

**insight™**  
**Year 12 Trial Exam Paper**

**2014**

**SPECIALIST MATHEMATICS**

**Written examination 1**

**STUDENT NAME:**

**Reading time: 15 minutes**

**Writing time: 1 hour**

**QUESTION AND ANSWER BOOK**

**Structure of book**

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring sheets of paper, notes of any kind or white out liquid/tape into the examination.
- Calculators are not permitted in this examination.

**Materials provided**

- The question and answer book of 15 pages with a separate sheet of miscellaneous formulas.
- Working space is provided throughout this book.

**Instructions**

- Write your **name** in the box provided.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

**Students are NOT permitted to bring mobile phones and/or any other electronic devices into the examination.**

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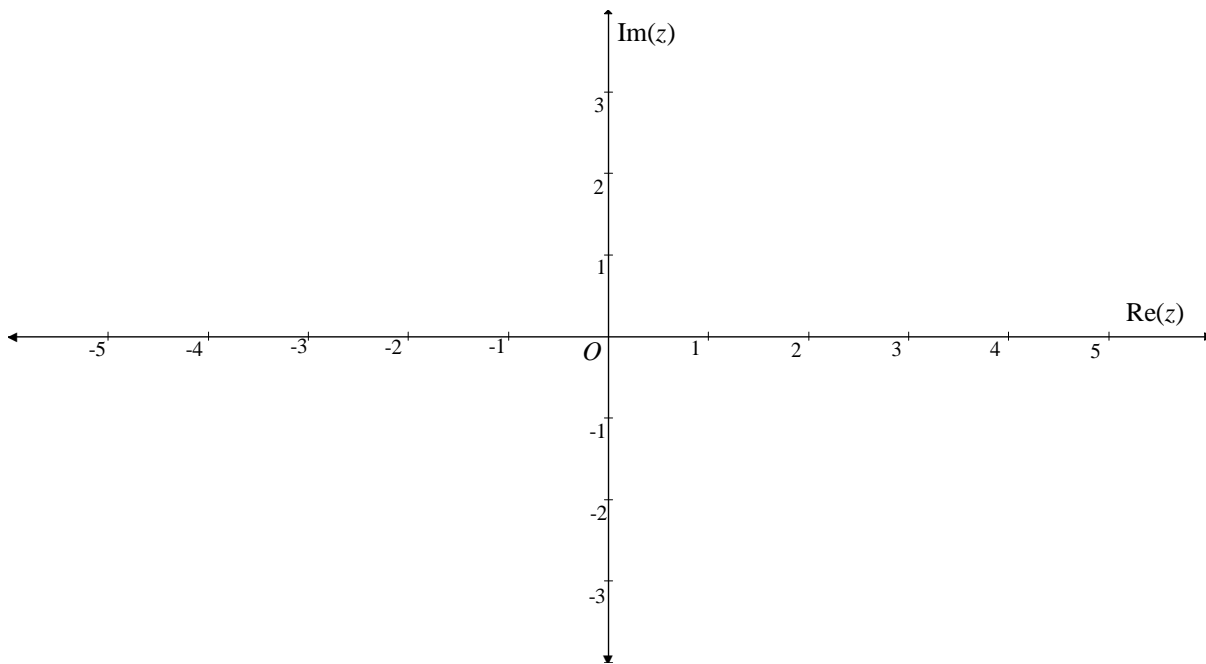
**Instructions**

Answer **all** questions in the spaces provided.  
Unless otherwise specified, an **exact** answer is required to a question.  
In questions where more than one mark is available, appropriate working **must** be shown.  
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.  
Take the **acceleration due to gravity** to have magnitude  $g \text{ m/s}^2$ , where  $g = 9.8$ .

**Question 1** (5 marks)

- a. Show the region represented by  $\{z : \text{Arg}(z - 2) = \text{Arg}(z + 1)\}$  on the Argand diagram below.

3 marks



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**QUESTION 1** – continued

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- b.**  $P(z) = z^3 - (4+i)z^2 + (5+4i)z - 5i$ , where  $z \in \mathbb{C}$ . Given that  $(z-i)$  is a factor of  $P(z)$ , find the other factors.

2 marks

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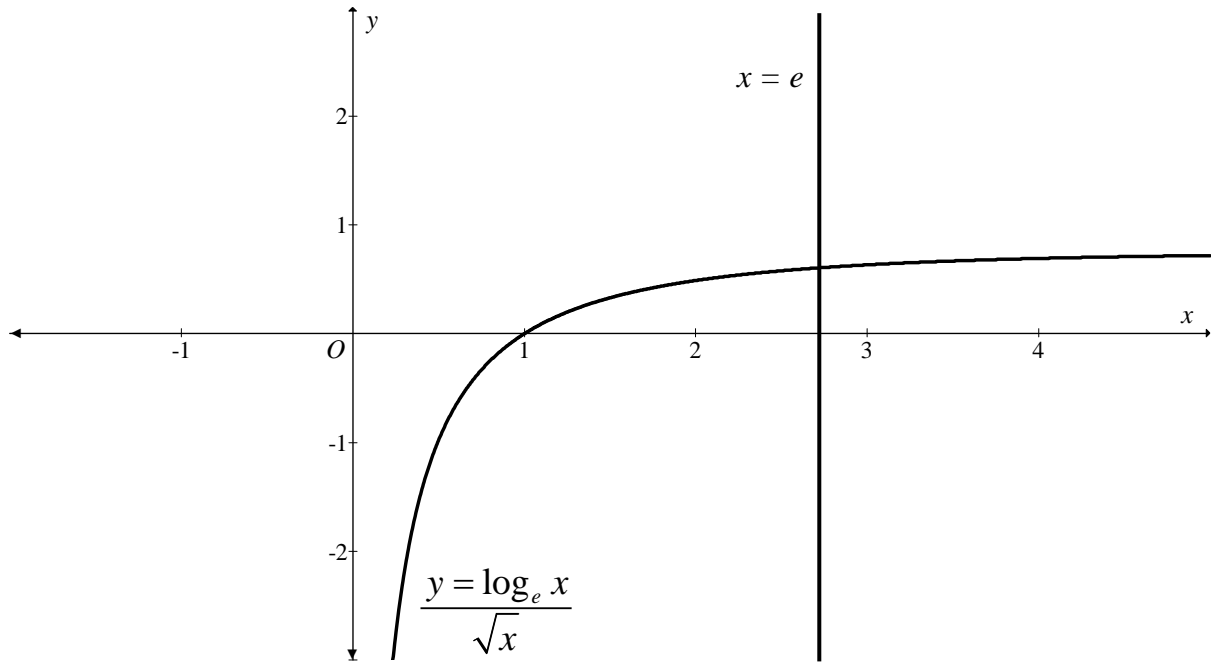
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**Question 2** (4 marks)

The area bounded by the curve  $y = \frac{\log_e x}{\sqrt{x}}$ , the  $x$ -axis and the straight line  $x = e$  is rotated about the  $x$ -axis. Calculate the volume.




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**Question 4** (5 marks)

The position vector of a particle is given by  $r = \tan(t)i + tj$ , where  $t \in \left[0, \frac{\pi}{2}\right)$ .

- a. Find the Cartesian equation of the path of this particle in the form  $y = f(x)$ . State the domain.

2 marks

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- b. Find the rule for the gradient function  $y = f'(x)$ .

1 mark

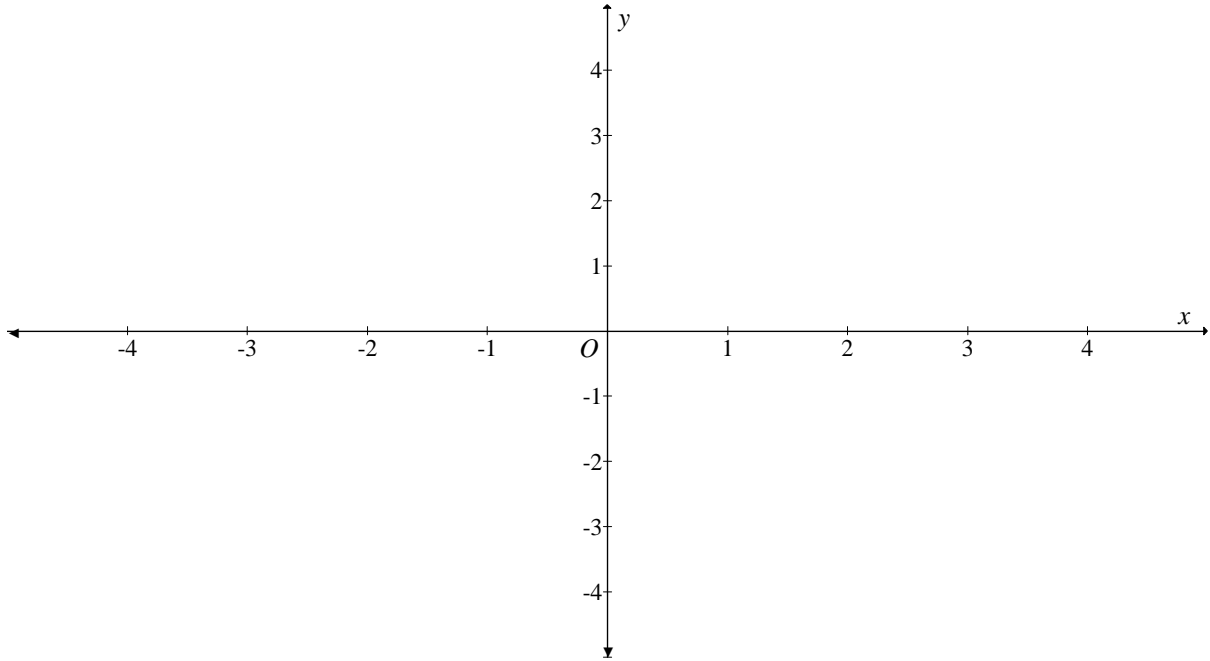
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- c. Sketch the graph of  $y = f'(x)$  on the set of axes below. Clearly indicate any axes intercepts, stationary points or asymptotes.

2 marks



**Question 5** (5 marks)

A person of mass 80 kg stands on a set of scales in a lift, which is moving up.

- a. i.** What is the reading on the scales, in kg, when it is accelerating at  $0.2 \text{ m/s}^2$ ?

Express the answer in the form  $\frac{a}{g}$ , where  $a \in \mathbb{Z}^+$  and  $g$  is the acceleration due to gravity.

1 mark

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- ii.** What is the acceleration when the reading on the scales is 60 kg?

1 mark

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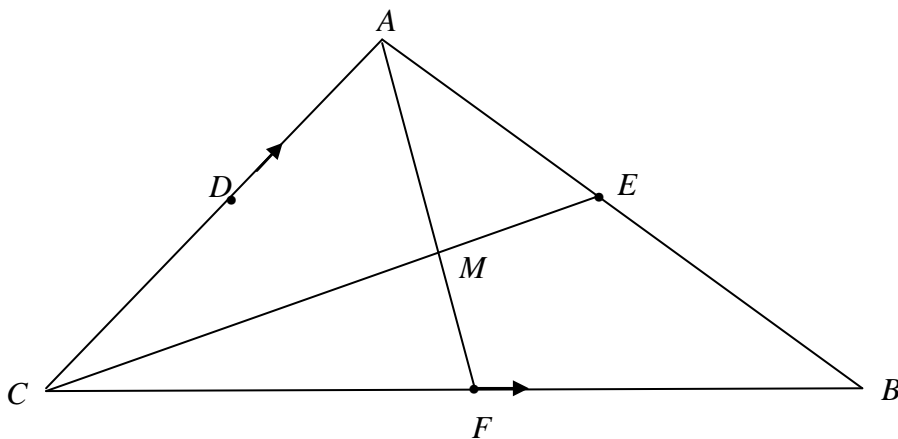
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**Question 6** (4 marks)

The points  $D$ ,  $E$  and  $F$  are the midpoints of the lines  $CA$ ,  $AB$  and  $CB$ , respectively, in the triangle  $ABC$  shown below. The medians  $AF$  and  $CE$  intersect at point  $M$ .



Let  $\overrightarrow{CA} = 2\mathbf{a}$  and  $\overrightarrow{CB} = 2\mathbf{b}$ .

Use vectors to show that  $M$  divides the median line  $\overrightarrow{CE}$  in the ratio  $2 : 1$ .

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**Question 8** (3 marks)

Given that  $\sec(4\theta) = 8$ ,  $0 \leq \theta \leq \frac{\pi}{8}$ , find the exact value of  $\tan(\theta)$ .

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**Question 9** (4 marks)

Given that  $f(x) = \frac{x}{1+x^2}$  :

a. Show that  $f'(x) = \frac{(1-x^2)}{(1+x^2)^2}$ .

1 mark

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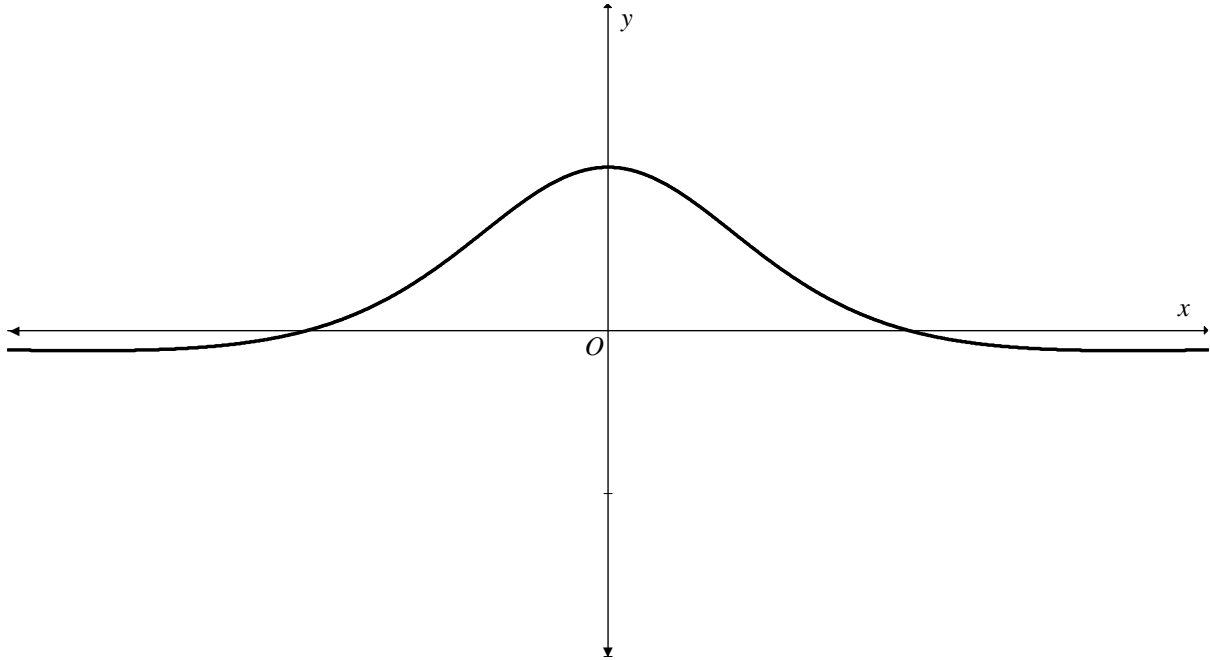
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**QUESTION 9** – continued  
**TURN OVER**

- b.** Hence, calculate the area bounded by the curve with equation  $y = \frac{a(1-x^2)}{(1+x^2)^2}$  (shown below) and the  $x$ -axis, where  $a \in \mathbb{R}^+$ .



3 marks

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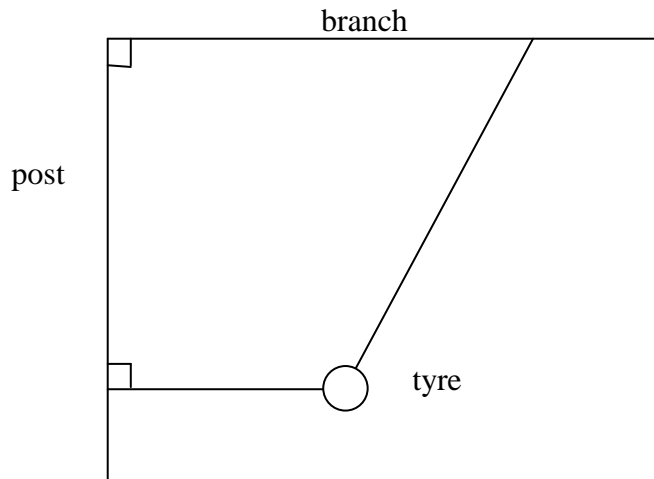
**Question 10** (3 marks)

A tyre swing of mass 8 kg is suspended from a tree branch by an inextensible rope. When it is not in use, the tyre is supported by a second, horizontal, inextensible rope that is attached to a post. Assume the masses of the ropes are negligible.

The tension in the rope attached to the branch is  $\sqrt{3}$  times the tension in the other rope.

- a. Label all the forces, in newtons, acting on the tyre in the diagram.

1 mark



- b. Calculate the tension in the rope attached to the post.  
Express the tension in the form  $a\sqrt{b}g$ , where  $a$  and  $b$  are positive integers.

2 marks

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**END OF QUESTION AND ANSWER BOOK**