Specialist Mathematics

2014

Trial Examination 1

Instructions

Answer **all** questions. Do **not** use calculators.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working or explanation **must** be shown.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

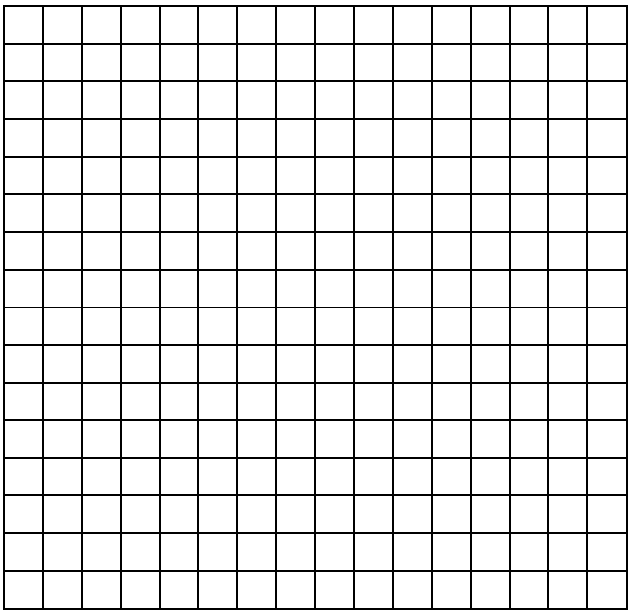
Question 1

Consider function f with the rule $f(x)=\left(\frac{1}{\sqrt{x}}-\sqrt{x}\right)^2+2$.

a. Simplify the rule of f. 1 mark

b. Find the range of f. 1 mark

c. Sketch the graph of f. Show and label the turning point(s) and the asymptote(s) of f. 3 marks



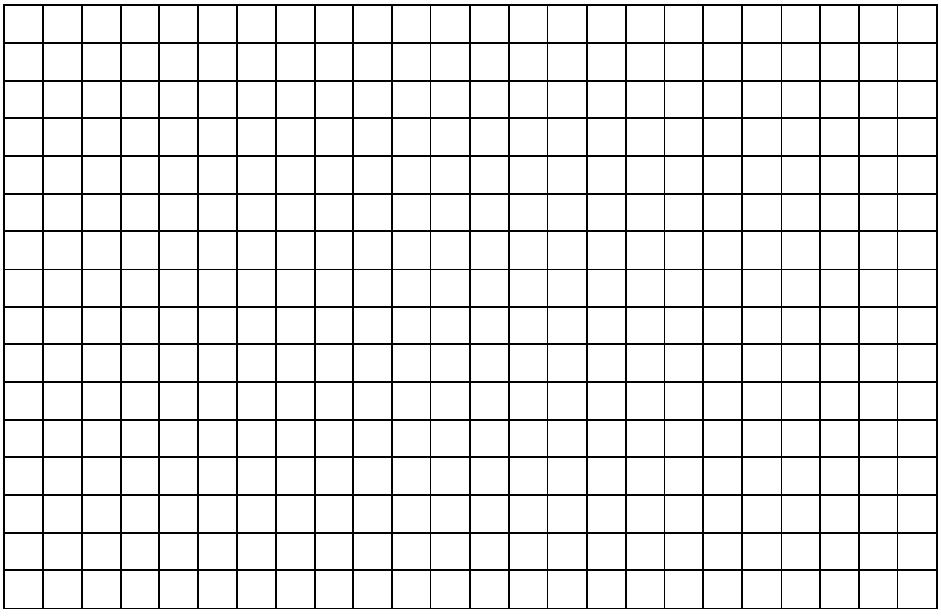
Question 2

Consider $g : R \to R$, $g(x) = \tan^{-1}(3x) + \tan^{-1}(2x) - \frac{\pi}{4}$.

a. Find the exact value(s) of x where $g(x) = 0$. 2 marks

b. Use the method of addition of ordinates to sketch the graph of g. Show and label the axis-intercept(s) and the asymptote(s) of g.

3 marks



Question 3

a. Solve $\sqrt{3}\,z - \sqrt{2}\,i = \sqrt{2}\,i\,z + \sqrt{3}$ for z. Express your answer in $x + y\,i$ form. 2 marks

b. $P(z)$ is a cubic polynomial in z with real coefficients.
Given $P(z) = (z - i)\,Q(z) + 1$ and $P(z) = (2z - 1)\,T(z) + 1$ where $Q(z)$ and $T(z)$ are polynomials in z, solve $P(z) = 0$ for z. 3 marks

Question 4

Consider $|z| - |3 - z| = 1$ where $z = x + y\,i$ and $x, y \in R$.

a. Express $|z| - |3 - z| = 1$ in the form $\dfrac{(x-h)^2}{a} - \dfrac{(y-k)^2}{b} = 1$. 2 marks

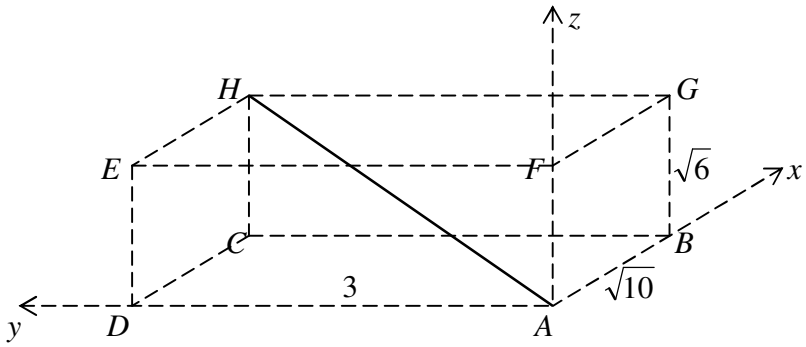
b. Hence or otherwise sketch the graph of $|z| - |3 - z| \leq 1$. 2 marks

Question 5

Show that $\tilde{p} = \tilde{i} - \tilde{j}$, $\tilde{q} = 2\tilde{i} + \tilde{j}$, $\tilde{r} = \tilde{i} + 2\tilde{j}$ and $\tilde{s} = 3\tilde{i} - 2\tilde{j}$ are linearly ***dependent***. 2 marks

Question 6

ABCDEFGH is a cuboid. Use vector method to find the shortest distance in surd form from vertex *G* to line *AH*. 3 marks



Question 7

The solution curve to the differential equation $\frac{dy}{dx}+\frac{y}{x}=0$ passes through $(1, 2)$.

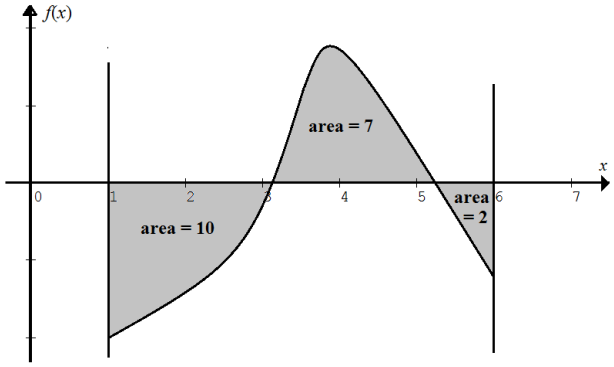
a. Use Euler's method (first order approximation) to estimate the value of y at $x=2.5$.
Choose 0.5 as the step size. 2 marks

b. Show that $xy=2$ is the equation of the solution curve. 1 mark

c. If both x and y are functions of λ and $\frac{dy}{d\lambda}=-1$, find $\frac{dx}{d\lambda}$ at $x=1$. 1 mark

Question 8

Consider $\frac{dy}{dx}=f(x)$ with $y=5$ when $x=1$. The graph of $\frac{dy}{dx}=f(x)$ for $x\in[1, 6]$ is shown below.
The areas of the regions (shaded) bounded by the curve, the x-axis, $x=1$ and $x=6$ are indicated in the graph.

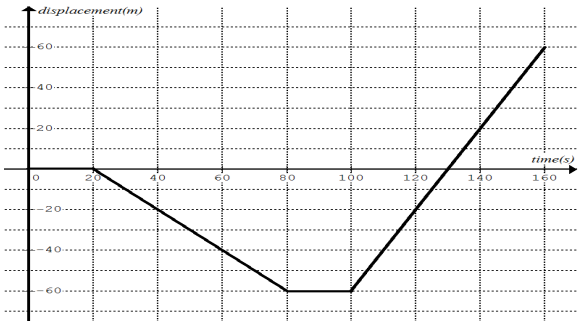


a. Evaluate $\int_1^6 f(x)\,dx$. 1 mark

b. Find y when $x=6$. 1 mark

Question 9

A sign is erected in the street which runs in the north-south direction. The displacement (m) of a cyclist from the sign as a function of time (s) is shown in the following graph. A positive displacement indicates a displacement to the north.



a. State the direction of motion of the cyclist at time $t = 120\,\text{s}$. 1 mark

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b. Calculate the average speed (in m s^{-1}) of the pedestrian between $t = 0$ and $t = 160\,\text{s}$. 1 mark

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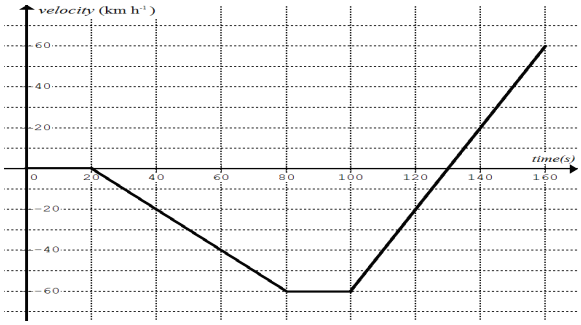
The velocity-time graph of a car is shown below. The car starts at 500 metres north of the street sign.



c. State the direction of motion of the car at $t = 120\,\text{s}$. 1 mark

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d. Calculate the average speed (in km h^{-1}) of the car between $t = 0$ and $t = 160\,\text{s}$.

2 marks

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e. How many times does the car pass the cyclist between $t = 0$ and $t = 160\,\text{s}$? 1 mark

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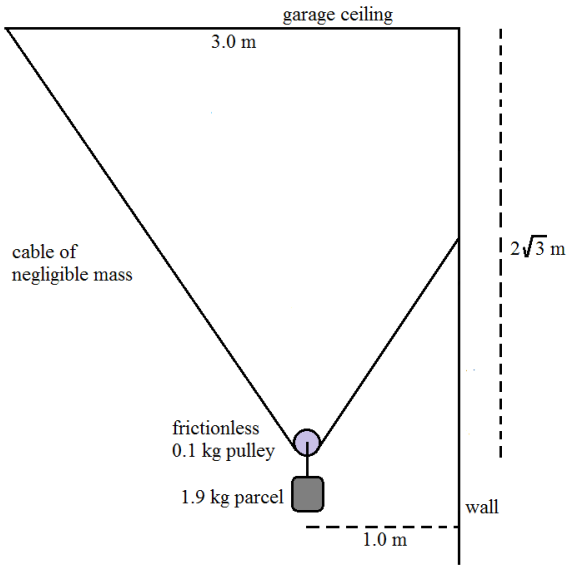
Question 10

A 1.9 kg parcel is attached to a frictionless 0.1 kg pulley. A cable of negligible mass is fastened to a garage ceiling and wall. The pulley is allowed to run along the cable until it comes to a stop as shown in the drawing below. *Take* $g = 10$ N kg^{-1}. *Assume the pulley is a point mass.*



a. Calculate the exact value in newtons of the tension in the cable. 2 marks

Now a horizontal force of F newtons is used to pull the pulley to the left until the section of the cable on the right is horizontal.

b. Calculate the exact value of F. 2 marks

End of Exam 1