



# Victorian Certificate of Education 2013

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

## STUDENT NUMBER

Figures

Words


Letter

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# SPECIALIST MATHEMATICS

## Written examination 2

Monday 11 November 2013

Reading time: 3.00 pm to 3.15 pm (15 minutes)

Writing time: 3.15 pm to 5.15 pm (2 hours)

## QUESTION AND ANSWER BOOK

### Structure of book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

### Materials supplied

- Question and answer book of 25 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.

### Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- All written responses must be in English.

### At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

**SECTION 1****Instructions for Section 1**

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Take the **acceleration due to gravity** to have magnitude  $g \text{ m/s}^2$ , where  $g = 9.8$ .

**Question 1**

The domain of the function with rule  $f(x) = \arcsin(3x)$  is

- A.  $[-1, 1]$
- B.  $[-3, 3]$
- C.  $\left[0, \frac{\pi}{3}\right]$
- D.  $\left[-\frac{1}{3}, \frac{1}{3}\right]$
- E.  $\left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$

**Question 2**

The rule of the relation determined by the parametric equations  $x = 2\operatorname{cosec}(t) + 1$  and  $y = 3\cot(t) - 1$  is

- A.  $\frac{(x-1)^2}{4} - \frac{(y+1)^2}{9} = 1$
- B.  $\frac{(y+1)^2}{9} - \frac{(x-1)^2}{4} = 1$
- C.  $\frac{(x-1)^2}{4} + \frac{(y+1)^2}{9} = 1$
- D.  $\frac{(y+1)^2}{3} - \frac{(x-1)^2}{2} = 1$
- E.  $\frac{(x-1)^2}{2} - \frac{(y+1)^2}{3} = 1$

**Question 3**

The graph of  $y = \frac{1}{ax^2 + bx + c}$  has asymptotes at  $x = -5$ ,  $x = 3$  and  $y = 0$ .

Given that the graph has one stationary point with a  $y$ -coordinate of  $-\frac{1}{8}$ , it follows that

- A.  $a = 1, b = 2, c = -15$
- B.  $a = \frac{1}{2}, b = -1, c = -\frac{15}{2}$
- C.  $a = -\frac{1}{2}, b = -1, c = 15$
- D.  $a = -1, b = -2, c = -15$
- E.  $a = \frac{1}{2}, b = 1, c = -\frac{15}{2}$

**Question 4**

The graphs of  $y = ax$  and  $y = \arctan(bx)$  intersect exactly **three** times if

- A.  $0 < b < a$
- B.  $a < b < 0$
- C.  $a = b$
- D.  $b < a < 0$
- E.  $0 < b^2 < a^2$

**Question 5**

The region in the complex plane that is **outside** the circle of radius  $b$  centred at the origin is given by the set of points  $z$ , where  $z \in C$ , such that

- A.  $|z| < b$
- B.  $|z| > b$
- C.  $|z| > b^2$
- D.  $|z| = b$
- E.  $|z| < b^2$

**Question 6**

Let  $z = a + bi, z \in C$ .

If the principal argument of  $z^3$  is in the second quadrant, then the complete set of values for  $\text{Arg}(z)$  is

- A.  $\left(\frac{\pi}{2}, \pi\right)$
- B.  $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$
- C.  $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$
- D.  $\left(-\frac{\pi}{2}, -\frac{\pi}{3}\right) \cup \left(\frac{\pi}{6}, \frac{\pi}{3}\right)$
- E.  $\left(\frac{\pi}{6}, \frac{\pi}{3}\right) \cup \left(\frac{\pi}{2}, \pi\right)$

**Question 7**

If  $z = r \operatorname{cis}(\theta)$ , then  $\frac{z^2}{\bar{z}}$  is equivalent to

- A.  $r^3 \operatorname{cis}(3\theta)$
- B.  $r^3 \operatorname{cis}(-\theta)$
- C.  $2 \operatorname{cis}(3\theta)$
- D.  $r^3 \operatorname{cis}(\theta)$
- E.  $r \operatorname{cis}(3\theta)$

**Question 8**

The principal arguments of the solutions to the equation  $z^2 = 1 + i$  are

- A.  $\frac{\pi}{8}$  and  $\frac{9\pi}{8}$
- B.  $-\frac{\pi}{8}$  and  $\frac{7\pi}{8}$
- C.  $-\frac{7\pi}{8}$  and  $\frac{\pi}{8}$
- D.  $\frac{7\pi}{8}$  and  $\frac{15\pi}{8}$
- E.  $-\frac{3\pi}{4}$  and  $\frac{\pi}{4}$

**Question 9**

The definite integral  $\int_{e^3}^{e^4} \frac{1}{x \log_e(x)} dx$  can be written in the form  $\int_a^b \frac{1}{u} du$  where

- A.  $u = \log_e(x)$ ,  $a = \log_e(3)$ ,  $b = \log_e(4)$
- B.  $u = \log_e(x)$ ,  $a = 3$ ,  $b = 4$
- C.  $u = \log_e(x)$ ,  $a = e^3$ ,  $b = e^4$
- D.  $u = \frac{1}{x}$ ,  $a = e^{-3}$ ,  $b = e^{-4}$
- E.  $u = \frac{1}{x}$ ,  $a = e^3$ ,  $b = e^4$

**Question 10**

The region bounded by the lines  $x = 0$ ,  $y = 3$  and the graph of  $y = x^{\frac{4}{3}}$  where  $x \geq 0$  is rotated about the **y-axis** to form a solid of revolution.

The volume of this solid is

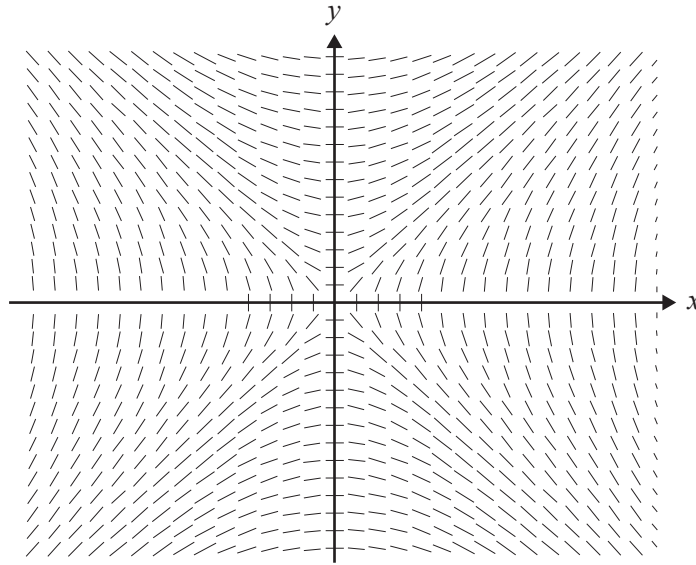
- A.  $\frac{81\pi 3^{\frac{2}{3}}}{11}$
- B.  $\frac{12\pi 3^{\frac{3}{4}}}{7}$
- C.  $\frac{27\pi 3^{\frac{1}{3}}}{7}$
- D.  $\frac{18\pi 3^{\frac{1}{2}}}{5}$
- E.  $\frac{6\pi 3^{\frac{1}{2}}}{5}$

**Question 11**

Consider the differential equation  $\frac{dy}{dx} = \frac{1}{3 + 3x + x^2}$ , with  $y_0 = 1$  when  $x_0 = 0$ .

Using Euler's method with a step size of 0.1, the value of  $y_2$ , correct to three decimal places, is

- A. 1.033
- B. 1.063
- C. 1.064
- D. 1.065
- E. 1.066

**Question 12**

The differential equation that best represents the above direction field is

- A.  $\frac{dy}{dx} = x^2 - y^2$
- B.  $\frac{dy}{dx} = y^2 - x^2$
- C.  $\frac{dy}{dx} = \frac{y}{x}$
- D.  $\frac{dy}{dx} = -\frac{x}{y}$
- E.  $\frac{dy}{dx} = \frac{x}{y}$

**Question 13**

Water containing 2 grams of salt per litre flows at the rate of 10 litres per minute into a tank that initially contained 50 litres of pure water. The concentration of salt in the tank is kept uniform by stirring and the mixture flows out of the tank at the rate of 6 litres per minute.

If  $Q$  grams is the amount of salt in the tank  $t$  minutes after the water begins to flow, the differential equation relating  $Q$  to  $t$  is

- A.  $\frac{dQ}{dt} = 20 - \frac{3Q}{25 + 2t}$
- B.  $\frac{dQ}{dt} = 10 - \frac{3Q}{25 + 2t}$
- C.  $\frac{dQ}{dt} = 20 - \frac{3Q}{25 - 2t}$
- D.  $\frac{dQ}{dt} = 10 - \frac{3Q}{25 - 2t}$
- E.  $\frac{dQ}{dt} = 20 - \frac{3Q}{25}$

**Question 14**

The distance from the origin to the point  $P(7, -1, 5\sqrt{2})$  is

- A.  $7\sqrt{2}$
- B. 10
- C.  $6 + 5\sqrt{2}$
- D. 100
- E.  $5\sqrt{6}$

**Question 15**

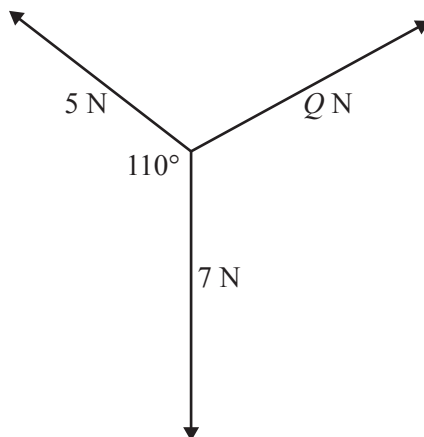
Let  $\underline{u} = 4\underline{i} - \underline{j} + \underline{k}$ ,  $\underline{v} = 3\underline{j} + 3\underline{k}$  and  $\underline{w} = -4\underline{i} + \underline{j} + \underline{k}$ .

Which one of the following statements is **not** true?

- A.  $|\underline{u}| = |\underline{v}|$
- B.  $|\underline{u}| = |-\underline{w}|$
- C.  $\underline{u}$ ,  $\underline{v}$  and  $\underline{w}$  are linearly independent
- D.  $\underline{u} \cdot \underline{v} = 0$
- E.  $(\underline{u} + \underline{w}) \cdot \underline{v} = 12$

**Question 16**

Forces of magnitude 5 N, 7 N and  $Q$  N act on a particle that is in equilibrium, as shown in the diagram below.



The magnitude of  $Q$ , in newtons, can be found by evaluating

- A.  $\sqrt{5^2 + 7^2 - 2 \times 5 \times 7 \cos(70^\circ)}$
- B.  $5^2 + 7^2 - 2 \times 5 \times 7 \cos(110^\circ)$
- C.  $\sqrt{5^2 + 7^2 - 2 \times 5 \times 7 \cos(110^\circ)}$
- D.  $5^2 + 7^2 - 2 \times 5 \times 7 \cos(70^\circ)$
- E.  $\sqrt{5^2 + 7^2 - 2 \times 5 \times 7 \cos(20^\circ)}$

**Question 17**

Consider the four vectors  $\underline{a} = \underline{j} + 3\underline{k}$ ,  $\underline{b} = \underline{i} - 4\underline{k}$ ,  $\underline{c} = 3\underline{j} - \underline{k}$  and  $\underline{d} = 2\underline{j} + \underline{k}$ .

Which one of the following is a **linearly dependent** set of vectors?

- A.  $\{\underline{a}, \underline{b}, \underline{c}\}$
- B.  $\{\underline{a}, \underline{c}, \underline{d}\}$
- C.  $\{\underline{a}, \underline{b}, \underline{d}\}$
- D.  $\{\underline{b}, \underline{c}, \underline{d}\}$
- E.  $\{\underline{a}, \underline{b}\}$

**Question 18**

A particle moves in a straight line such that its acceleration is given by  $a = \sqrt{v^2 - 1}$ , where  $v$  is its velocity and  $x$  is its displacement from a fixed point.

Given that  $v = \sqrt{2}$  when  $x = 0$ , the velocity  $v$  in terms of  $x$  is

- A.  $v = \sqrt{2 + x}$
- B.  $v = 1 + |x + 1|$
- C.  $v = \sqrt{2 + x^2}$
- D.  $v = \sqrt{1 + (1 + x)^2}$
- E.  $v = \sqrt{1 + (x - 1)^2}$

**Question 19**

A tourist in a hot air balloon, which is rising at 2 m/s, accidentally drops a camera over the side and it falls 100 m to the ground.

Neglecting the effect of air resistance on the camera, the time taken for the camera to hit the ground, correct to the nearest tenth of a second, is

- A. 4.3 s
- B. 4.5 s
- C. 4.7 s
- D. 4.9 s
- E. 5.0 s

**Question 20**

A 5 kg parcel is on the floor of a lift that is accelerating downwards at  $3 \text{ m/s}^2$ .

The reaction, in newtons, of the floor of the lift on the parcel is

- A.  $-15 + 5g$
- B.  $15 + 5g$
- C.  $-15 + 3g$
- D.  $-15 - 5g$
- E.  $15 + 3g$



**Question 21**

A particle of mass 2 kg moves in a straight line with an initial velocity of 20 m/s. A constant force opposing the direction of the motion acts on the particle so that after 4 seconds its velocity is 2 m/s.

The magnitude of the force, in newtons, is

- A. 4.5
- B. 6
- C. 9
- D. 18
- E. 36

**Question 22**

A parachutist of mass  $m$  kg falls towards Earth and is slowed by air resistance of  $kv^2$  newtons, where  $v$  is the velocity of the parachutist  $t$  seconds after commencing the fall.

The equation of motion for the parachutist is

- A.  $\frac{dv}{dt} = g - kv^2$
- B.  $m \frac{d^2x}{dt^2} = g - kv^2$
- C.  $m \frac{d(v^2)}{dx} = \frac{mg - kv^2}{2}$
- D.  $v \frac{dv}{dx} = 1 - \frac{kv^2}{mg}$
- E.  $\frac{dv}{dx} = \frac{g}{v} - \frac{kv}{m}$

**SECTION 2****Instructions for Section 2**

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude  $g \text{ m/s}^2$ , where  $g = 9.8$ .

**Question 1** (11 marks)

A curve is defined by the parametric equations

$$\begin{aligned}x &= 1 + 3\cos(t) \\ y &= -2 + 2\sin(t)\end{aligned}$$

for  $t \in [0, 2\pi]$ .

- a.** Find the cartesian equation of the curve.

2 marks

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- b.** Find the values of  $t$  for which the gradient of the curve is  $-\frac{2\sqrt{3}}{3}$ .

3 marks

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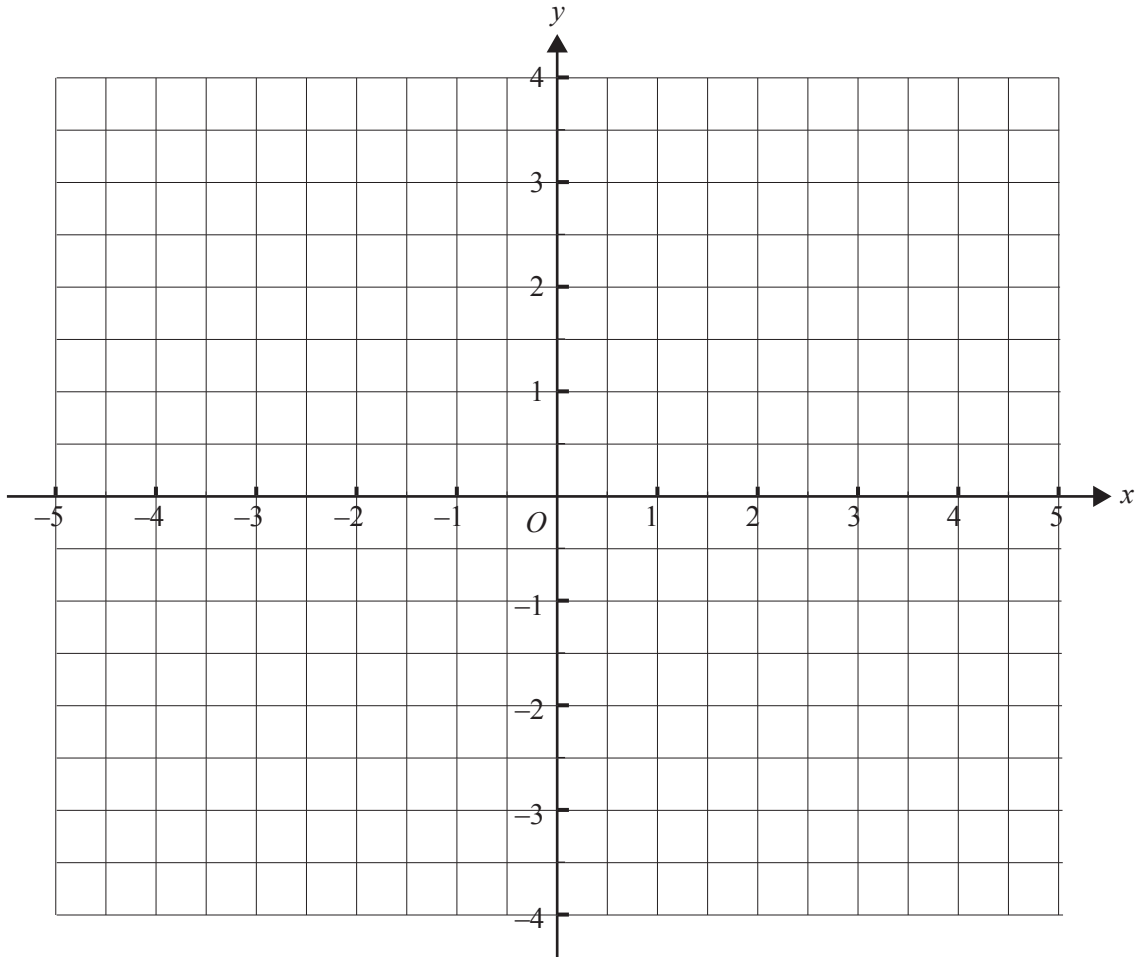
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Consider a **different** relation  $\frac{(x-1)^2}{9} + \frac{y^2}{4} = 1$ .

- c. Sketch the graph of  $\frac{(x-1)^2}{9} + \frac{y^2}{4} = 1$  and find the values of the  $x$ -axis and  $y$ -axis intercepts. 3 marks




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The region in the first quadrant enclosed by the graph of  $\frac{(x-1)^2}{9} + \frac{y^2}{4} = 1$ , the  $x$ -axis, and the lines  $x = 1$  and  $x = 3$  is rotated about the  $x$ -axis to form a solid of revolution.

- d. i.** Write down a definite integral, in terms of  $x$ , that gives the volume of this solid of revolution.

2 marks

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- ii.** Find the volume of this solid of revolution.

1 mark

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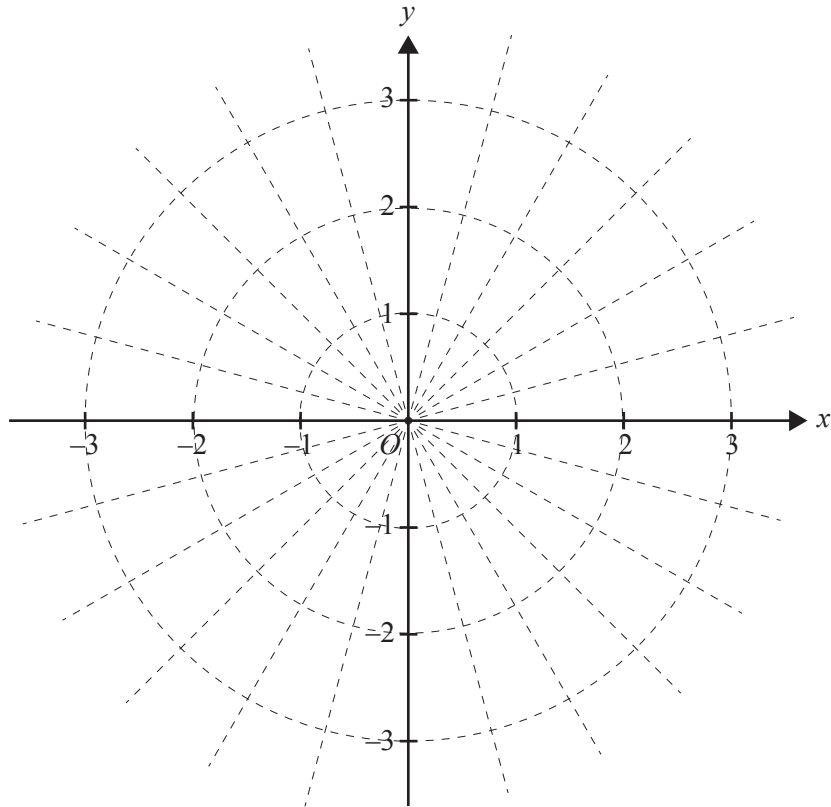
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**Question 2** (12 marks)

a. On the Argand diagram below, sketch  $\{z : z\bar{z} = 4, z \in C\}$  and sketch  $\{z : |z + \bar{z}| = |z - \bar{z}|, z \in C\}$ . 3 marks




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b. Find all elements of  $\{z : z\bar{z} = 4, z \in C\} \cap \{z : |z + \bar{z}| = |z - \bar{z}|, z \in C\}$ , expressing your answer(s) in the form  $a + ib$ .

3 marks

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- c. One of the roots of the equation  $z^4 + 16 = 0$  is  $z = \sqrt{2} + i\sqrt{2}$ .

Write down the other roots in cartesian form.

Plot and label all of these roots on the Argand diagram provided in **part a**.

2 marks

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- d. Express  $z^4 + 16$  as the product of four linear factors in terms of  $z$ .

1 mark

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- e. On the Argand diagram provided in **part a**., shade the region defined by

$$\{z : |z| \leq 2, z \in \mathbb{C}\} \cap \{z : \operatorname{Re}(z) \geq \sqrt{2}, z \in \mathbb{C}\}$$

1 mark

- f. Find the area of the shaded region in **part e**.

2 marks

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**Question 3** (11 marks)

The number of mobile phones,  $N$ , owned in a certain community after  $t$  years, may be modelled by  $\log_e(N) = 6 - 3e^{-0.4t}$ ,  $t \geq 0$ .

- a. Verify by substitution that  $\log_e(N) = 6 - 3e^{-0.4t}$  satisfies the differential equation

$$\frac{1}{N} \frac{dN}{dt} + 0.4 \log_e(N) - 2.4 = 0.$$

2 marks

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- b. Find the initial number of phones owned in the community.

Give your answer correct to the nearest integer.

1 mark

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- c. Using this mathematical model, find the limiting number of mobile phones that would eventually be owned in the community.

Give your answer correct to the nearest integer.

2 marks

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The differential equation in **part a.** can also be written in the form  $\frac{dN}{dt} = 0.4N(6 - \log_e(N))$ .

**d. i.** Find  $\frac{d^2N}{dt^2}$  in terms of  $N$  and  $\log_e(N)$ .

2 marks

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**ii.** The graph of  $N$  as a function of  $t$  has a point of inflection.

Find the values of the coordinates of this point.

Give the value of  $t$  correct to one decimal place and the value of  $N$  correct to the nearest integer.

2 marks

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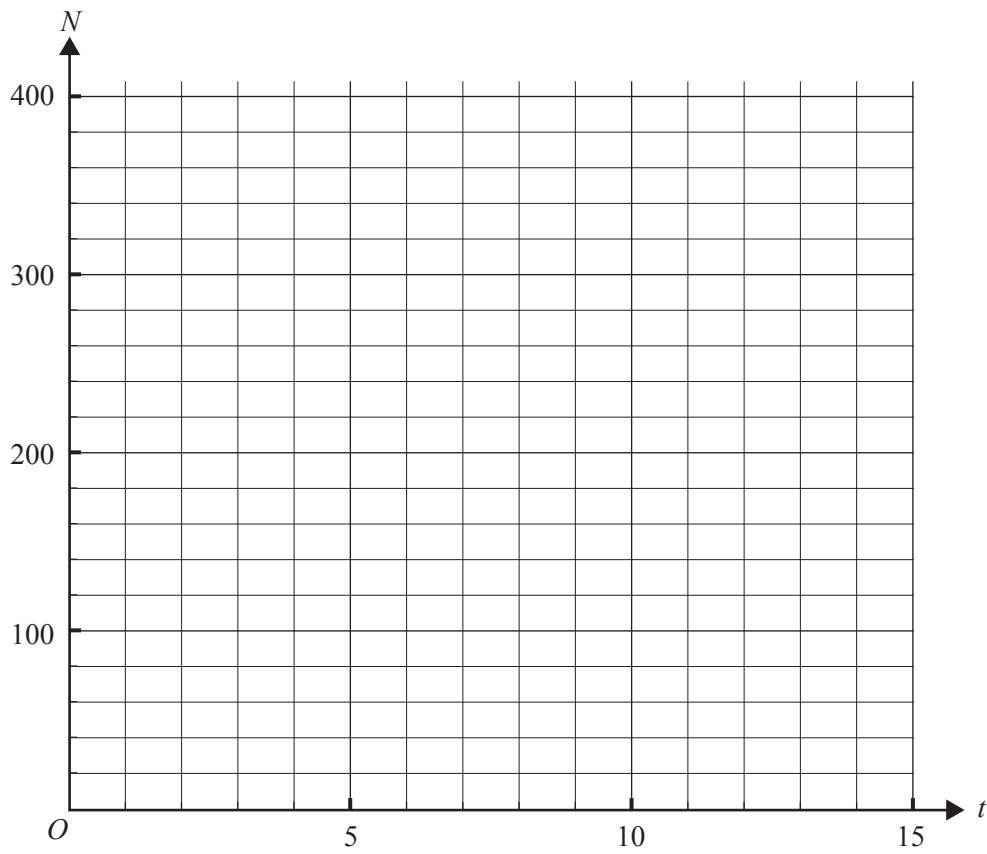
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e. Sketch the graph of  $N$  as a function of  $t$  on the axes below for  $0 \leq t \leq 15$ .

2 marks



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**Question 4** (12 marks)

Let  $\underline{a} = -\frac{7\sqrt{3}}{3}\underline{i} + \underline{j} - 2\underline{k}$  and  $\underline{b} = \underline{i} + \sqrt{3}\underline{j} + 2\sqrt{3}\underline{k}$ .

- a. Find a unit vector in the direction of  $\underline{b}$ . 1 mark

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- b. Resolve  $\underline{a}$  into two vector components, one that is parallel to  $\underline{b}$  and one that is perpendicular to  $\underline{b}$ . 3 marks

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- c. Find the value of  $m$  such that  $\underline{c} = m\underline{i} + \underline{j} - 2\underline{k}$  makes an angle of  $\frac{2\pi}{3}$  with  $\underline{b}$  and where  $\underline{c} \neq \underline{a}$ . 2 marks

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- d. Find the angle, in degrees, that  $\underline{c}$  makes with  $\underline{a}$ , correct to one decimal place. 2 marks

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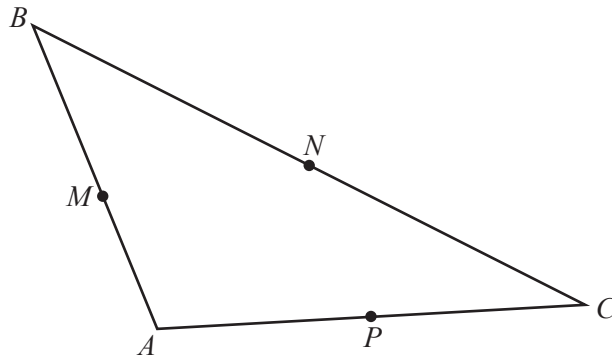
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For the triangle  $ABC$  shown below, the midpoints of the sides are the points  $M$ ,  $N$  and  $P$ .

Let  $\overrightarrow{AC} = \underline{u}$  and  $\overrightarrow{CB} = \underline{v}$ .



- e. i. Express  $\overrightarrow{AN}$  in terms of  $\underline{u}$  and  $\underline{v}$ . 1 mark

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- ii. Express  $\overrightarrow{CM}$  and  $\overrightarrow{BP}$  in terms of  $\underline{u}$  and  $\underline{v}$ . 2 marks

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- iii. Hence simplify the expression  $\overrightarrow{AN} + \overrightarrow{CM} + \overrightarrow{BP}$ . 1 mark

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**Question 5** (12 marks)

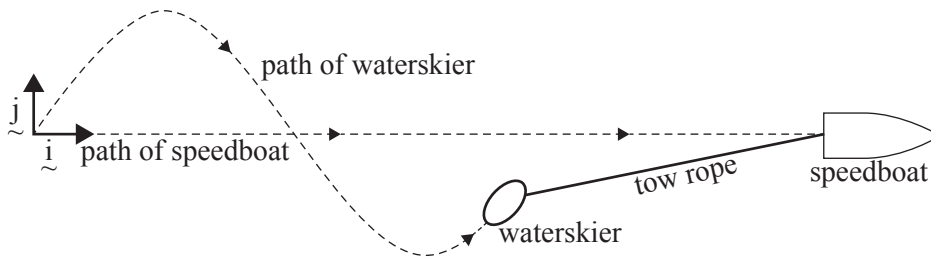
A waterskier moves from side to side behind a speedboat that is travelling in a straight line along a still lake.

The position vector of the waterskier relative to a fixed observation point on the shore of the lake is given by

$$\underline{r}(t) = 7.5t\underline{i} + \left(50 - 10\sin\left(\frac{\pi t}{6}\right)\right)\underline{j}$$

$t$  seconds after passing a marker buoy.

The components of the position vector are measured in metres, where  $\underline{i}$  is a unit vector in the direction of motion of the speedboat, and  $\underline{j}$  is a unit vector perpendicular to  $\underline{i}$  to the left of the direction of motion of the speedboat. The diagram below represents the view from above.



- a. Find  $\dot{\underline{r}}(t)$  and **hence** determine the minimum and maximum speeds of the waterskier.

Give your answers in metres per second, correct to one decimal place.

3 marks

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- b. Find the times when the acceleration of the waterskier is zero.

2 marks

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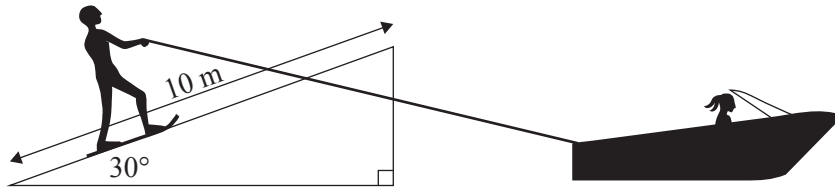


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Later, the waterskier performs a jump from the top of a 10 m long ramp, which is inclined at  $30^\circ$  to the horizontal. To do this, the speedboat travels beside the ramp and keeps the waterskier's speed at 15 m/s directly up the ramp, until the waterskier reaches the top. Then the speedboat slows, so that the tow rope slackens and remains slack after the waterskier leaves the top of the ramp. Assume negligible air resistance on the waterskier, who is then subject only to the force of gravity.



- c. Find the time it takes for the waterskier to hit the water after leaving the top of the ramp. Give your answer in seconds, correct to the nearest one hundredth of a second. 2 marks

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- d. After leaving the top of the ramp, how far in the horizontal direction does the waterskier travel before hitting the water? Give your answer in metres, correct to the nearest metre. 1 mark

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When attempting a second jump, the waterskier accidentally drops the tow rope when beginning to ascend the ramp and skids directly up the ramp with an initial velocity of 10 m/s.

- e. Find how far up the ramp the waterskier travels before coming to a stop, if the coefficient of friction between the skis and the ramp is  $\mu = \frac{1}{8\sqrt{3}}$ .

Give your answer in metres, correct to one decimal place.

3 marks

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- f. The waterskier comes to a stop on the ramp.

Find the minimum coefficient of friction that would be needed to prevent the waterskier from sliding back down the ramp.

1 mark

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# **SPECIALIST MATHEMATICS**

## **Written examinations 1 and 2**

### **FORMULA SHEET**

#### **Directions to students**

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

## Specialist Mathematics formulas

### Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc \sin A$
sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab \cos C$

### Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
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### Circular (trigonometric) functions

$$\cos^2(x) + \sin^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$$

$$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$$

$$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$

$$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$$

$$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	$R$
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

**Algebra (complex numbers)**

$$z = x + yi = r(\cos \theta + i \sin \theta) = r \operatorname{cis} \theta$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$

$$z^n = r^n \operatorname{cis}(n\theta) \quad (\text{de Moivre's theorem})$$

$$-\pi < \operatorname{Arg} z \leq \pi$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

**Calculus**

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e|x| + c$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule:

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Euler's method:

$$\text{If } \frac{dy}{dx} = f(x), x_0 = a \text{ and } y_0 = b, \text{ then } x_{n+1} = x_n + h \text{ and } y_{n+1} = y_n + hf(x_n)$$

acceleration:

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$\text{constant (uniform) acceleration: } v = u + at \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as \quad s = \frac{1}{2}(u+v)t$$

**TURN OVER**

## Vectors in two and three dimensions

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\vec{r}_1 \cdot \vec{r}_2 = r_1 r_2 \cos \theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\vec{r}} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k}$$

## Mechanics

momentum:

$$\vec{p} = m\vec{v}$$

equation of motion:

$$\vec{R} = m\vec{a}$$

friction:

$$F \leq \mu N$$