

Question 7 (3 marks)

$\overrightarrow{BA} = \underset{\sim}{a} - \underset{\sim}{b}$ and $\overrightarrow{CB} = \underset{\sim}{a} + \underset{\sim}{b}$ A1

$\overrightarrow{BA} \cdot \overrightarrow{CB} = (\underset{\sim}{a} - \underset{\sim}{b}) \cdot (\underset{\sim}{a} + \underset{\sim}{b})$

$= |\underset{\sim}{a}|^2 - |\underset{\sim}{b}|^2$ A1

$\overrightarrow{BA} \cdot \overrightarrow{CB} = 0$ since $|\underset{\sim}{a}| = |\underset{\sim}{b}|$ and $|\underset{\sim}{a}|, |\underset{\sim}{b}| \neq 0$. A1

Hence $\angle ABC$ is a right angle.

Question 8 (6 marks)

a. Substituting $x = k$ and $y = 1$ into $x^3y - \sin\left(\frac{\pi y}{2}\right) = 7$ M1

$k^3 - \sin\left(\frac{\pi}{2}\right) = 7$

$k^3 = 8$ $\left(\text{as } \sin\left(\frac{\pi}{2}\right) = 1\right)$

$k = 2$ A1

b. Using implicit differentiation to differentiate $x^3y - \sin\left(\frac{\pi y}{2}\right) = 7$. M1

$3x^2y + x^3\frac{dy}{dx} - \frac{\pi}{2}\cos\left(\frac{\pi y}{2}\right)\frac{dy}{dx} = 0$ A1

At (2,1), $12 + 8\frac{dy}{dx} = 0$ M1

$\frac{dy}{dx} = -\frac{3}{2}$ A1

Question 9 (4 marks)

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{2x}{x^2 + 1}$ and so $\int \frac{d}{dx}\left(\frac{1}{2}v^2\right) = \int \frac{2x}{x^2 + 1}dx$

$\frac{1}{2}v^2 = \int \frac{2x}{x^2 + 1}dx$ A1

$v^2 = 2\log_e(x^2 + 1) + c$ where c is a constant of integration. M1

When $x = 0$, $v = 1$. Hence $c = 1$. M1

$v^2 = 2\log_e(x^2 + 1) + 1$

$v = \sqrt{2\log_e(x^2 + 1) + 1}$ (as $v = 1$ when $x = 0$) A1

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Question 10 (5 marks)

a. Given $\frac{(z+i)^n}{z^n} = 1$.

$\left(\frac{z+i}{z}\right)^n = 1 \Rightarrow \left(1 + \frac{i}{z}\right)^n = 1$

$1 + \frac{i}{z} = \text{cis}\left(\frac{0 + 2k\pi}{n}\right), k = 1, 2..., n - 1$ M1

$\frac{i}{z} = \text{cis}\left(\frac{2k\pi}{n}\right) - 1$

$z = \frac{i}{\text{cis}\left(\frac{2k\pi}{n}\right) - 1}$ A1

b. $z = \frac{i}{\text{cis}\left(\frac{2k\pi}{n}\right) - 1}$

$= \frac{i\,\text{cis}\left(-\frac{k\pi}{n}\right)}{\text{cis}\left(\frac{k\pi}{n}\right) - \text{cis}\left(-\frac{k\pi}{n}\right)}$ (multiplying by $\frac{\text{cis}\left(-\frac{k\pi}{n}\right)}{\text{cis}\left(-\frac{k\pi}{n}\right)}$) M1

$= \frac{i\left(\cos\left(\frac{k\pi}{n}\right) - i\sin\left(\frac{k\pi}{n}\right)\right)}{2i\sin\left(\frac{k\pi}{n}\right)}$ A1

$= \frac{1}{2}\left(\frac{\cos\left(\frac{k\pi}{n}\right)}{\sin\left(\frac{k\pi}{n}\right)} - \frac{i\sin\left(\frac{k\pi}{n}\right)}{\sin\left(\frac{k\pi}{n}\right)}\right)$

So $z = \frac{1}{2}\left(\cot\left(\frac{k\pi}{n}\right) - i\right)$. A1