Year 2014

VCE

Specialist Mathematics

Trial Examination 2

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Victorian Certificate of Education 2014

STUDENT NUMBER

									Letter
Figures									
Words									

SPECIALIST MATHEMATICS
Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	4	4	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer booklet of 36 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Answer sheet for multiple choice questions.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Write your **name** and **student number** on your answer sheet for multiple-choice questions and sign your name in the space provided.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section I

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1 mark, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

If $a \in R^+$ and $f(x) = a^2 - x^2$ then the graph of $y = \dfrac{a}{f(x+a)}$, has

A. only the x and y axes as asymptotes.

B. the x and y axes and $x = 2a$ as asymptotes and a minimum turning point at $\left(a, \dfrac{1}{a}\right)$.

C. the x and y axes and $x = 2a$ as asymptotes and a maximum turning point at $\left(a, \dfrac{1}{a}\right)$.

D. the x and y axes and $x = -2a$ as asymptotes and a minimum turning point at $\left(-a, \dfrac{1}{a}\right)$.

E. the x and y axes and $x = -2a$ as asymptotes and a maximum turning point at $\left(-a, \dfrac{1}{a}\right)$.

Question 2

The graph of $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$ touches the y-axis at $y = -1$ and one of the asymptotes has the equation $y = -\dfrac{x}{2}$. It follows that

A. $h = 2$, $k = -1$, $a = 2$, $b = 1$

B. $h = 2$, $k = -1$, $a = 1$, $b = 2$

C. $h = 2$, $k = 1$, $a = 1$, $b = -1$

D. $h = -2$, $k = -1$, $a = 2$, $b = 1$

E. $h = -2$, $k = -1$, $a = 1$, $b = 2$

Question 3

The graph of $y = a\tan^{-1}(bx) + c$ has a range equal to $(-3,3)$. Then

A. $b = \dfrac{\pi}{6}$

B. $b = \dfrac{6}{\pi}$

C. $a = \dfrac{\pi}{6}$, $c = 0$

D. $a = \dfrac{6}{\pi}$, $c = 0$

E. $a = 3$, $c = \dfrac{\pi}{2}$

Question 4

The quadratic $z^2 + bz + c = 0$ has a root $\alpha + i$ where *b*, *c* and α are all non-zero real constants. The quadratic with real coefficients with one of its roots $\alpha + 1 - i$ is

A. $z^2 + (b-2)z + c + b + 1 = 0$

B. $z^2 + (b+2)z + c + b + 2 = 0$

C. $z^2 + (b-2)z + c - b + 1 = 0$

D. $z^2 - (b+2)z + c + b + 1 = 0$

E. $z^2 + (b+1)z + c + 1 = 0$

Question 5

A cubic equation $P(z)$ with real coefficients has $z = a$ and $z = -2ai$ amongst its roots, where *a* is a non-zero real number. Then $P(z)$ is equal to

A. $z^3 - az^2 + 2a^2z - 2a^3$

B. $z^3 - az^2 - 2a^2z + 2a^3$

C. $z^3 + az^2 + 4a^2z + 4a^3$

D. $z^3 - az^2 - 4a^2z + 4a^3$

E. $z^3 - az^2 + 4a^2z - 4a^3$

Question 6

If $a+bi=\sqrt{a^2+b^2}\,\text{cis}(\theta)$ where a and b are positive real constants, then $\text{Arg}\left(2ab+\left(a^2-b^2\right)i\right)$ is equal to

A. $\pi-2\theta$

B. $\frac{\pi}{2}-2\theta$

C. $2\theta-\pi$

D. $2\theta-\frac{\pi}{2}$

E. $2\theta-\frac{3\pi}{2}$

Question 7

Consider the definite integral $\int_0^{\frac{\pi}{4}} \sin^3(2x)\cos^3(2x)\,dx$

Three students stated some ideas about how to evaluate this definite integral.

Alex used the substitution $u=\sin(2x)$ and reduced the definite integral to $\frac{1}{2}\int_0^1 u^3\left(1-u^2\right)du$. Brenda used the substitution $u=\cos(2x)$ and reduced the definite integral to $\frac{1}{2}\int_0^1 u^3\left(1-u^2\right)du$. Claire made some other substitution and reduced the definite integral $\frac{1}{32}\int_{-1}^1 \left(1-u^2\right)du$.

Then

A. Only Alex and Claire are correct.

B. Only Brenda and Claire are correct.

C. Only Claire is correct.

D. Only Alex and Brenda are correct.

E. Alex, Brenda and Claire are all correct.

Question 8

If a is a positive real number, then the graphs of $\frac{2a}{\pi}\sin^{-1}\left(\frac{x}{a}\right)$ and $\frac{x^2}{4a^2}+\frac{y^2}{b^2}=1$ intersect exactly twice if

A. $b > \sqrt{2}\,a$

B. $b = 2a$

C. $b > a$

D. $a > \frac{\sqrt{3}\,b}{2}$

E. $\frac{\sqrt{3}\,b}{2} > a$

Question 9

The set of points in the Argand plane defined by $\{z: |z+2a| = 2|z-ai|\}$ where $a \in R^+$ represents

A. a straight line.

B. a circle.

C. a parabola.

D. an ellipse.

E. a hyperbola.

Question 10

The area between the graphs of $y=\sqrt{x}$, $y=1$ and $x=4$ is rotated about the *x*-axis to form a solid of revolution. This volume is equal to

A. 4π

B. $\frac{9\pi}{2}$

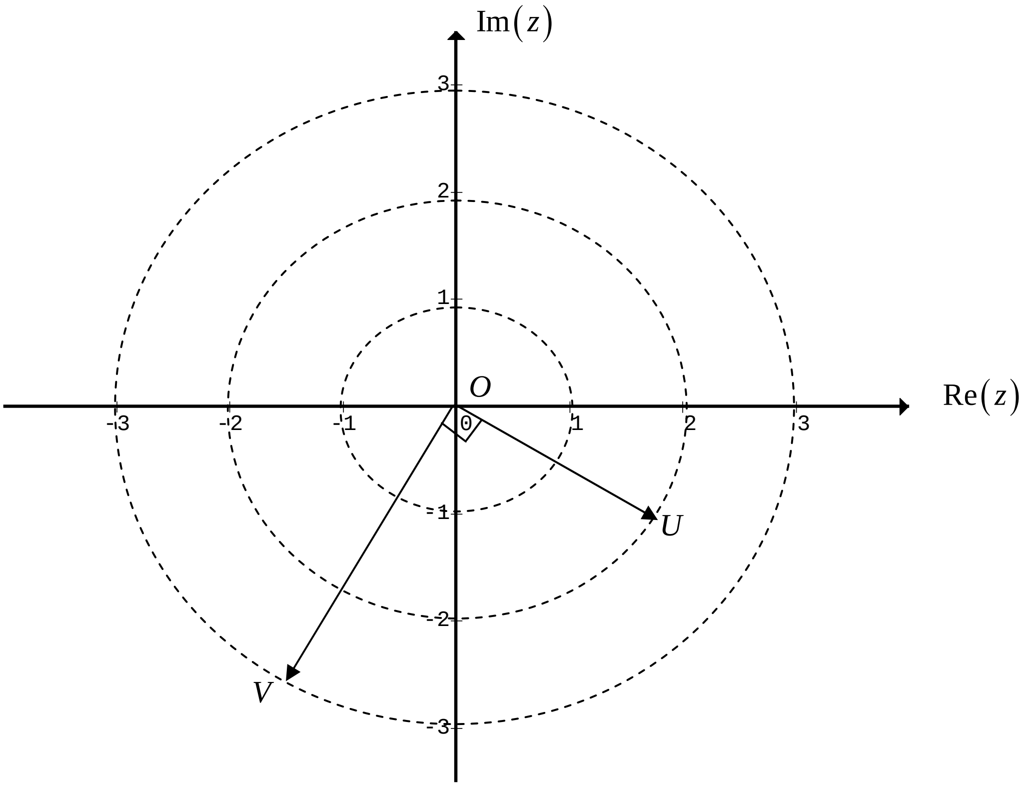
C. 7π

D. $\frac{28\pi}{3}$

E. 9π

Question 11

In the diagram below, the points U and V represent the complex numbers u and v respectively. The angle UOV is a right angle, where O is the origin.



Which of the following is the correct relationship between u and v?

A. $3v + 2\overline{u} = 0$

B. $3vi - 2\overline{u} = 0$

C. $2v + 3ui = 0$

D. $2vi + 3u = 0$

E. $2vi - 3u = 0$

Question 12

Two vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are such that $|\underset{\sim}{u}| = 3$ and $|\underset{\sim}{v}| = 4$. The angle between the vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$ is 120^0. Then which of the following is **not** true?

A. $\underset{\sim}{u}.\underset{\sim}{v} = -6$

B. The scalar resolute of $\underset{\sim}{u}$ in the direction of $\underset{\sim}{v}$ is equal to $-\frac{3}{2}$.

C. The scalar resolute of $\underset{\sim}{v}$ in the direction of $\underset{\sim}{u}$ is equal to -2.

D. $|\underset{\sim}{u} + \underset{\sim}{v}| = \sqrt{13}$

E. $|\underset{\sim}{v} - \underset{\sim}{u}| = 1$

Question 13

The equation of the normal to the curve defined by $x^2 + 2\tan^{-1}\left(\frac{y}{2}\right) + y^2 = 5 + \frac{\pi}{2}$ at the point $(-1, 2)$ is given by

A. $y = \frac{4x}{9} + \frac{22}{9}$

B. $y = -\frac{4x}{9} - \frac{14}{9}$

C. $y = \frac{9x}{4} + \frac{17}{4}$

D. $y = -\frac{9x}{4} - \frac{1}{4}$

E. $y = -\frac{9x}{4} - \frac{7}{4}$

Question 14

Given the vectors $\underset{\sim}{a} = x\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{b} = \underset{\sim}{i} - 2\underset{\sim}{j} + 2\underset{\sim}{k}$.

Then which of the following is **not** true?

A. If $x = 4$ then the vector $\underset{\sim}{a}$ is perpendicular to the vector $\underset{\sim}{b}$

B. If $x = -\frac{1}{2}$ then the vector $\underset{\sim}{a}$ is parallel to the vector $\underset{\sim}{b}$.

C. If $x = \pm\sqrt{7}$ then the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are equal in length.

D. If $x = 0$ then $\left|\underset{\sim}{a} + \underset{\sim}{b}\right| = 2\sqrt{3}$.

E. If $x = \sqrt{2}$ then $\left|\underset{\sim}{a}\right| + \left|\underset{\sim}{b}\right| = 5$.

Question 15

If $\frac{d^2y}{dx^2} = 5x^4 - 5$ and $\frac{dy}{dx} = 0$ at $x = 1$. Then the graph of y has a

A. stationary point of inflexion at $x = 1$ and no other stationary points.

B. stationary point of inflexion at $x = 1$ and a local minimum at $x = -1$.

C. stationary point of inflexion at $x = 1$, a point of inflexion at $x = -1$ and another minimum turning point.

D. stationary points at $x = \pm 1$ only.

E. maximum turning point at $x = 1$ and a minimum turning point at $x = -1$.

Question 16

When Euler's method, with a step size of $\frac{\pi}{8}$, is used to solve the differential equation $\frac{dy}{dx} = \cos^2(2x)$ with $x_0 = 0$ and $y_0 = 2$, the value of y_3 is equal to

A. $\frac{\pi}{8}$

B. $2 + \frac{\pi}{8}$

C. $2 + \frac{3\pi}{16}$

D. $\frac{3\pi}{16} - \frac{1}{8}$

E. $\frac{\pi}{8} + \frac{7}{4}$

Question 17

The velocity v ms^{-1} of a particle is given by $e^{\sqrt{t}}$ at a time t seconds, where $t \geq 0$.

If $x = 3$ when $t = 1$, then the value of x when $t = 2$ can be found by evaluating

A. $\int_1^2 e^{\sqrt{u}}\,du$

B. $\int_1^2 e^{\sqrt{u}}\,du + 3$

C. $\int_1^2 \left(e^{\sqrt{u}} + 3\right)du$

D. $\int_1^2 e^{\sqrt{u}}\,du - 3$

E. $\int_1^2 \left(e^{\sqrt{u}} - 3\right)du$

Question 18

A car is moving with constant acceleration has its speed reduced from $3V\ \text{ms}^{-1}$ to $V\ \text{ms}^{-1}$, over a distance of D m when the driver applies the brakes. The car travels a further distance of S m until it comes to rest. The time T seconds represents the time when the driver applies the brakes until the car comes to rest. Then

A. $D = 8S$ and $T = \dfrac{2(D+S)}{3V}$

B. $D = 4S$ and $T = \dfrac{2(D+S)}{3V}$

C. $D = 8S$ and $T = \dfrac{S}{V}$

D. $D = 4S$ and $T = \dfrac{S}{V}$

E. $D = 2S$ and $T = \dfrac{D}{2V}$

Question 19

A particle of mass M kg is on a horizontal table and is connected by a light string to a particle of mass 2 kg hanging vertically at the edge of the table. The coefficient of friction between the table and the mass M is equal to $\frac{1}{3}$. Then if

A. $M > 6$ both masses move with constant acceleration.

B. $0 < M < 6$ both masses move with constant acceleration.

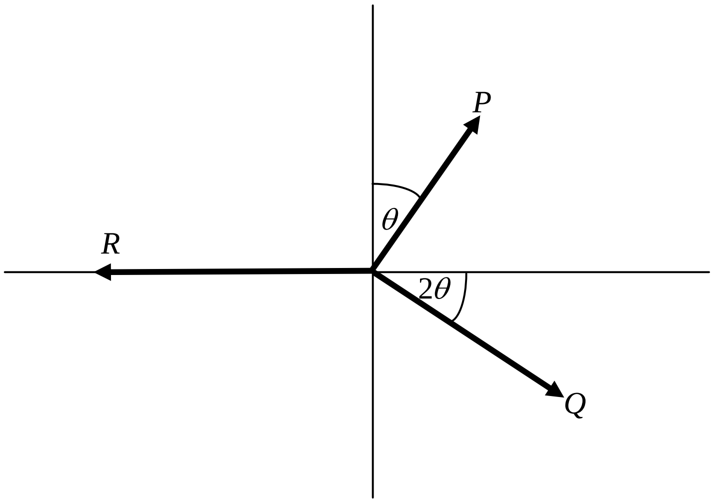
C. $0 < M \leq 6$ the system is in limiting equilibrium.

D. $M > 6$ both masses move with constant velocity.

E. $0 < M < 6$ both masses move with constant velocity.

Question 20

Three coplanar forces of magnitudes P, Q and R newtons act on a particle that is in equilibrium as shown in the diagram below.



Then

A. $Q = R$ and $P = 2R\sin(\theta)$

B. $Q = R$ and $P = 2R\cos(\theta)$

C. $Q = 2P$ and $R = P\sin(\theta)$

D. $P = Q$ and $R = 2P\sin(\theta)$

E. $P + Q + R = 0$

Question 21

A particle of mass 10 kg travels in a straight line with velocity v ms^{-1} when its displacement is x metres, where $v = 2\log_e\left(\sqrt{x^2+1}+x\right)$ for $x \geq 0$. The maximum force in newtons acting on the particle is closest to

A. 1.5

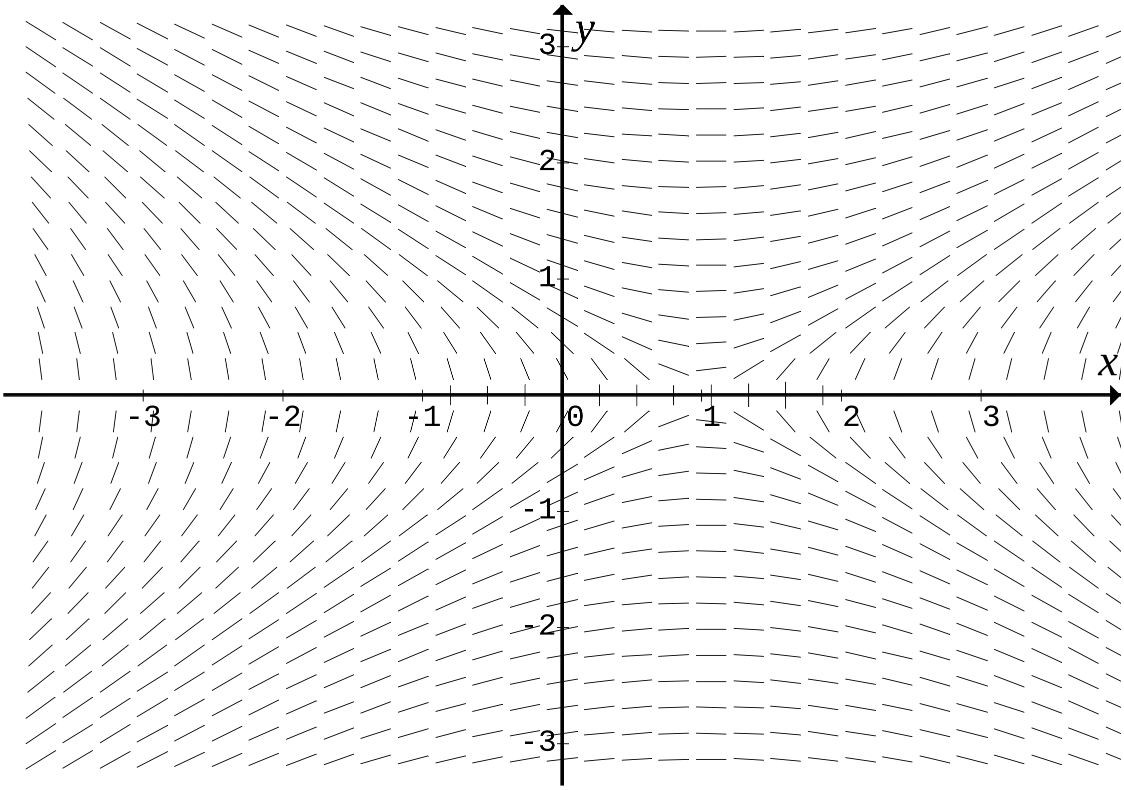
B. 2.7

C. 20

D. 24

E. 26.5

Question 22



The differential equation which best represents the above direction field is

A. $\frac{dy}{dx} = \frac{x-1}{y}$

B. $\frac{dy}{dx} = \frac{1-x}{y}$

C. $\frac{dy}{dx} = \frac{y}{x-1}$

D. $\frac{dy}{dx} = \frac{y}{1-x}$

E. $\frac{dy}{dx} = y(1-x)$

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
Unless otherwise specified an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1 (17 marks)

a. Show that $\cos\left(\frac{5\pi}{12}\right) = \frac{1}{4}\left(\sqrt{6} - \sqrt{2}\right)$ 2 marks

b. A and B are two points in the plane, with coordinates $(1,1)$ and $\left(-1, \sqrt{3}\right)$ respectively. O is the origin and $\underset{\sim}{i}$ and $\underset{\sim}{j}$ are two unit vectors in the x and y directions respectively.

i. Find the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

1 mark

ii. Using vectors find the angle in degrees between $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

2 marks

iii. Find the area of the triangle *OAB*.

2 marks

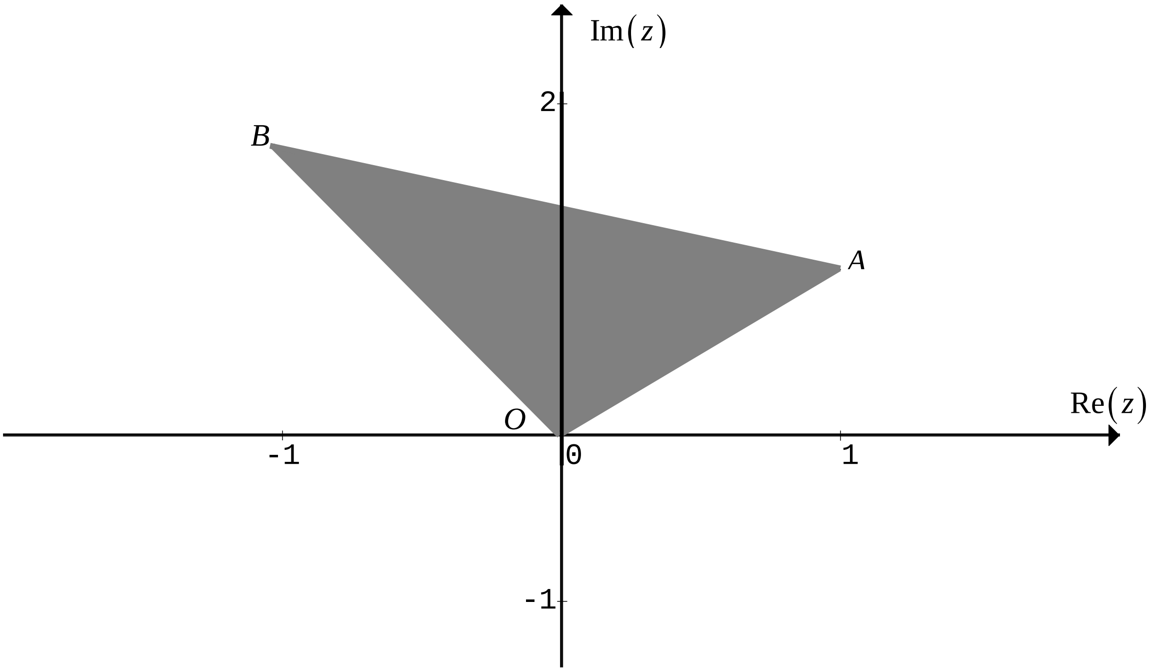
iv. Let *C* be the mid-point of *AB*, find the vector $\overrightarrow{OC}$.

2 marks

c. Let $a = 1 + i$ and $b = -1 + i\sqrt{3}$ be two complex numbers.

i. The complex numbers *a* and *b* can be expressed in polar form as $a = |a|\text{cis}(\alpha)$ and $b = |b|\text{cis}(\beta)$, state the values of $|a|, |b|, \alpha$ and β.

2 marks



ii. The shaded triangular region R, in the Argand diagram above, shows the complex numbers a and b represented by the points A and B respectively and the origin O. The shaded region can be represented as a subset of the complex plane by $\{ z : \alpha \le \text{Arg}(z) \le \beta \} \cap \{ z : \text{Im}(z) \le m\,\text{Re}(z) + k \}$.

Determine the values of m and k.

2 marks

iii. Show that the area of the shaded region *R* is given by $\frac{1}{4}\left(a\bar{b}-\bar{a}b\right)i$

2 marks

iv. A circle is drawn to pass through the points *A* and *B*, with its centre at a point *C*, which is the midpoint of *AB*. This circle is represented in the Argand diagram as $\{z:|z-c|=r\}$, state the value of the real number *r* and the complex number *c*.

2 marks

Question 2 (12 marks)

A biologist is studying the growth of two different strands of bacteria. He has two different experimental models and both start at exactly the same time.

Model 1

The number of type 1, bacteria cells, N after a time t hours, is modelled by

$N = N(t) = \dfrac{1000}{1+9e^{-kt}}$ for $t \geq 0$, where k is a positive real constant.

a. Verify by substitution that $\dfrac{dN}{dt} = kN\left(1 - \dfrac{N}{1000}\right)$ 2 marks

b. Find the initial number and the limiting number of type 1 bacteria cells, that could eventually be present.

1 mark

c. It is observed that 900 bacteria are present after 20 hours, determine the value of k.

1 mark

d. Express $\frac{d^2N}{dt^2}$ in terms of N and hence find the coordinates of the point of inflexion.

3 marks

Model 2

The number of type 2, bacteria cells, N after a time t hours, is modelled by the differential equation $\frac{dN}{dt} = rN$ for $t \geq 0$, where r is a positive real constant. Initially there are 50 of these type 2 cells present and after 20 hours, the number grows to 450.

e. Express the number N of type 2 bacteria cells present in terms of t.

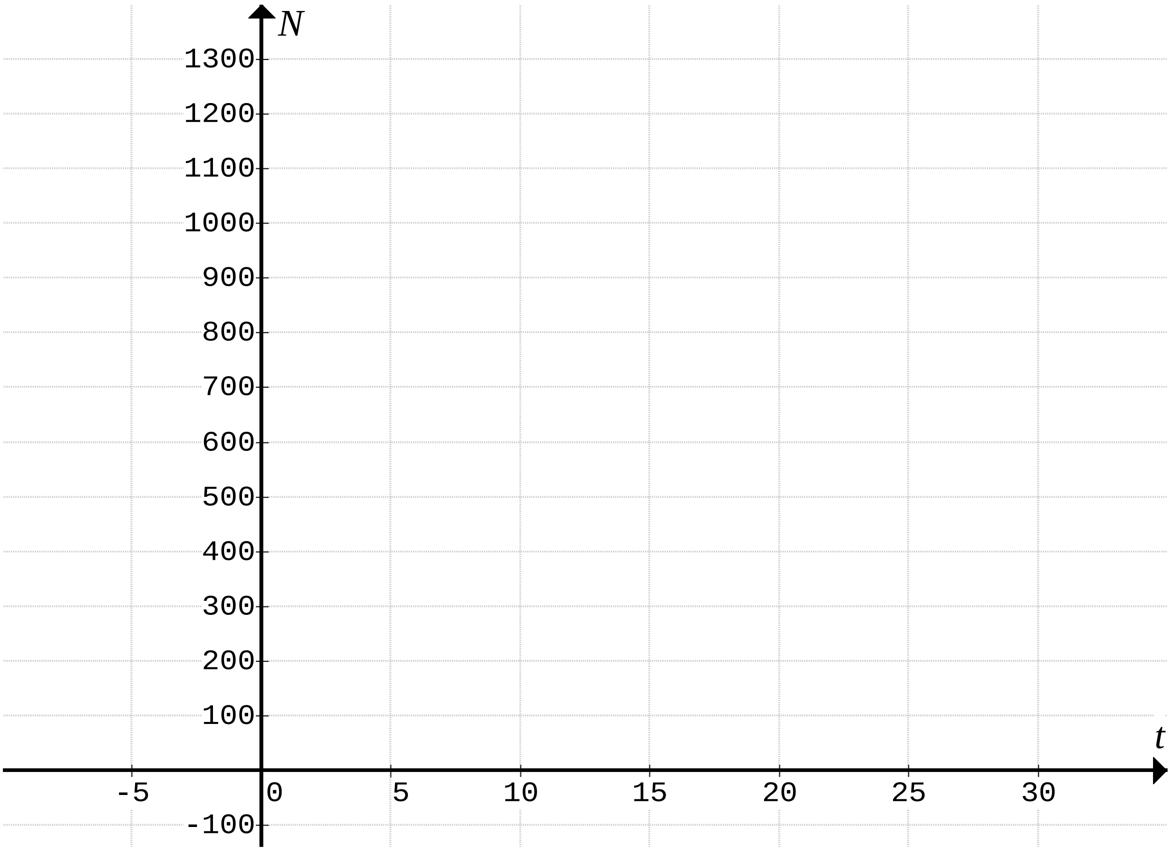
2 marks

f. Find the time correct to one decimal place, when the number of type 1 and type 2 bacteria are equal, and state this actual number of bacteria cells.

1 mark

g. Sketch the graphs of the number of both models of bacteria, type 1 and 2, on the axes below, for the first 30 hours.

2 marks



Question 3 (14 marks)

A curve is defined by the parametric equations

$x = 9\cos(t) \quad y = \dfrac{9\sin^2(t)}{2+\sin(t)}$ for $t \in [0, 2\pi]$.

a. Show that the curve satisfies the implicit equation $y^2\left(81 - x^2\right) = \left(x^2 + 18y - 81\right)^2$

3 marks

b. Find the values of t for which the gradient of the curve is zero, and hence find the coordinates of the turning points on the graph of $y^2\left(81-x^2\right)=\left(x^2+18y-81\right)^2$

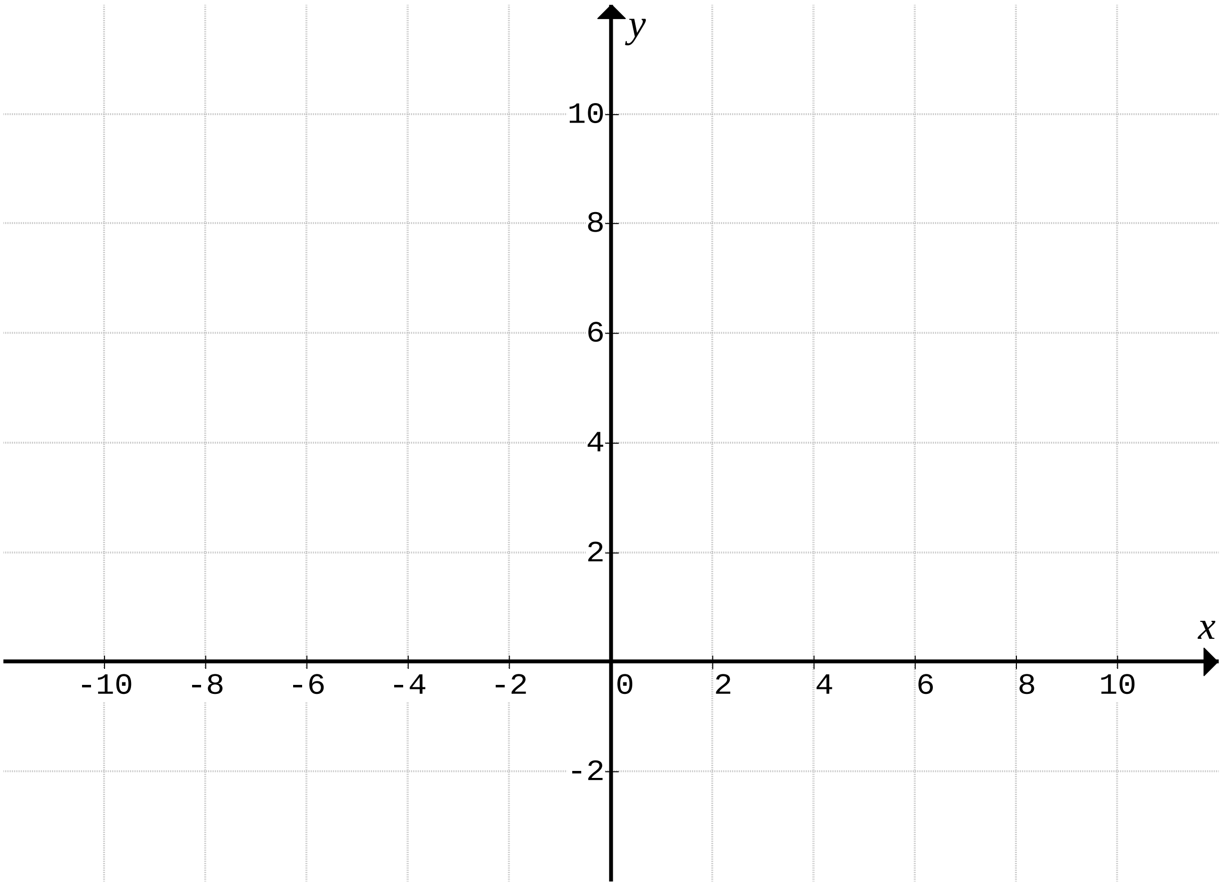
3 marks

c. The implicit relation, $y^2\left(81-x^2\right)=\left(x^2+18y-81\right)^2$ can be represented as two functions, $y=f(x)$ and $y=g(x)$. Given the function

$f:[-9,9]\to R$, $f(x)=\dfrac{81-x^2}{18-\sqrt{81-x^2}}$, find the function g.

2 marks

d. Sketch the graph of the relation $y^2\left(81-x^2\right)=\left(x^2+18y-81\right)^2$ on the axes below, clearly showing all axial intercepts. 2 marks



e. The area A, between the functions $y = f(x)$ and $y = g(x)$, can be expressed as $A = \int_0^9 \frac{4\left(81 - x^2\right)^n}{x^2 + b}\,dx$. Determine the values of b and n.

2 marks

The area A is rotated about the *y*-axis, to form a solid of revolution.

f.i. Write down a definite integral which gives the volume of this solid of revolution.

1 mark

ii. Find the volume of this solid of revolution, giving your answer correct to one decimal place.

1 mark

Question 4 (15 marks)

A football is kicked in a vertical plane and has a position vector given by $\underset{\sim}{r}(t) = x(t)\underset{\sim}{i} + y(t)\underset{\sim}{j}$ for $y \ge 0$ and $0 \le t \le T$. Where t is the time in seconds, $\underset{\sim}{i}$ is a unit vector of one metre horizontally forward and $\underset{\sim}{j}$ is a unit vector of one metre vertically upwards, above ground level. Initially, the football is kicked from a point one metre above the ground, and is kicked into a headwind, so that its velocity vector is given by $\dot{\underset{\sim}{r}}(t) = (12 - 0.1x(t))\underset{\sim}{i} + (12 - gt)\underset{\sim}{j}$.

a. Using integration, show that $x(t) = 120\left(1 - e^{-\frac{t}{10}}\right)$.

3 marks

b. Show that $y(t) = 1 + 12t - 4.9t^2$.

1 mark

c. Find the time T, in seconds, when the football hits the ground.
Give your answer correct to three decimal places.

1 mark

d. Find the range in metres, that is, the horizontal distance that the football travels before hitting the ground. Give your answer correct to three decimal places.

1 mark

e. Find the time in seconds when the football reaches its maximum height. Find the maximum height in metres reached and the horizontal distance travelled in metres at this time. Give all answers correct to three decimal places.

2 marks

f. Determine the speed and angle at which the football hits the ground. Give your answer for the speed in ms^{-1} correct to three decimal places, and the angle in degrees and minutes.

2 marks

g. After a while, when the football is kicked, it bounces and goes over the boundary line. It comes to rest on grassy slope which is inclined at angle of θ degrees to the horizontal. When a spectator pushes the football with force of P newtons, upwards and parallel to the line of greatest slope with the grass, the football is on the point of moving down the grassy slope. When the spectator pushes the football with force of Q newtons, upwards and parallel to the line of greatest slope with the grass, the football is on the point of moving up the grassy slope, where $Q > P$. The football has a mass of m kg and the coefficient of friction between the football and the grass is μ. Express μ in terms of m, P and Q only.

5 marks

END OF EXAMINATION

EXTRA WORKING SPACE

END OF QUESTION AND ANSWER BOOKLET

SPECIALIST MATHEMATICS

Written examination 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.   This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of triangle:	$\frac{1}{2}bc\sin(A)$
sine rule:	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$$\cos^2(x) + \sin^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x) \qquad \cot^2(x) + 1 = \operatorname{cosec}^2(x)$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y) \qquad \sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y) \qquad \cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$

$$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)} \qquad \tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$$

$$\sin(2x) = 2\sin(x)\cos(x) \qquad \tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1,1]$	$[-1,1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0,\pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex Numbers)

$z = x + yi = r\left(\cos(\theta) + i\sin(\theta)\right) = r\,\text{cis}(\theta)$

$|z| = \sqrt{x^2 + y^2} = r$

$-\pi < \text{Arg}(z) \le \pi$

$z_1 z_2 = r_1 r_2\,\text{cis}(\theta_1 + \theta_2)$

$\frac{z_1}{z_2} = \frac{r_1}{r_2}\,\text{cis}(\theta_1 - \theta_2)$

$z^n = r^n\text{cis}(n\theta)$ (de Moivre's theorem)

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

sliding friction: $F \le \mu N$

constant (uniform) acceleration:

$v = u + at$ $\quad s = ut + \frac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \frac{1}{2}(u + v)t$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\tfrac{1}{2}v^2\right)$

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$

$\int x^n dx = \frac{1}{n+1}x^{n+1} + c \ , \ n \neq -1$

$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$

$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$

$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$

$\int \frac{1}{x} dx = \log_e(x) + c, \text{ for } x > 0$

$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$

$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$

$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$

$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$

$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$

$\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$

$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$

$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c, \ a > 0$

$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$

$\int \frac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + c, \ a > 0$

$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$

$\int \frac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Euler's method If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

END OF FORMULA SHEET

ANSWER SHEET

STUDENT NUMBER

Figures										Letter
Words										

SIGNATURE ______________________________

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