# The Mathematical Association of Victoria

# Trial Exam 2014 SPECIALIST MATHEMATICS Written Examination 1

# STUDENT NAME

# Reading time: 15 minutes Writing time: 1 hour

# **QUESTION AND ANSWER BOOK**

#### **Structure of Book**

Number of questions	Number of questions to be answered	Number of marks
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers,
- Students are NOT permitted to bring into the examination room: notes of any kind, a calculator of any type, blank sheets of paper and/or white out liquid/tape.

#### Materials supplied

• Question and answer book of 13 pages with a detachable sheet of miscellaneous formulas in the centrefold.

#### Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your **name** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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#### Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Take the acceleration due to gravity to have magnitude  $g \text{ m/s}^2$ , where g = 9.8

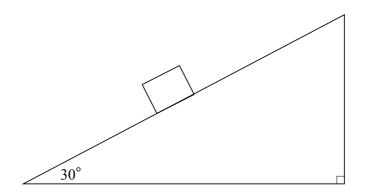
#### Question 1 (5 marks)

A body of mass 6 kg is projected at a speed of 3 m/s up a **rough** plane inclined at  $30^{\circ}$  to the horizontal.

The coefficient of friction between the body and the plane is  $\frac{2\sqrt{3}}{3}$ .

**a.** On the diagram below, show all the forces acting on the body while it is moving up the plane and label them.

1 mark

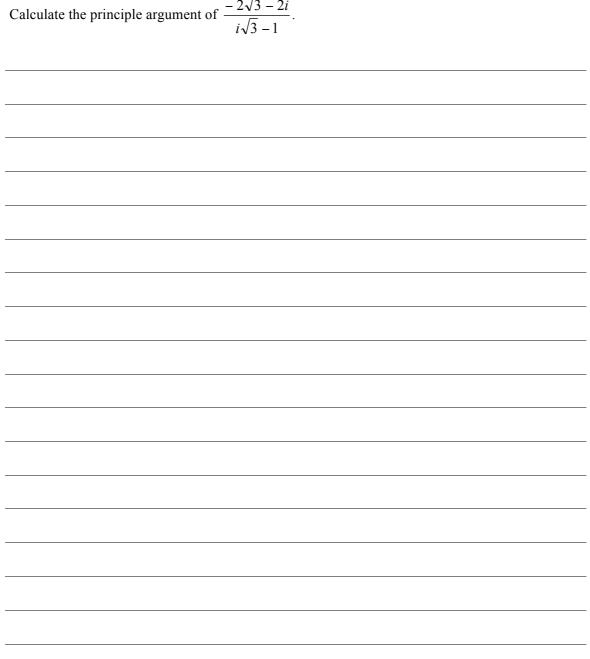


Find in terms of $g$ the total distance travelled <b>up</b> the plane by the body.	4

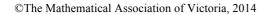
# Question 2 (6 marks)

**a.** Calculate the principle argument of 
$$\frac{-2\sqrt{3}-2i}{i\sqrt{3}-1}$$
. 3 marks

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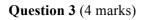


**b.** Let  $p(z) = 4z^3 + az^2 + bz - 34$  where  $a, b \in R$  and  $z \in C$ . If  $p\left(-\frac{3}{2} + i\sqrt{2}\right) = 0$ , find the values of a and b.





3 marks



Evaluate 
$$\int_{\frac{1}{2}}^{1} \frac{x-1}{\sqrt{3-2x}} dx$$
 and express your answer in the form  $\frac{\sqrt{a}-b}{c}$  where *a*, *b* and *c* are positive

integers.



# Question 4 (6 marks)

A particle of mass 3 kg is acted on by a variable force so that its velocity v m/s when the particle is x m from the origin is given  $v = 1 - x^2$ .

a.	Find the force in newtons acting on the particle when $x = \frac{2}{3}$ .	2 marks
b.	Find the time it takes for the particle to travel from the origin to $x = \frac{2}{3}$ .	4 marks

# Question 5 (4 marks)

For the curve defined by the relation

$$3(y+1)^2 - 2xy - x^2 = 7,$$

find the equation of all tangents to the curve that are parallel to the *x*-axis.

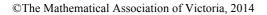
# Question 6 (4 marks)

Consider the vectors

$$a = 2i - j - 2k$$
 and  $b = -2i + tj - k$ 

where  $t \in R$ . Let  $\theta$  be the angle between a and b.

Find the values of t such that  $\sin(\theta) = \frac{4\sqrt{5}}{9}$ .



#### Question 7 (3 marks)

The path of a particle is given by

$$\mathbf{r}(t) = t^2 \cos(t) \mathbf{i} - t \sin(t) \mathbf{j}, \ t \ge 0.$$

If the particle leaves the origin at t = 0, find the speed of the particle at time  $t = \pi$ .

#### Question 8 (3 marks)

Find the equation of the normal to the curve  $y = \cos^{-1}\left(\frac{3}{x}\right)$  at the point where  $x = -2\sqrt{3}$ .


#### Question 9 (5 marks)

Consider the function

$$f: [1,a) \cup (a,b) \rightarrow R, f(x) = 1 - 2\operatorname{cosec}\left(\frac{\pi x}{3}\right).$$

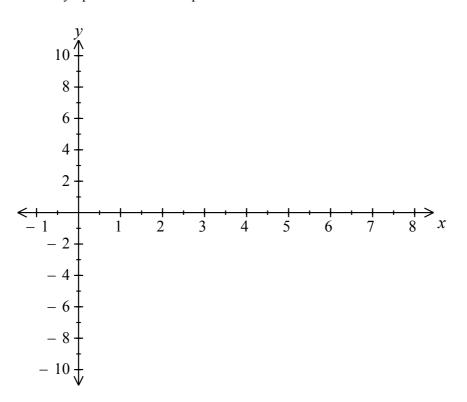
where the values of a and b are the largest for which f is defined.

**a.** State the values of *a* and *b*.

1 mark

**b.** Sketch the graph of y = f(x). Label the turning points and any endpoints with their **coordinates** and the asymptotes with their equations.





#### END OF QUESTION AND ANSWER BOOK