2013 Specialist Mathematics GA 2: Examination 1

GENERAL COMMENTS

In the 2013 Specialist Mathematics 1 exam, students were required to answer nine short-answer questions worth a total of 40 marks. This was a technology-free examination.

In the comments on specific questions in this report, common errors are highlighted. These should be brought to the attention of students so that they can develop strategies to avoid them. Students are reminded to read questions carefully as responses to several questions indicated that students had not done so.

The quality of students' handwriting and the manner in which they set out their mathematics continues to be of concern. Students should be reminded that if an assessor cannot read a student's writing or is not certain as to what it is conveying, marks cannot be awarded. Furthermore, students are expected to set out their work properly. If an assessor is unable to follow a student's working (or reasoning), full marks will not be awarded. Equals signs should be placed between quantities that are equal – the working should not appear to be a number of disjointed statements. If there are logical inconsistencies in the student's working, full marks will not be awarded. For example, if an equals sign is placed between quantities that are not equal, full marks will not be awarded.

Areas of weakness included

- not reading the question carefully enough – this included not answering the question, proceeding further than required or not giving the answer in the specified form. The latter was common and particularly evident in Questions 4a., 4b., 8 and 9. Students should be reminded that good examination technique includes rereading the question after it has been answered to ensure that they have answered what was required and that they have given their answer in the correct form
- algebraic skills – Difficulty with algebra was evident in several questions. The inability to simplify expressions often prevented some students from completing a question. Incorrect attempts to factorise, expand and simplify, and the poor use of brackets were common
- arithmetic skills – Difficulty with arithmetic was evident in several questions. The inability to evaluate expressions, especially those involving fractions, was common
- showing a given result – This was required in Questions 5a. and 7a. In such questions, students need to include sufficient relevant working to demonstrate that they know how to derive the result. Students should be reminded that they can use a given value in the remaining part(s) of the question whether or not they were able to derive it
- recognising the need to use the chain rule when differentiating implicitly (Question 6)
- recognising the need to use the quotient rule when differentiating (Question 6)
- recognising the need to use the chain rule when differentiating (Questions 4c., 6 and 7c.)
- recognising the need to use the product rule when differentiating (Question 6)
- recognising the method of integration required (Questions 2, 5a. and 9)
- using a constant of integration when performing an indefinite integration (Question 5)
- knowing the exact values for circular functions (Questions 1b., 7c., 8 and 9)
- consideration of the quadrant for values of circular functions (Question 9)
- giving answers in the required form (Questions 4a. and 5a.).

In this examination, students should be able to apply techniques, routines and processes, involving rational, real and complex arithmetic, and evaluate arithmetic expressions. Many students were unable to do this easily and missed out on marks as a consequence. Many students made algebraic or numerical slips at the end of an answer, which meant that the final mark could not be awarded. This was especially unfortunate in cases where students had a correct answer and there was no need for further simplification.

There were several cases where incorrect working fortuitously led to a correct answer. Students should be reminded that in such cases, the final answer mark will not be awarded if it is not supported by relevant and correct working.

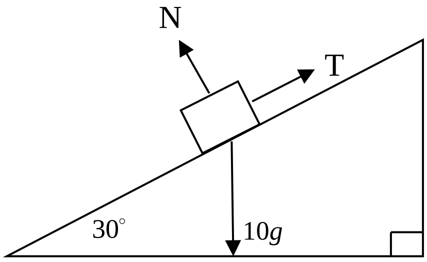
SPECIFIC INFORMATION

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding errors resulting in a total less than 100 per cent.

Question 1a.

Marks	0	1	Average
%	16	84	**0.9**



This question was quite well done. The most common errors involved showing extra forces. This was usually a friction force, but often a force down the plane was shown. Where a force is resolved, students should not show the components as bold line segments with arrows. Instead, dashed line segments should be shown to make it clear that these are components and not extra forces. Other errors included a vertical direction for the normal reaction, the weight force drawn perpendicular to the plane, and the weight force labelled as 10 rather than 10*g*.

Question 1b.

Marks	0	1	2	Average
%	11	14	75	**1.7**

$T = 5g = 49$

This question was reasonably well done. The main error was that friction was included in the equation of motion (and not removed or not properly set to zero). Sometimes acceleration was included in the equation of motion (and not removed or not set to zero) and often $10g\cos(30^\circ)$ was used for the component of the weight force along the plane. There were several errors made with exact trigonometric values such as $\sin(30^\circ) = \frac{\sqrt{3}}{2}$, and with numerical calculations such as 5*g* = 490.

Question 2

Marks	0	1	2	3	4	Average
%	14	4	13	22	47	**2.8**

$2\log_e(3) - 5\log_e(2) = \log_e\left(\frac{9}{32}\right)$ (other equivalent answers were accepted)

Most students identified the need to use partial fractions. However, quite a few students were unable to successfully factorise the quadratic in the denominator, often giving $x^2 - 5x + 6 = (x-6)(x+1)$. Most students who had the correct factors managed to obtain the correct partial fractions. The most common error was the lack of modulus signs leading to the logarithms of negative numbers. A number of students were unable to apply the log laws and simplified incorrectly. For instance, the following 'simplification' occurred often: $2\log_e(3) - 5\log_e(2) = \frac{2}{5}\log_e\left(\frac{3}{2}\right)$. A significant number of students increased the work required either by first writing the given fraction as $\frac{x-5}{x^2-5x+6}=\frac{x}{x^2-5x+6}-\frac{5}{x^2-5x+6}$ and then using partial fractions on each term, or by using $\frac{x-5}{x^2-5x+6}=\frac{1}{2}\left(\frac{2x-5}{x^2-5x+6}\right)-\frac{1}{2}\left(\frac{5}{x^2-5x+6}\right)$ and then using substitution and partial fractions. Few students who used an unnecessarily complicated approach were successful.

Question 3a.

Marks	0	1	Average
%	14	86	**0.9**

$\overrightarrow{AB}=2\underset{\sim}{i}-2\underset{\sim}{j}+\underset{\sim}{k}$

This question was very well done by most students. The most typical errors were finding $\overrightarrow{BA}$ or simply adding the components. Some students gave their answer as an ordered triple without indicating that it was a vector rather than a point.

Question 3b.

Marks	0	1	2	Average
%	19	5	75	**1.6**

Students were required to show that there was a right angle at *A* using the dot product or Pythagoras's theorem.

Most students used one of the correct approaches. Many students made a mistake in finding $\overrightarrow{AC}$ (or $\overrightarrow{CA}$). In a small number of cases the dot product was evaluated as a vector. A significant number of students used $\cos A=\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$, wasting time finding the modulus values in the denominator.

Question 3c.

Marks	0	1	Average
%	28	72	**0.7**

$\sqrt{38}$

This question was reasonably well done. Most errors were arithmetic. Other typical errors included giving 38, which is the square of the length, and finding the lengths of all three sides but not stating which was the length of the hypotenuse.

Question 4a.

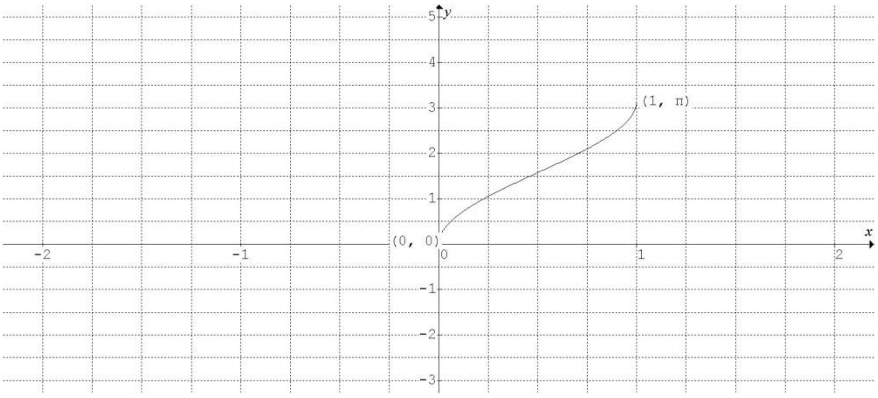
Marks	0	1	2	Average
%	20	28	52	**1.3**

Domain $[0, 1]$, range $[0, \pi]$

There were many incorrect answers given for the domain (for example, [–1, 1], [–1, 3], [–0.5, 1.5]), with poor attempts to solve the inequality $-1 \le 1-2x \le 1$. Some students reached $-2 \le -2x \le 0$ and then divided by –2 to write $1 \le x \le 0$. The most successful technique appeared to be to solve the equation $1-2x=\pm 1$ to find the endpoints of the domain. The range was generally given correctly, but many students did not state it. Students should always review their work to make sure they have answered all parts of a question. A small number of students omitted the endpoints, writing $(0, \pi)$.

Question 4b.

Marks	0	1	2	Average
%	24	30	46	**1.2**



This question was reasonably well done by students who successfully completed part a. Many students showed the incorrect orientation of the graph (i.e. pairing the endpoints of the domain and range the wrong way), perhaps using the shape of $y = \arccos(x)$ and ignoring the reflection. Several incorrectly had a stationary rather than non-stationary point of inflection in the middle of the graph (where the gradient is 2), while others gave the correct endpoint but incorrect shape (with the gradient near zero at the endpoint). A few students sketched the correct graph but did not label both endpoints with their coordinates as required – usually (0, 0) was missing.

Question 4c.

Marks	0	1	2	Average
%	40	13	47	**1.1**

$$\frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$$

This question was not done well, with many students not recognising the need to use the chain rule to find the derivative. The -2 was commonly missing. Students who used the chain rule often made errors in substitution and/or numerical simplification.

Question 5a.

Marks	0	1	2	Average
%	19	16	65	**1.5**

Students had to show the given result $e^{-5k} = \frac{3}{4}$.

Most students made a reasonable attempt at this question, although many did not have a constant of integration. Some students who did have a constant of integration were unable to evaluate it. Nevertheless, this did not stop them from 'showing' the given result. Some students made errors of arithmetic. A small number of students were able to use definite integration successfully.

Question 5b.

Marks	0	1	2	3	Average
%	23	14	22	41	**1.8**

$T = 65$

Surprisingly, the result of Question 5a. was rarely used directly. The majority of students expressed k in logarithmic form and often struggled with the algebra. In many cases, their solution to the differential equation had not been expressed with T as the subject but was left with t as the subject. Only a small number of students who did have

$T = 20 + 80e^{-kt}$ were able to write $e^{-10k} = (e^{-5k})^2$ in order to solve this part efficiently. Some had difficulties with the exponentials and logarithms. There was some poor arithmetic at the end; for example, $T - 20 = 45$, so $T = 25$ was quite common. Several students multiplied first rather than cancelling and made their working more complicated. A small number of students left their answer in the form $T = \frac{1040}{16}$, $T = \frac{720}{16} + 20$ or $T = \frac{130}{2}$ (unsimplified).

Question 6

Marks	0	1	2	3	4	Average
%	13	7	12	28	40	**2.8**

$c = -\frac{3}{4}$

This question was reasonably well done. Most students recognised the need for implicit differentiation and so wrote $2y\frac{dy}{dx}$. A reasonable number realised that they needed the quotient rule (or product rule) and the chain rule, although a number had difficulties with algebra. Some students forgot that the derivative of a constant was 0, so a '*c*' remained on the right-hand side after differentiation, meaning that no significant progress was then possible. Some students chose to multiply through by $(x - 2)$ before differentiation. These students were rarely able to make good progress (though a few were able to correctly complete the question this way). Those who attempted to make *y* the subject often omitted the $\pm$. Typical errors included having a negative sign error in finding *y* (which nevertheless gave the correct value for *c*), incorrect differentiation such as $\frac{d}{dx}(3e^{x-1}) = 3(x-1)e^{x-1}$ and errors in algebra.

Question 7a.

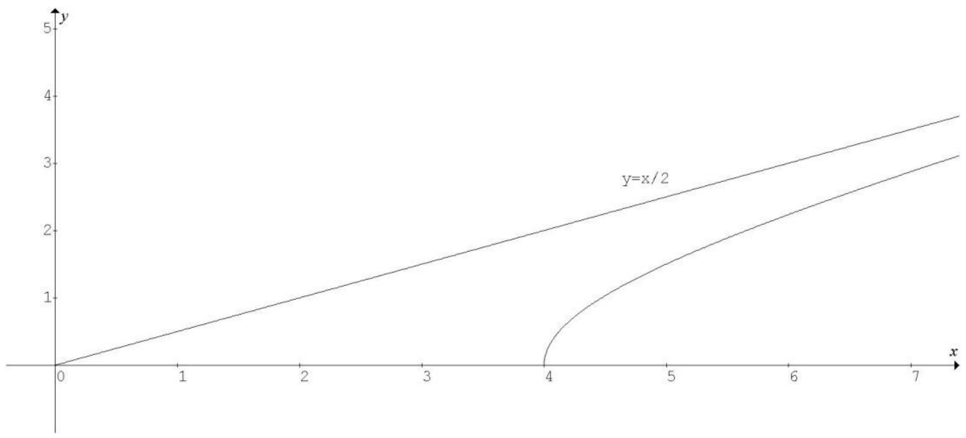
Marks	0	1	2	Average
%	16	4	80	**1.7**

Students had to show the given result $\frac{x^2}{16} - \frac{y^2}{4} = 1$

Most students recognised the trigonometric identity required and answered this question very well. Some did not show sufficient working or clearly indicate what they were doing (stating the identity makes this clear). A few students managed to express everything in terms of cosine and sine and eventually answered correctly. Students should be reminded that the formula sheet often has a suitable formula for use in questions such as this.

Question 7b.

Marks	0	1	2	Average
%	32	46	22	**0.9**



This question was not well done by most students. A large number of students drew a complete hyperbola or the complete right-hand branch. A few students had the branch in the first quadrant but inexplicably indicated that it stopped at about $x = 8$. The equation of the asymptote was often correctly found, although a few students did not label it as required. There were, however, many incorrect equations, including $y = \frac{1}{4}x$, $y = \frac{1}{2}$ and $y = 2x$. There were also several graphs that did not exhibit asymptotic behaviour. A variety of graphs other than hyperbolas were often seen, including ellipses.

Question 7c.

Marks	0	1	2	Average
%	52	19	29	**0.8**

$\sqrt{48} = 4\sqrt{3}$

This question was poorly done by most students, with most unable to differentiate nec(G) to obtain $\sec(t)\tan(t)$. Some used $\frac{4}{\cos(t)}$ and gave the derivative as $-\frac{4}{\sin(t)}$ or used the quotient rule with $\frac{d}{dt}(4) = 4$. A small number of students used $4\cos^{-1}(t)$ and then found the derivative of the inverse trigonometric function. A few students were able to find the correct derivatives, but then stopped working with vectors. Several students thought that $\frac{dy}{dx}$ gave the speed, while some found the magnitude of $\frac{dy}{dx}$ rather than $\frac{dr}{dt}$. There were also a large number of sign errors and some difficulties with simplifying surds.

Question 8

Marks	0	1	2	3	4	Average
%	23	13	26	14	23	**2**

$\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}i, -\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}i, \frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}i, -\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}i$

There were many good attempts at this question, using either completing the square or the quadratic formula, so many students were able to find that $z^2 = 1 \pm i\sqrt{3}$. The most frequently occurring answer from that point was $z = \pm\sqrt{1 \pm i\sqrt{3}}$, which is not of the form $z = x \pm iy$. Those who used polar form often achieved correct answers, although a few forgot to change back to Cartesian form. Some students made errors in the sine and cosine of standard angles. A few students solved the question successfully by other methods and did not need to use polar form. Solving $(x+iy)^2 = 1 \pm i\sqrt{3}$ was a viable approach but most who used this approach struggled with the algebra. There were some interesting attempts including treating the original expression as a perfect square as well as working that involved the factor theorem.

Question 9

Marks	0	1	2	3	4	Average
%	27	13	11	20	29	**2.1**

$\frac{\pi^4}{9} - \frac{\pi^2}{6} + \frac{\pi\sqrt{3}}{8}$

This question was well done by some students and poorly by others. Common errors included π being omitted from the outset, an incorrect expression for the volume – using $(3x - \sin x)^2$ as the integrand was common – the double angle formula not being used or being used incorrectly (for example, a sign error), occasional mistakes with the integration step (for example, a sign error), algebraic simplification errors at the end and finding the area rather than the volume. Several students did not distribute a negative through the brackets correctly. Too many students were unable to evaluate

$\sin\left(\frac{2\pi}{3}\right)$ and many errors were made when expanding brackets, especially sign errors. Also, $(3x)^2 = 6x^2$ was often seen. A few students treated the shape to be rotated as a triangle. Some students arrived at the correct answer but then made errors when attempting to use a common denominator (which was unnecessary).